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MODEL ESTIMATION OF TOTAL ELECTRON CONTENT CALCULATION ERRORS BY THE METHOD OF SPACED RECEPTION OF SATELLITE SIGNALS

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Abstract. A technique for model estimation of the accuracy of the total electron content calculation in the ionosphere from the data of spaced receivers of low Earth orbit satellite signals is proposed, and errors in obtaining total electron content using spaced receiver data for quiet ionosphere conditions and in the presence of a plasma ring in the observation region are estimated. To estimate the errors, a dynamic model of the electron concentration of the Earth's ionosphere was used. For the selected input parameters, a two-dimensional model electron concentration distribution was calculated from this model. Using the obtained distribution, model quantities (differential phase and total electron content) were calculated, which were used for a model estimation of total electron content calculation errors by the spaced reception method.

Introduction. Radiophysical methods play an important role in the study of the upper atmosphere properties and solar-terrestrial relations. For the purposes actively used low Earth orbit satellites. Modern low Earth orbit (LEO) satellites (for example, the constellations of SpaceX, Telesat, LeoSat, Kepler satellites) have altitudes up to 2000 km and are designed for various purposes, but some of them can be used to study the ionosphere, since they have transmitters on board that emit coherent frequencies [*Ge et al.*, 2022]. Low Earth orbit satellites are actively used for the purposes of communication, development of the global Internet etc, and their number will increase. At the same time, some of them will carry on board transmitters intended for navigational purposes. The advantages of using such satellites in comparison with satellites of global navigation satellite systems include better coverage of the earth by the constellation of satellites and stronger signals received by navigation receivers.

One of the actively used parameters is the total electron content (TEC). To obtain TEC, methods based on various physical principles and equipment are used. Phase methods are the simplest in terms of hardware and data processing [*Kunitsyn et al.*, 2007]. The resulting phase difference Φ of the emitted satellite coherent frequencies depends on the integral of the electron density along the line of sight, i.e., on the TEC. The general formula for the relationship between the difference phase and TEC can be written as

$$\Phi = C \int_R^S N ds,$$

where C is a constant, $\int_R^S N ds$ is the total electron content along the line of sight from the satellite S to the receiver R. It can be seen that the difference phase directly depends on the total electron content. In the future, by phase we mean the difference phase. Considering the experimental discrete time series of phase changes Φ_i obtained along the satellite flight path, the phase calculation formula can be written as follows:

$$\Phi_i + \Phi_0 = C\,T_i\,, \qquad i = 1, 2, \ldots, k, \tag{1}$$

where Φ_i is the value of the observed difference phase at the time t_i, Φ_0 is the unknown initial phase, C is a constant, T_i is the TEC along the line of sight at the time t_i, k is the number of reference points along the satellite trajectory. The main difficulty in calculating the total electron content along the line of sight of the receiver-satellite is that we do not know the phase Φ_0, which we have during the passage of the signal from the satellite to the receiver at the initial moment of recording, the so-called initial phase constant. Thus, in the experiment we cannot calculate the TEC for the receiving point. Therefore, in fact, only the change in TEC can be calculated from the experimental phase during the flyby of the satellite. In order to obtain a dataset of TEC values between the receiver and the satellite according to formula (1), it is necessary to know the initial phase constant Φ_0. The TEC value along the line of sight is commonly referred to as the slope TEC.

The slope TEC value is calculated by integrating the electron density along the line of sight from the receiver antenna to the satellite. In this form, the TEC is used to solve some problems, but if it is necessary to localize disturbances recorded in the TEC, it can't be done. To eliminate the uncertainty, the TEC is tied to some height on the line of sight. To do this, the thin phase screen approximation is used, according to which changes in the signal occur only on this screen located at a certain height. For most conditions in the ionosphere, the choice of a fixed altitude of 400 km is sufficient. At the intersection of the line of sight of the receiver-satellite with this height, which is called the ionospheric point, a geometric conversion of the slant TEC into the vertical total electron content (VTEC) is performed by simply multiplying the TEC by the cosine of the angle χ at the height between the vertical and the line of sight from the satellite to the receiver. VTEC, therefore, is the calculated total electron content in a vertical column passing through the ionospheric point. In this case, formula (1) will take the form:

$$\Phi_i + \Phi_0 = CD_iI_i \, , \qquad (2)$$

where $D_i = \sec\chi_i$, I_i is the vertical total electron content.

Simultaneous observations of satellite signals during the satellite flybys were carried out at different stations mainly for the purpose of comparing the results of TEC calculations using different methods and correcting the obtained values, since it was assumed that in the ionospheric point, which was seen from both stations, the obtained values of VTEC should be the same. Further development of phase measurements at several receiving points was proposed in [*Leitinger et al., 1975*]; it underlies the satellite signal spaced reception method under consideration. If we use the least squares method for datasets at two stations, then we can determine these unknown initial phases at the stations based on the minimum of the following expression:

$$\sum_{i=1}^{k}(\Phi_{01}/D_{i1} + \Phi_{i1}/D_{i1} - \Phi_{02}/D_{i2} - \Phi_{i2}/D_{i2})^2,$$

where k is the number of common ionospheric points in the experimental datasets. The authors assumed that estimation of the obtained values of the initial phase constants Φ_{01} and Φ_{02} would be significantly improved.

Model estimation. As previous studies have shown, the use of the satellite signal spaced reception method is an informative method for obtaining data on the structure and dynamics of the ionosphere [*Brunelli et al.*, 1992; *Cherniakov and Shulgina*, 1995]. The matter of the accuracy of the data obtained by the method always remains significant. Comparison of the results of the method with the data of independent experimental methods showed its applicability with good accuracy for studying the high-latitude ionosphere under conditions of various disturbances [*Cherniakov et al.*, 1992; 1993]. The formation of electron density structures using an ionospheric model makes possible to check the accuracy of one or another experimental method under the conditions under study. In our case, using the ionospheric model, we can estimate the accuracy of determining the TEC when using the method of spaced reception of signals from low Earth orbit navigation satellites.

The dynamic model of the Earth's ionosphere electron concentration at geomagnetic latitudes higher than 60°, in the altitude range from 100 to 1000 km, was taken as the initial model of the ionosphere [*Models*, 1986]. Its advantage is also that it is completely described in the form of simple equations and does not require much time to calculate the values of the electron concentration. For the given coordinates of a point (latitude, longitude, height), by setting the input parameters describing the time and geomagnetic situation, one can calculate the value of the electron concentration at the chosen point and, consequently, a two-dimensional distribution of concentration in the flyby plane of a low Earth orbit satellite for a given moment of time.

The model estimation of the TEC calculation accuracy using the spaced reception method was based on the data of real satellite flybys and the location of the satellite signal receiving points used in the experiments. For the selected satellite flyby, a model two-dimensional distribution of the electron concentration was calculated, as well as the parameters of the satellite orbit (latitude, longitude, and altitude) at the chosen times of the flyby. Knowing the coordinates of the satellite and the receiving points allows us to calculate the distance to the satellite along the line of sight of the receiving point-satellite, and thus the slant TEC. Knowing the value of the obtained TEC, you can calculate the total phase F along the line of sight of the receiver-satellite using equation (1). By setting the real time period of a satellite flyby and its coordinates, it is possible to obtain for the selected point of satellite signal reception the model values of the total phase F_i along the line of sight of the receiver-satellite for each moment of time of the satellite flyby, i.e., ultimately, a dataset of values of the total model phases F_i. Subtracting from the obtained phase dataset F_i the phase value at the initial moment of calculation F_0, one can obtain datasets of phases Φ_i, which correspond to the phases obtained in the experiment (zero first phase and its subsequent changes) for each receiving point. The value of the initial model total phase F_0 is an unknown initial phase, the determination of which is the main difficulty in the experimental determination of TEC. Using the datasets of model phases Φ_{i1} and Φ_{i2} obtained for each receiving point, it is possible to calculate the initial phases for each receiving point (Φ_{01}, Φ_{02}) using the spaced reception method. Comparison of the initial phases obtained as a result of the calculation by the spaced reception method (Φ_{01}, Φ_{02}) with the model values of the total phases at the initial moment (F_{01}, F_{02}) will make it possible to estimate the errors of the method for each receiving point.

Let us estimate the accuracy of determining the TEC using the example of two model spatial distributions of the electron concentration similar to those shown in Fig. 1a and Fig. 2a. The first distribution (Fig. 1a) corresponds to a quiet condition in the ionosphere in the daytime, the second can be observed during a substorm when a plasma ring appears in the observation region (Fig. 2a) [*Cherniakov and Shulgina, 1995*]. Integrating the electron concentration over height at the latitudes used for calculation of model distribution gives the model VTECm for each of the latitudes. This makes a possibility to obtain model VTECm profiles along latitudes (Fig. 1b,c, 2b,c, black lines). This VTECm profile does not depend on the choice of the observation point, and is calculated directly from the two-dimensional electron concentration distribution.

To calculate the total phases F and VTEC, the times and trajectory of the satellite were chosen, corresponding to the actual satellite flyby in the observation region. The positions of the receiving points in the model calculations are close to the positions of the real receiving points, in which the observations of signals from low Earth orbit navigation satellites were performed in the experiments.

For each observation point and for each distribution of the electron concentration for the flyby of the satellite, we calculated our own datasets of total phases F and, consequently, our own slant TEC. In the figures below, phases F_{i1} and VTEC1m calculated for point R1 are marked in blue, and phases F_{i2} and VTEC2m (in red) - for point R2. Examples of the obtained phases F_i are shown in the upper figures Fig. 1b and Fig. 2b. Using formula (2), the slant TEC, obtained from these model total phases F, was recalculated into the VTEC. The altitude of 400 km was taken to calculate the VTEC. The bottom figures Fig. 1b and Fig. 2b show the model VTEC1m and VTEC2m obtained for different observation points R1 and R2 for the selected electron concentration distributions. It can be seen that the latitudinal profiles of the model VTEC1m and VTEC2m obtained at different reception points depend on the observation point. In addition, they differ from the model VTECm, which does not depend on a particular observation point (black line). The slant TEC calculation takes into account regions of the ionosphere, that are outside the vertical ones at the recalculation point, which introduces errors in the resulting VTEC relative to the model VTEC.

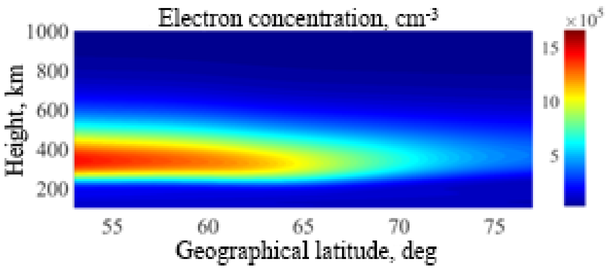

Figure 1a. Model distribution of electron concentration (quite conditions).

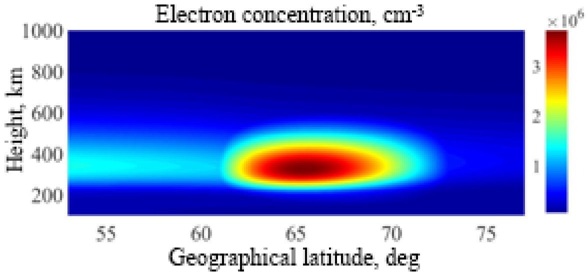

Figure 2a. Model distribution of electron concentration (plasma ring).

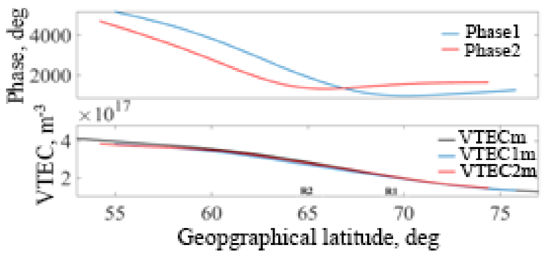

Figure 1b. View of the calculated model phases (top) and VTEC (bottom) from observation points R1 and R2, the black line in the lower figure is the model VTEC along latitudes (quite conditions).

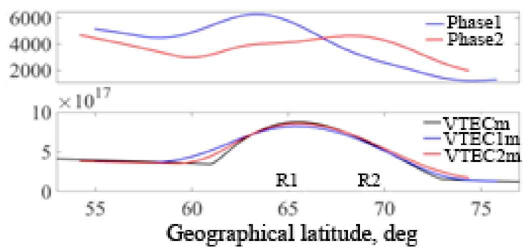

Figure 2b. View of the calculated model phases (top) and VTEC (bottom) from observation points R1 and R2, the black line in the lower figure is the model VTEC along latitudes (plasma ring).

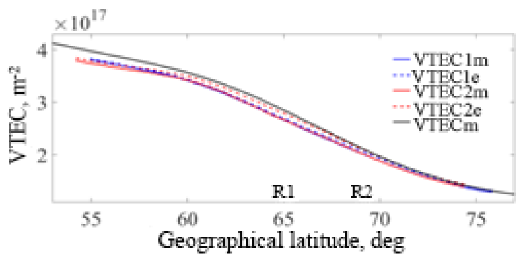

Figure 1c. View of the calculated model VTEC (black line) and the calculated VTEC from observation points R1 and R2 (quite conditions).

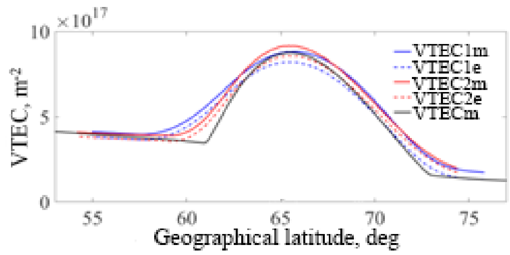

Figure 2c. View of the calculated model VTEC (black line) and the calculated VTEC from observation points R1 and R2 (plasma ring).

We also note that the choice of the recalculation height affects the value of the resulting VTEC. Thus, comparison of the model VTECm with the model VTEC1m and VTEC2m obtained at the observation point can be used to estimate the errors associated with the position of the receiving station relative to the considered structures in the ionosphere and the choice of the height of recalculation of the slant TEC to the VTEC. In the case of a quiet conditions in the ionosphere (Fig. 1b), the model VTECm and the model VTEC1m and VTEC2m determined at the observation points are close to each other. So, in quite geophysical conditions, the behavior of VTEC is restored quite well at the receiving points. In the presence of isolated irregularities in the ionosphere (Fig. 2b), the latitudinal profiles of the model VTEC1m and VTEC2m obtained at different receiving points have differences from the model latitudinal profile of the VTECm. The greatest discrepancy is observed in places with a sharp change of the electron concentration.

To obtain a dataset of model phases Φ_i, corresponding to the phases obtained during experimental observations, at each receiving point the value of the first phase F_0 was subtracted from the dataset of model total phases F_i calculated for this receiving point. Ideally, the initial phase constants Φ_{01} and Φ_{02}, calculated from the model datasets of phases Φ_{i1} and Φ_{i2} using the spaced reception method, should be equal, respectively, to the first phases F_{01} and F_{02} in the datasets of the model total phases F, from which the Φ_{i1} and Φ_{i2} datasets were obtained. The difference in the values of Φ_0 and F_0 will indicate the error in determining the initial phase constant when calculating by the spaced receiving method, and, therefore, about the error in determining TEC.

The initial phase constants (initial phases) Φ_{01} and Φ_{02} from the phases Φ_{i1} and Φ_{i2} were calculated by the spaced reception method for the cases of a quiet condition and a plasma ring. VTEC, determined at the receiving points using Φ_{01} and Φ_{02}, we will be called as “experimental” results, since these calculations are similar to the calculations with datasets of experimental phases. Using the obtained values of the calculated initial phases Φ_{01} and Φ_{02} and the datasets of phases Φ_{i1} and Φ_{i2}, according to formula (2) it is possible to obtain an "experimental" VTEC1e and VTEC2e for the receiving points, which includes the error in determining the initial phase constants when calculating by the spaced reception method. This makes it possible to compare the behavior of the VTEC1m and VTEC2m obtained from the model at the receiving point with the "experimental" VTEC1e and VTEC2e obtained from the spaced reception method, i.e. evaluate the influence of the error in determining the initial phase constant on the error in calculating the VTEC (Fig. 1c, 2c).

Similar calculations for the phases at the receiving point R1 showed the following: a) for quiet conditions, the difference between the initial phase constants Φ_0 and F_0 is 33.3°, b) for the case of a plasma ring, the difference between Φ_0 and F_0 is 186.2°. Thus, we can say that for the considered cases, the spaced reception method gives a satisfactory estimate of the initial phase constant.

By adapting the dynamic model of the ionosphere, it is possible to obtain a two-dimensional electron concentration distribution close to that observed in the experiment, and, after carrying out model calculations, to obtain estimated values of the errors in determining the initial phase constant for the actual configuration of receiving points and conditions in the ionosphere. The advantage of this approach to estimating the accuracy of the resulting initial phase constant is that by changing the location of the receiving points relative to each other, it is possible to estimate the effect of the distance between the receiving points on the accuracy of determining the experimental VTEC, as well as the spatial resolution of the electron concentration structures. This will allow, when preparing the experiment, choosing the distances between the receiving points that correspond to the proposed task.

Conclusion. A technique is proposed that makes possible to estimate the error in obtaining the initial phase constant by the spaced reception method of signals from low Earth orbit satellites. The values of the errors in determining the initial phase constant and VTEC are estimated for the cases of a quiet ionosphere and a plasma ring. For quiet conditions in the ionosphere, the relative error in determining the initial phase constant did not exceed 3 %, and in the case of a plasma ring, 13 %; for VTEC, the relative error in quiet conditions did not exceed 7 %, and in the case of a plasma ring, it did not exceed 60 %. Large values of the relative error were in the places of sharp changes of model VTECm. Nevertheless, the latitudinal profiles of the VTECe obtained by the spaced reception method described the model situation quite adequately. The proposed method makes possible to estimate the probable error in choosing the position of the receiving points relative to each other, as well as the errors in choosing the height of recalculation of the slant TEC to the VTEC when considering the situation of interest in the ionosphere.

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