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SOME CONVERGENCE ASPECTS OF A VARIATIONAL APPROACH IN THE IONOSPHERIC RAY TRACING PROBLEM

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Abstract. Optimization methods for point-to-point ionospheric ray tracing is discussed. Early it was shown that variational approach can be applied for point-to-point ray tracing using ionospheric models. One of the problems is automatic points control along the ray path to provide accurate convergence. This problem can be overcomes taking into account artificial spring forces acting between points along the ray path. The comparative analysis of the convergence and accuracy of direct ray tracing obtained by using variational principle are present in our investigation.

Introduction

There are two main approaches of the two point ionospheric ray tracing. The most popular approach is shooting method (homing-in) [1-3]. This approach consist of two parts: the first one is finding of direction of emission radio waves and the second is construction trajectory of radio path. But sometimes shooting method has serious disadvantages [4] which are associated high computational effort and the absence of methods for finding exact direction of radio waves emission. In the other hand, alternative approach is based on direct utilization on Fermat's principle [5]. The main idea is transforming initial trajectory to an optimal configuration, while its end-points are fixed in accordance to the boundary condition. According to the task of ionospheric ray tracing, early a variant of the direct approach has been developed. The statement of the ray tracing problem using direct variation method is presented by [6]. The calculations results discussed in [6] for different model media showed that the standard optimization method based on variational principle can lead to unsatisfactory results. In most cases it can be overcomes using the projection force which solve the problem of points down-sliding near the maximum of medium parameter. In the improved variational method, a chain of points which gives a discrete representation of the ray is brought to an optimal configuration by iterative, transverse displacements, while the endpoints of the path are kept fixed according with boundary condition. The main challenging problem of the method application in modeled ionosphere is automatic points control along the ray path to provide true convergence. This problem overcomes by adding spring forces between points along the ray path that was described by [7]. The goal of this paper is to demonstrate the main advantages of spring forces. The comparative analysis of the convergence and accuracy of two point ray tracing by direct variational principle are also present in current work.

Method

In terms of variational theory the functional of the ray path in isotropic medium is defined by

$$S[\gamma] = \int_A^B n(\vec{r})\, dl. \tag{1}$$

Integration is performed along the curve γ, which joins boundary points A и B; $n(\vec{r})$ — refractive indexes in each point of curve γ with $\vec{r} = (x, y, z)$; dl — the length of the element along γ. According with Fermat's principle the functional of the ray path must satisfies to the expression

$$\delta S = 0. \tag{2}$$

Simplify curve γ can be represented as a polygonal line joining N points in chosen space. Consequently we have a discrete representation of curve $\gamma = [\vec{r}_1, \vec{r}_2, \dots, \vec{r}_N]$ while the end points A and B are keep fixed according with boundary condition. Thus the problem adduct to the problem of finding minimum of the functional. For compute calculation the integral (1) can be rewritten using trapezoidal or Simpson rule. The most of computational methods based on antigradient of objective function:

$$\boldsymbol{F} = -\nabla S = \left(\vec{F}^2, \vec{F}^3, \dots, \vec{F}^{N-1}\right). \tag{3}$$

However, the optimization methods can be improved by two procedures: force projection and elastic forces. The first one is described in detail in [7] and solve the problem of points down-sliding near the maximum of medium parameter. This approach is the base of variational methods performed for ionospheric ray tracing described in [6]. The second one is implementing of points interaction which extremely important for control of points distribution along the ray-path and described in [7]. These two approaches are the base of NEB method. According with NEB method the full force acting on each point i of the curve can be defined as

$$\vec{F}^i = \vec{F}_\perp^i + \vec{F}_{spring}^i. \tag{4}$$

Thus, the force on each point contains only the parallel component of the spring force, and perpendicular component of the true force.

Results

As discussed above application of variational method may lead to incorrect results, since in this case the results of the calculations depend on the initial sampling of ray-path. The example of such situation is shown on Fig. 1a. In this example the functional of refractive index has a simple parabolic form. Inhomogeneous distribution of points in initial sampling of ray-path leads to result which is not in an agreement with known analytical solution. To solve this problem, we include the elastic forces which acting between points along the path and consequently do not influence on optimization process. Fig 1b shows that the calculation results using the method converge to an analytical solution.

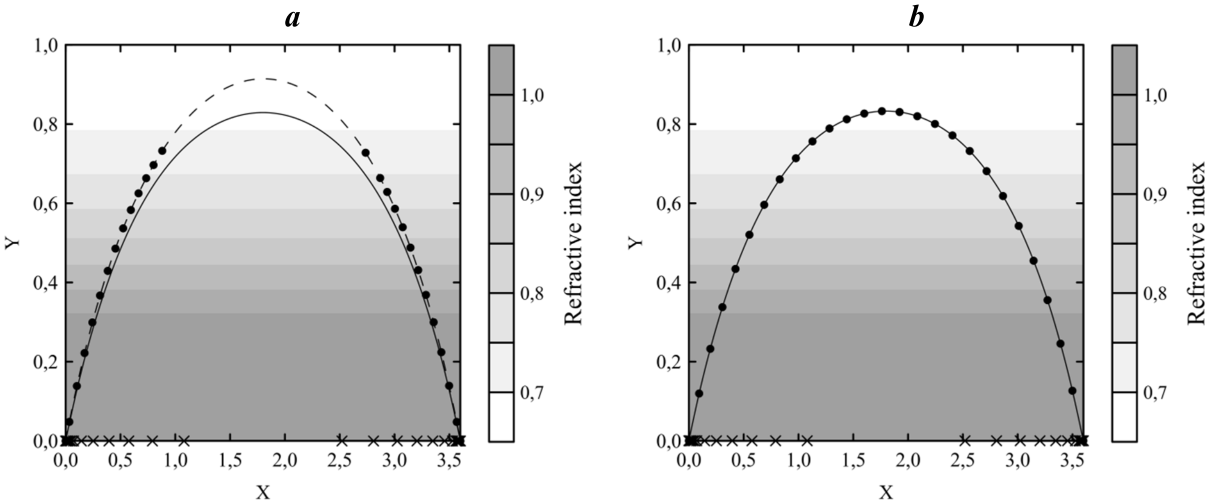

Figure 1. Simulation of the shortest ray-path for the parabolic layer of refractive index using only projected forces (***a***) and taking into account elastic forces (***b***) (discretization is equal to 27 point). The solid line shows the analytical solution, crosses - the initial location of the discretization points of ray-path; solid circles connected by dashed line - calculation results.

The convergence of optimization method with trapezoidal and Simpson rules is also investigated. It is shown on Fig. 2 (a, b) that for accurate ray tracing the Simpson rule is preferred. Calculation results for medium with parabolic layer shows that the optimization with high accuracy can receive using more than 10 points. Nevertheless the trapezoidal rule can be useful when the computational time is more important for the multiple calculations (fig. 2b).

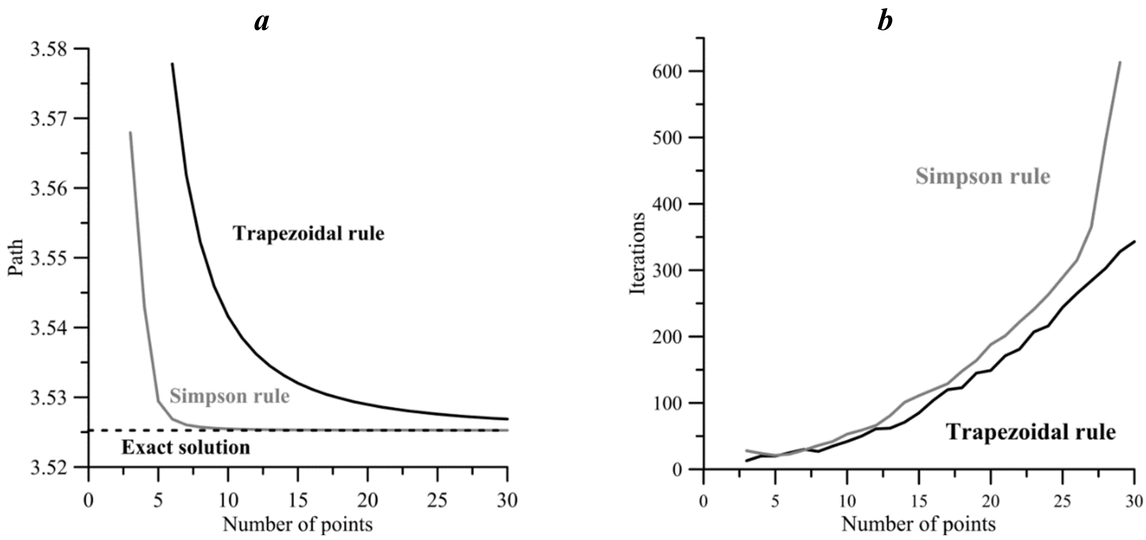

Figure 2. Convergence of variational method in parabolic layer for path length (***a***) and iteration numbers (***b***). Gray solid line correspond to the Simpson rule of path approximation; black line – trapezoidal rule.

Conclusions

This work is devoted to solving some practical aspects associated with the application of the optimization method in modeled mediums. It was shown that variational method can effectively search solutions of the boundary problem of propagation HF radio waves if control of point's distribution along the ray path is present. Convergence of the optimization method was also discussed for the two path approximations: trapezoidal and Simpson's rules. Application of Simpson's approximation due to high accuracy and efficiency is more preferred.

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