"Physics of Auroral Phenomena", Proc. XL Annual Seminar, Apatity, pp. 58-61, 2017

Polar Geophysical Institute

THEORY OF A RECEIVING ANTENNA APPLIED TO THE SPACECRAFT OBSERVATIONS OF QUASI-ELECTROSTATIC WHISTLER MODE WAVES

E.A. Shirokov[1], A.G. Demekhov[1,2], Yu.V. Chugunov[1,3,4], A.V. Larchenko[2]

[1] *Institute of Applied Physics RAS, Nizhny Novgorod, Russia*
[2] *Polar Geophysical Institute, Apatity, Russia*
[3] *Lobachevsky State University of Nizhny Novgorod, Nizhny Novgorod, Russia*
[4] *Deceased 24 August 2016*
e-mail: evshirok@gmail.com

Abstract. In this paper, we develop the known theory of a quasi-static receiving antenna in a magnetoplasma and apply it to the calculation of the antenna effective length in case of reception of quasi-electrostatic whistler mode waves in plasmas. Also we analyze several typical chorus events detected by THEMIS spacecraft and show that the effective length of receiving antenna can be more than an order of magnitude greater than the geometric length.

Introduction

As it is known, the effective length of a receiving antenna in a magnetoplasma can significantly differ from its geometric length especially for the quasi-electrostatic whistler mode waves (see [*Chugunov and Shirokov*, 2016] and the references therein). Such waves propagate near the resonance cone. A wave packet composed of them has a resonance structure, i.e., it is a superposition of plane monochromatic waves having a continuous and fairly wide spectrum of wave numbers which are large compared to the inverse wave length $\lambda_{\mathrm{em}}^{-1}$ of a parallel propagating electromagnetic whistler mode wave.

For the resonance waves, the problem of calculating the complex amplitude E of the electric field from the voltage complex amplitude U induced on the receiving antenna terminals is nontrivial. We introduce the effective (or electrical) length l_{eff} of a receiving antenna according to the formula

$$U = El_{\mathrm{eff}} \cos\Upsilon, \tag{1}$$

where Υ is the angle between the antenna axis and vector $\vec{E}$ of the detected wave. Quantity l_{eff} is determined by the reradiation efficiency of the antenna [*Balanis*, 2016]. Importantly, it is not constant and can depend rather strongly on the mutual orientation of the electric field vector and the antenna.

Some general relationships of the receiving antenna theory for magnetized plasmas were developed in the earlier works (see [*Chugunov and Shirokov*, 2016] and the references therein). In this paper, we apply this theory for emissions of natural origin, namely the very low frequency chorus in the Earth's magnetosphere. Typically, chorus emissions propagate quasi-parallel to the ambient magnetic field in their source region [*Santolík et al.*, 2014]. However, recent analysis of satellite data shows that chorus can also propagate in the quasi-electrostatic mode with wave normal angles θ close to the resonance cone ($\theta = \theta_{\mathrm{res}}$) [*Agapitov et al.*, 2014].

Expression for the Effective Length

The effective length calculation is based on the reciprocity theorem [*Chugunov and Shirokov*, 2016]

$$\int_{\mathrm{pl}} \rho(\vec{r},t)\Phi_0(\vec{r},t)\mathrm{d}\vec{r} = \int_{\mathrm{ant}} \rho_0(\vec{r},t)\Phi(\vec{r},t)\mathrm{d}\vec{r}, \tag{2}$$

where the integrals are over the plasma ("pl") and antenna ("ant") volumes, $\Phi(\vec{r},t)$ is the scalar potential of the incident wave, $\rho(\vec{r},t)$ is the charge fluctuation in the plasma which induces voltage on the antenna terminals, and $\Phi_0(\vec{r},t)$ is the potential of a field due to the charge distribution $\rho_0(\vec{r},t)$ on the antenna. Applying spectral approach, one finally comes to the resulting expression for the effective length of the antenna receiving a quasi-monochromatic wave with a carrier frequency $\omega=\omega_0$ [*Shirokov et al.*, 2017]:

$$l_{\mathrm{eff}}(\omega) = \frac{64\lambda_{\mathrm{em}}^2}{\sqrt{\varepsilon(\varepsilon+|\eta|)}\, l_{\mathrm{tr}}^2 \sin\theta_{\mathrm{res}}\, |\cos\Upsilon|} \cdot \frac{\left| \int_0^{+\infty}\int_0^{2\pi} k^{-1/2}\rho_{\mathrm{tr}}(k)\rho_{0\vec{k}}(k,\psi)e^{iq(\omega-\omega_0)\tau_0 k}\,\mathrm{d}k\,\mathrm{d}\psi \right|}{\left| \int_0^{+\infty} k^{1/2}\rho_{\mathrm{tr}}(k)e^{iq(\omega-\omega_0)\tau_0 k}\,\mathrm{d}k \right|}. \tag{3}$$

Here $q = (\partial\mu/\partial\omega)|_{\omega=\omega_0}\left(1+[\mu(\omega_0)]^2\right)^{-1}$, $\mu = \cot\theta_{\rm res}$, $\varepsilon = \varepsilon_{xx} = \varepsilon_{yy}$ and $\eta = \varepsilon_{zz}$ are the transverse and longitudinal components of relative permittivity tensor calculated for the carrier frequency, respectively (z-axis is parallel to the local geomagnetic field), k is the wavenumber, ψ is the azimuth angle in $\vec{k}$ -space, $\rho_{0\vec{k}}(k)$ is the spectrum (calculated on the resonance cone) of the trial charge distribution on the receiving antenna; $l_{\rm tr}$, τ_0, and $\rho_{\rm tr}(k)$ are the length of a fictituous electric dipole source, its distance to the receiver along the group velocity resonance direction, and the spectrum (calculated on the resonance cone) of its charge distribution $\rho_{\rm tr}(\vec{r})$. The reason why we use this source model is that we need to specify the incident wave field. Since chorus generation is a complicated nonlinear process, it is quite difficult to write down an expression for its electric field. However, it is possible just to introduce a fictituous (or effective) electric dipole producing the detected field characteristics, and that is what has been done. Two main properties of wave packets we want to model are (i) the field should have wave normal angles close to the resonance cone, and (ii) the wave number spectrum is assumed fairly broad which seems to be a natural property of resonant whistler-mode emissions. We make this simplifying assumption since we do not know actual wave number spectrum of quasi-electrostatic chorus waves. These two assumptions allowed us to specify a simple electric dipole model of effective source of the measured radiation field. Such a source does not need to coincide with an actual chorus source.

According to the above discussion, we choose $\rho_{\rm tr}(\vec{r})$ in the form that corresponds to a thin dipole of length $l_{\rm tr}$, directed along the z-axis:

$$\rho_{\rm tr}(\vec{r}) = -\frac{8Q_{\rm tr}}{l_{\rm tr}^2} z\exp\left(-\frac{4z^2}{l_{\rm tr}^2}\right)\delta(x)\delta(y), \tag{4}$$

where $Q_{\rm tr}$ is the total half-dipole charge on the effective transmitter. Then

$$\rho_{\rm tr}(k) = \frac{i}{2}Q_{\rm tr}\sqrt{\pi}kl_{\rm tr}\cos\theta_{\rm res}\exp\left(-\frac{k^2l_{\rm tr}^2\cos^2\theta_{\rm res}}{16}\right). \tag{5}$$

The described shape of effective source is chosen for the sake of symmetry and simplicity. The charge distribution along z is smooth which means that the source region has no sharp boundaries. We limit ourselves by the dipole approximation and do not consider any multipoles of higher orders, because the dipole charge distribution relatively easily provides the wave field with measured parameters if this field corresponds to a quasi-electrostatic wave packet with a spread in wave vectors. Indeed, length $l_{\rm tr}$ of this effective transmitter is determined by the wavenumber $k_{\rm obs}$ that corresponds to the observed spectral maximum:

$$k_{\rm obs}l_{\rm tr}\cos\theta_{\rm res} = 2\sqrt{2}. \tag{6}$$

Let us now deal with the trial charge spectrum $\rho_{0\vec{k}}$. If the receiving dipole consists of 2 thin straight rods with a gap between them, then the current distribution along it can be chosen as a triangular one [*Chugunov et al.*, 2015], and

$$\rho_{0\vec{k}}(k,\psi) = -\frac{8i}{\gamma(\psi)kl_{\rm rec}}\exp[-ikR_0(\psi)]\sin^2\left[\frac{\gamma(\psi)kl_{\rm rec}}{4}\right]. \tag{7}$$

Here $\gamma(\psi) = \sin\alpha\sin\theta_{\rm res}\cos(\psi-\beta)+\cos\alpha\cos\theta_{\rm res}$, $R_0(\psi) = x_0\sin\theta_{\rm res}\cos\psi + y_0\sin\theta_{\rm res}\sin\psi + z_0\cos\theta_{\rm res}$; x_0, y_0, and z_0 are the receiver Cartesian coordinates (see Fig. 1):

$$x_0 = \tau_0\cos\theta_{\rm res}\cos\varphi_{\rm obs},\quad y_0 = \tau_0\cos\theta_{\rm res}\sin\varphi_{\rm obs},\quad z_0 = \tau_0\sin\theta_{\rm res}; \tag{8}$$

$\varphi_{\rm obs}$ is the azimuth angle of the incident wave, α and β are the receiver orientation angles: α is the angle between the geomagnetic field and the dipole axis, and β is the azimuth angle of the dipole. If the receiver consists of 2 small, as compared to the distance between them, spherical conductors placed on the thin metal rod, then it may be represented as 2 point charges:

$$\rho_{0\vec{k}}(k,\psi) = -2i\exp[-ikR_0(\psi)]\sin\left[\frac{\gamma(\psi)kl_{\rm rec}}{2}\right]. \tag{9}$$

In the following, we will consider quasi-monochromatic (at each moment of time) wave packets, and therefore will choose $\omega=\omega_0$ which simply means appropriate choice of ω_0 for each spectral component. Importantly, this does not prevent k to vary in a wide range due to the resonance wave dispersion.

Consequently, the only effective source parameters that determine the receiver effective length are $l_{\rm tr}$ and τ_0. Length $l_{\rm tr}$ is determined by the wave and plasma parameters ($k_{\rm obs}$ and $\theta_{\rm res}$) according to (6), and τ_0, generally speaking, is a free parameter. Let us discuss its choice. As it was shown in the previous studies using ray tracing [*Chum and Santolik*, 2005], the chorus wave normal angle changes significantly due to refraction on the distance corresponding to the geomagnetic latitude $\lambda_{\rm m}$ change of 1°. Therefore, in order to neglect the refraction effects on the entire emitter—receiver line, we will choose τ_0 in the interval $\tau_0 \le \tau_{0\max} = \Lambda$, where Λ corresponds to the geomagnetic latitude change of 0.1° along the geomagnetic field line at given $\lambda_{\rm m}$ and McIlwain parameter L. The minimum estimate is obviously determined by the source half-size: $\tau_{0\min} = 0.5 l_{\rm tr}$.

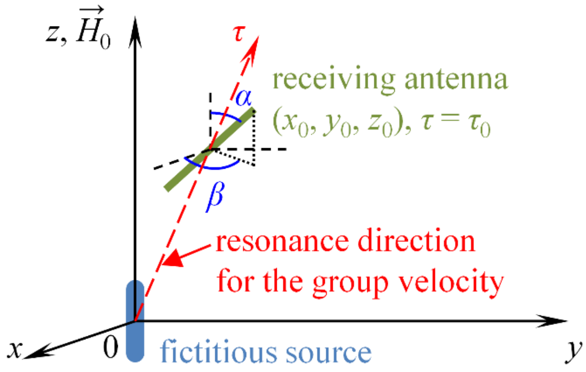

Figure 1. Geometry of the problem.

Calculation Results

In this section, we apply the general formulas obtained above to some measurements of chorus wave electric fields onboard THEMIS spacecraft [*Burch and Angelopoulos*, 2009]. Each THEMIS spacecraft is equipped with the Electric Field Instrument (EFI) that consists of the three dipole orthogonal antennas. These antennas have a half-length of 24.8 m, 20.2 m, and 3.47 m. We will refer to them as dipoles A, B, and C, respectively. Dipoles A and B consist of 2 small spheres (see (9)), and dipole C consists of 2 straight thin rods (see (7)). Since the dipoles are orthogonal to each other, they allow one to measure three components of the incident wave electric field and can be treated independently from each other. The electric field value should be calculated from these three components, and each of them should be obtained according to (1). The antennas orientation angles α and β have been found from the Flux Gate Magnetometer (FGM) data that provide the results of geomagnetic field measurements.

For our analysis we chose some of the events which were previously considered by [*Agapitov et al.*, 2014] in relation to electron energization. The two chorus events that we have analyzed in depth are described in Table 1. Angle $\theta_{\rm obs}$ that corresponds to the observed spectrum maximum was found using a singular value decomposition method [*Santolik et al.*, 2003] from the Search Coil Magnetometer (SCM) data only.

Table 1. The analyzed chorus events detected by THEMIS

Event	1	2
THEMIS	C	A
Date (yyyy-mm-dd)	2007-08-28	2008-11-26
UT (hh:mm:ss)	15:51:48	03:18:23
$\lambda_{\rm m}$ (deg)	15	0
L	5.4	5.0
ω_0 (s^{-1})	9425	15708
$\theta_{\rm res}$ (deg)	78.0	64.7
$\theta_{\rm obs}$ (deg)	75.0	58.0
$l_{\rm tr}$ (km)	13	7.6
α (deg) [a]	78.6, 110, 23.7	62.1, 116, 40.1
β (deg) [a]	62.5, 148, 180	51.0, 126, 180
$l_{\rm eff} / l_{\rm rec}$ [a] ($\tau_0 = \tau_{0\max}$)	2.7, 2.7, 0.4	13, 12, 0.8
$l_{\rm eff} / l_{\rm rec}$ [a] ($\tau_0 = \tau_{0\min}$)	9.0, 9.0, 1.9	50, 49, 5.8

[a] The three values correspond to dipoles A, B, and C, respectively.

The results of calculations, performed for these events, are presented in the last two lines of Table 1. One can see that in case $\tau_0 = \tau_{0\min}$ the receiver effective length is several times larger as compared to case $\tau_0 = \tau_{0\max}$. However, in both cases $l_{\mathrm{eff}} / l_{\mathrm{rec}}$ can significantly exceed unity.

Conclusions

In this paper, we proposed a method for calculating the effective length of an electrical receiving antenna in case of the quasi-electrostatic chorus waves. We applied the obtained general relations to several cases of chorus electric field measurements onboard THEMIS spacecraft, and found that the antenna effective length could be up to an order (or more) of magnitude greater than the geometric length l_{rec}. Therefore, the actual value of the electric field component that is parallel to the antenna can be less and even much less than quantity U/l_{rec}, which is conventionally used as the measured electric field, and it is important to take properly into account the resonance nature of quasi-electrostatic whistler mode waves when interpreting the results of chorus electric field measurements.

Acknowledgements. This work was supported by the Russian Science Foundation under grant 15-12-20005. We acknowledge NASA contract NAS5-02099 and V. Angelopoulos for use of data from the THEMIS Mission, specifically: J. W. Bonnell and F. S. Mozer for use of EFI data; A. Roux and O. LeContel for use of SCM data; K. H. Glassmeier, U. Auster and W. Baumjohann for the use of FGM data provided under the lead of the Technical University of Braunschweig and with financial support through the German Ministry for Economy and Technology and the German Center for Aviation and Space (DLR) under contract 50 OC 0302.

References

Agapitov, O.V., A.V. Artemyev, D. Mourenas, V. Krasnoselskikh, J. Bonnell, O.L. Contel, C.M. Cully, and V. Angelopoulos (2014), The quasi-electrostatic mode of chorus waves and electron nonlinear acceleration, *J. Geophys. Res. Space Physics*, 119, 1606–1626.

Balanis, C.A. (2016), *Antenna Theory: Analysis and Design*, 4th ed., Wiley, Hoboken, N. J.

Burch, J.L., and V. Angelopoulos (2009), *The THEMIS Mission*, Springer-Verlag, New York.

Chugunov, Y.V., and E.A. Shirokov (2016), Quasistatic dipole in magnetized plasma in resonance frequency band. Response of the receiving antenna, and charge distribution on the antenna wire, *Cosmic Res.*, 54(3), 198–204.

Chugunov, Y.V., E.A. Shirokov, and I.A. Fomina (2015), On the theory of a short cylindrical antenna in anisotropic media, *Radiophys. Quantum Electron.*, 58(5), 318–326.

Chum, J., and O. Santolík (2005), Propagation of whistler-mode chorus to low altitudes: Divergent ray trajectories and ground accessibility, *Ann. Geophys.*, 23(12), 3727–3738.

Santolík, O., M. Parrot, and F. Lefeuvre (2003), Singular value decomposition methods for wave propagation analysis, *Radio Sci.*, 38(1), 1010.

Santolík, O., E. Macúšová, I. Kolmašová, N. Cornilleau-Wehrlin, and Y. de Conchy (2014), Propagation of lower-band whistler-mode waves in the outer Van Allen belt: Systematic analysis of 11 years of multi-component data from the Cluster spacecraft, *Geophys. Res. Lett.*, 41, 2729–2737.

Shirokov, E.A., A.G. Demekhov, Y.V. Chugunov, and A.V. Larchenko (2017), Effective length of a receiving antenna in case of quasi-electrostatic whistler mode waves: Application to spacecraft observations of chorus emissions, *Radio Sci*., 52, 884–895.