

## APPLICATION OF MATHEMATICAL MORPHOLOGY TO DATA OF GROUND-BASED AURORAL OBSERVATIONS

Boris V. Kozelov (*Polar Geophysical Institute, Apatity, Murmansk region, 184209 Russia*)

**Abstract.** Methods of mathematical morphology (MM) have been actively developed due to the practical need to solve problems of automatic recognition of objects on TV images, in cartography and computer vision. Ground-based optical auroral observations have a number of features (strong aspect-angle distortion, low contrast, lack of a clear boundary, etc.) that must be considered when spatial and temporal structure of the auroral glow are analyzing by the MM methods. The report discusses the basic MM definitions and application of the MM methods to obtain the spatio-temporal characteristics of pulsating auroral arcs.

### 1. Introduction

Mathematical morphology is a theory which gives useful tools aimed to analysis of plane and spatial shapes. It widely applied to analysis of different data sets [1-3]. Mathematical morphology stands somewhat apart from traditional linear image processing, since the basic operations of morphology are non-linear in nature, and thus make use of a totally different type of algebra than the linear algebra. So, it can be used in complement to more traditional methods.

The basic idea of the MM in simplest binary case is to probe an image in each pixel with a simple, pre-defined shape, drawing conclusions on how this shape fits or misses the local shapes in the image. This simple pre-defined shape is called the *structuring element*. Different combinations of the procedure with different structuring elements and procedures from set theory used to define all MM operations.

This report presents basic MM definitions, illustrates the main operations by elementary examples and compares with other known numerical methods. Then we show an example of application of the MM methods to obtain characteristics of pulsating auroral arcs.

### 2. Mathematical background

The theoretical foundations of morphological image processing lies in set theory and the mathematical theory of order (mathematics of *lattices*). In practical image processing, it is sufficient to know that morphology can be applied to a finite set  $E$  if

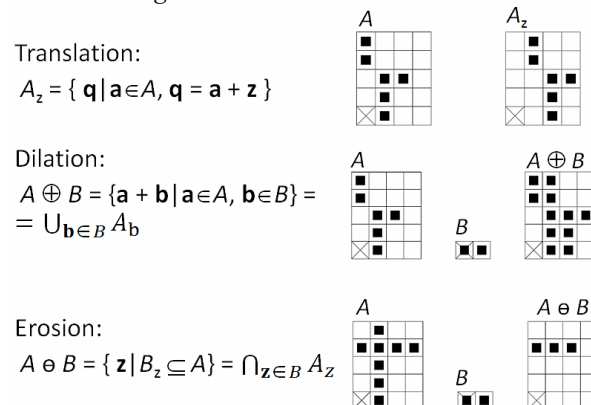
i) we can *partially order* its elements, (where the ordering is denoted by " $\leq$ "), i.e., for all  $x, y, z \in E$ :

$$\begin{aligned} x &\leq x \\ x \leq y, y \leq x &\Rightarrow x = y \\ x \leq y, y \leq z &\Rightarrow x \leq z \end{aligned}$$

ii) each non-empty subset of  $E$  has a maximum and minimum.

For example, any finite set of real or integer numbers is a suitable set  $E$ . The ordering " $\leq$ " can be defined as in ordinary calculus ( $2 \leq 3, 3 \leq 8$ , etc.). The maximum and minimum are also defined in the usual sense (e.g.,  $\max\{7, 1, 3\} = 7$ ). This means we can apply morphology not only to binary images, but and to grey-valued one (or subsets of images), because the collection of grey values can be viewed as a finite set  $E$  with ordering, maximum, and minimum well defined.

All MM operations are constructed by basics operations named *translation*, *dilation* and *erosion*, see definitions in Fig.1 illustrated by simple binary grid examples. Here  $A$  is a plane set under study and  $B$  is the *structuring element*.



**Figure 1.** Definition of basics mathematical morphology operations.

Combination of dilation and erosion give us the *opening* and *closing* operations:

$$\text{opening: } X \circ B = (X \ominus B) \oplus B \quad (1)$$

$$\text{closing: } X \bullet B = (X \oplus B) \ominus B \quad (2)$$

Sequential applications opening and closing with the same structuring element can be used to eliminate small-scale noise structures (so-called *open-close filter*).

To separate structures in same range of scales we can use two different structuring elements,  $B_{r1}$  and  $B_{r2}$ . In this case we should firstly use an open-close filter with a small-size structuring element, followed by an open-close filter with a structuring element of larger size:

$$F_r(X) = (X \circ B_{r1}) \bullet B_{r1} - (((X \circ B_{r1}) \bullet B_{r1}) \circ B_{r2}) \bullet B_{r2} \quad (3)$$

In this way we can implement an *alternating sequential filter*, which gradually eliminates the structure components from smallest to largest.

### 3. Simple numerical example

To illustrate the above definitions by numerical examples let us consider the test data set generated by function

$$f(x) = 50 \left( \sin \frac{\pi x}{8} \sin \frac{\pi x}{400} + \sin \frac{\pi x}{40} + \varepsilon \right) + 130 \quad (4)$$

Here  $x \in \{0, 1, \dots, 400\}$ ,  $\varepsilon$  - random number (Gaussian noise with zero mean and unit standard deviation). Now we define *subgraph of a function* as a set of points that lies between the graph of the function and above the X axis of the graph

$$SG(f) = \{(x, y) \in \mathbf{N} * \mathbf{R} \mid 0 \leq y \leq f(x)\} \quad (5)$$

Fig. 2 illustrates the basic MM operations by applications to this data set. In Fig. 2 (a,b) the subgraph  $SG(f)$  considered as a set  $A$ , and horizontal segment  $[0, 10]$  is taken as structuring element  $B$ . Fig. 2(c) illustrates filtering by open-close filter with segments  $[0, 4]$  (red) and  $[0, 16]$  (blue). One can see that the red line is higher the blue one when the graph has a peak which is not thinner in  $X$  axis than 4. The red line is

lower the blue one when the graph has a ‘valley’ of the correspondent range of scales.

Using the filtering with sequentially increasing structuring element it is possible to obtain a morphological spectrum, as it is illustrated by Fig. 3. The original data set is shown in panel ‘a’. The filter (3) has been applied sequentially with structuring elements  $[0, t_1]$  and  $[0, t_2]$ , where  $t_2 = t_1 + 1$ ,  $t_1 \in \{1, \dots, 100\}$ . Location of “peaks” and “valleys” at correspondent scale is shown in panel ‘b’ by white and black segments. Positions of center of each segment are shown in panel ‘c’ (Really, an operation called *thinning* has been applied.). Now we can calculate intervals between points on each scale and to obtain a statistical distribution presented in panel ‘d’. Integration this distribution over all scales gives us the integrated morphological spectrum shown in panel ‘e’. One can see that despite the short data set and random noise, the spectrum obviously demonstrates two main periods (16 and 80 data points).

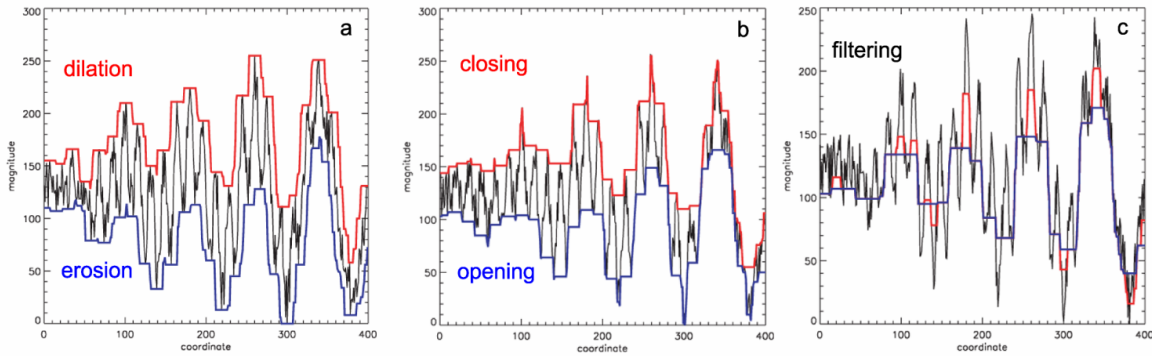


Figure 2. Application of MM operations to data set generated by Eq.4 (see text).

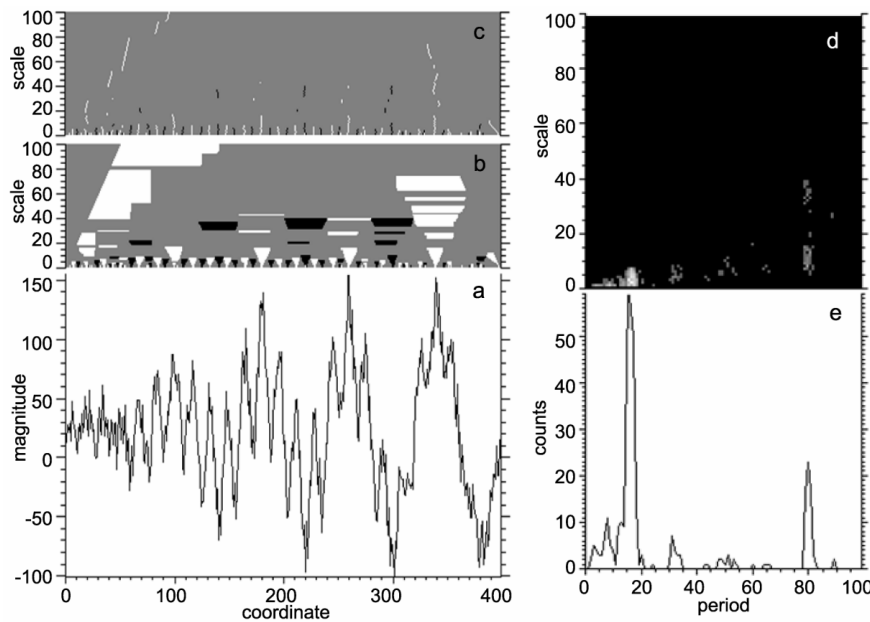
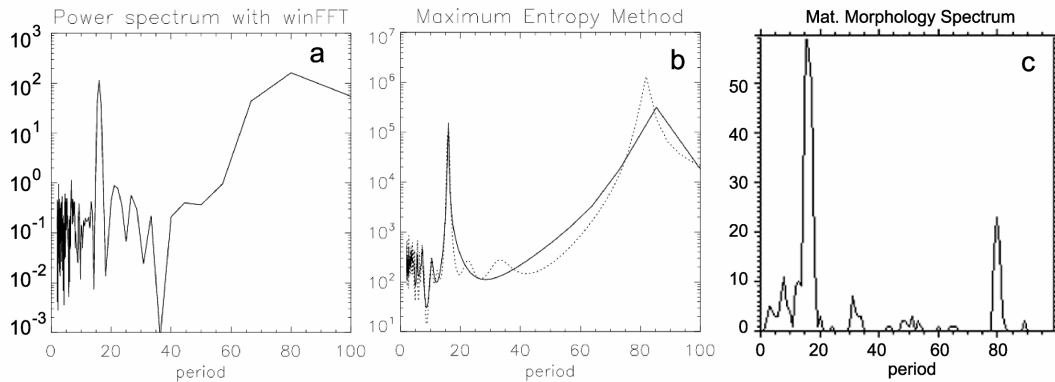


Figure 3. Construction of the MM spectrum for data set generated by Eq.4: a – data set; b – results of filtering at different scales (white – maximums, black - minimums); c – results of thinning operation applied to panel (b); d – spectra of intervals between elements in panel (c) at different scales; e – integrated MM spectrum.

#### 4. Comparison with traditional methods

The same data set (4) has been analysed on periodicity by more known methods traditionally included in numerical libraries. Fig. 4 presents spectra obtained by three methods: i) windowed Fast Fourier Transformation (winFFT), ii) Maximum Entropy Method (MEM), and iii) MM method described in previous section. One can see that all three methods perfectly found the short period of 16 points. The main difference of the methods is observed in separation of large period (80 points). The winFFT method gives a wide maximum in spectrum near correct period ( $80 \pm 20$ ), but the data set is too short to locate the

position by this method with good precision. The MEM method indicates that the maximum at 80-90 points exists, but position of the maximum depends on number of degrees of freedom assumed in the method. Larger number of degrees of freedom usually allows us to obtain better located steeper maximum (see dashed line in the panel 'b'), however it is known that this way sometimes can lead to artifacts. The MM method shows the best results: both main periods are perfectly located. Other peaks observed in the MM spectrum are also understandable: one can see at least first harmonic ( $\sim 8$  points) and subharmonic ( $\sim 32$  points) of the short period.

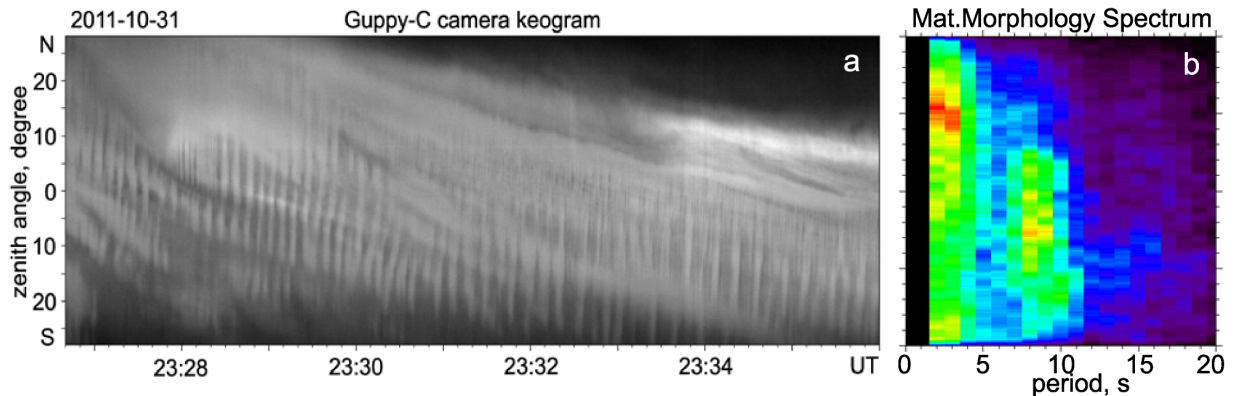


**Figure 4.** Extracting periodicity in the test set by different methods: a) windowed FFT, b) maximum entropy method, c) mathematical morphology spectrum.

#### 5. Example of data analysis

The methods described above have been applied to analyse data of optical observations of aurora by system MAIN [4]. Fig. 5-a presents a typical North-South keogram obtained by images observed by Guppy-C camera in Apatity with time resolution 1 frame per second. Angular resolution of this camera is  $\sim 0.14$  degree per pixel. The aurora intensity is shown by grayscale. One can see auroral pulsation in different locations on zenith angle with different intervals between pulses. To obtain a statistic of the intervals we

applied the MM formalism described above. Each line for given zenith angle in the keogram has been considered as numerical set like in described in previous section example, and the corresponded MM spectrum has been obtained. As a result we have the MM spectra as a function of zenith angle presented in Fig. 5-b by rainbow color coding. One can see that: i) there are two separated band of pulsations of periods 2-3 seconds and 8-9 seconds; ii) the shorter periods are more often observed at poleward boundary of the aurora region.



**Figure 5.** Application of the MM to aurora observations: a – N-S keogram of pulsing multiplet arc; b – MM spectrum of pulsations as a function of zenith angle.

## 6. Conclusions

The report presents basic definitions of mathematical morphology and illustrates the main operations by elementary examples. Dataset with two hidden periodicities, trend and random noise was used as a typical test case for analysis. The procedure for obtaining the integrated MM spectrum for this dataset has been described. The spectrum has been compared with results obtained by other known numerical methods: windowed Fast Fourier Transformation and Maximum Entropy Method. It is shown that the MM method gives the best results: both main periods in the test dataset are perfectly located, while other methods have known problems with location of long period.

Then we show an example of application of the MM methods to obtain characteristics of pulsating auroral arcs. It is shown that: i) there are two separated band of pulsations of periods 2-3 seconds and 8-9 seconds; ii) the shorter periods are more often observed at poleward boundary of the aurora region.

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