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ANALYSIS OF DISPERSION EQUATIONS FOR MAGNETOGRAVITY WAVES IN REALISTIC IONOSPHERE

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Abstract. In the framework of the magnetohydrodynamic approach we obtain the dispersion equations for magnetogravity waves in the ionosphere taking into account the finite conductivity and the combined effect of magnetic field and gravity. The source of magnetogravity waves propagating in the equatorial direction the auroral electrojet is selected. Experimental observation of magnetogravity waves was based on data analysis of ionospheric F2 layer vertical sounding and on the values of geomagnetic field horizontal component. Based on the data of vertical sounding characteristic frequencies ($10^{-4} \div 4 \bullet 10^{-4}$ Hz) and velocities (over 2000 m / s) of magnetogravity waves were estimated. In this frequency range such values are consistent with the analytical parameters of fast and slow propagating magnetogravity waves modes.

1 Introduction

Among the possible sources of wave disturbances in the ionosphere, including traveling ionospheric disturbances (TIDs), often the auroral electrojet is considered. Well known is moving of ionospheric disturbances from auroral region to middle and low latitudes by acoustic-gravity waves (AGW) of different spatial scales. The ionosphere is a conductive space, so there in addition to AGW magnetogravity waves (MGW) can propagate. Characteristic velocities of MGW are greater than AGW, but lower than magnetohydrodynamic (MHD) waves [Sorokin and Fedorovich, 1982]. Experimental confirmation of MGW existence in the ionosphere at altitudes of the F2 layer are obtained in [Barkhatova et al, 2009]. In this work the MGW dispersion equations for infinite conductivity space are analyzed.

In this paper the MGW dispersion equations in the ionosphere with finite conductivity are obtained. For this purpose the combined effect of magnetic field and gravity in hydrodynamics equations is considered. This must be done in case of magnetic pressure is comparable to or higher than the hydrostatic pressure and the frequencies of studied waves is much smaller than frequency of neutrals-ions collisions. These conditions are satisfied in the ionosphere from a height about 250 km.

2 The basic system of MGW equations

Analysis of MGW propagation conditions for finite conductivity space s with absence of regular winds may be conducted on the basis of magnetohydrodynamic equations. The initial system of linearized equations in this case was chosen as:

$$r_0 \frac{\partial \boldsymbol{\upsilon}}{\partial t} = -\nabla p + r\ \mathbf{g} + \frac{1}{c}(\mathbf{j}\times\mathbf{H_0})\,, \qquad \frac{\partial r}{\partial t} + r_0\,\mathrm{div}\,\boldsymbol{\upsilon} + (\boldsymbol{\upsilon}\nabla)\,r_0 = 0\,,$$

$$\frac{\partial p}{\partial t} + (\boldsymbol{\upsilon}\nabla)\,p_0 = V_s^{\,2}[\frac{\partial r}{\partial t} + (\boldsymbol{\upsilon}\nabla)\,r_0]\ ,\quad \mathbf{j} = s\,\{\mathbf{E} + \frac{1}{c}(\boldsymbol{\upsilon}\times\mathbf{H}_0)\} \tag{1}$$

$$\mathrm{rot}\,\mathbf{h} = \frac{4p}{c}\,\mathbf{j},\qquad \mathrm{div}\ \mathbf{h} = 0\,,\quad \mathrm{rot}\,\mathbf{E} = -\frac{1}{c}\frac{\partial \mathbf{h}}{\partial t}$$

Here r - medium density, p - pressure, $\boldsymbol{\upsilon}$ - velocity, $\mathbf{h}$ - magnetic field, V_s^2 - the square of adiabatic sound speed, g - the adiabatic constant, $\mathbf{g}$ - gravitational acceleration, c - light velocity. The values marked with the subscript "0" are undisturbed parameters of considered medium and magnetic field $\mathbf{H_0}$.

In our calculations we adopted isothermal atmosphere: $T_0(z) = const$. In this case the pressure and density are:

$$p_0(z),\, r_0(z) \square \exp(-z/H)\,,$$

where H - scale height of uniform atmosphere.

The solution of the linearized equations for magnetogravity waves in finite conductivity space is the following dispersion equation:

$$(w^2+Q)\{w^2[w^2+Q-(V_s^2+V_{AM}^2)(k_x^2+k_y^2)]-V_s^2(k_z^2+\frac{1}{4H^2})[w^2+Q-V_{AM}^2(k_x^2+k_y^2)]+$$
$$+(g-1)g^2(k_x^2+k_y^2)\}=0,$$

where $V_A^2=\frac{H_0^2}{4pr_0}$ – square of Alfven velocity and

$V_{AM}^2=\frac{V_A^2}{1+i\frac{c^2}{4psw}(k_x^2+k_y^2+k_z^2-\frac{1}{4H^2}-\frac{ik_z}{H})}$ - square of modified Alfven velocity in finite conductivity conditions, $Q=V_{AM}^2\left(\frac{1}{2H}+ik_z\right)^2$.

Thus, the modified Alfven waves are described by the equation

$$w^2+Q=0,$$

and MGW are described by the equation

$$(w^2+Q)[w^2-V_s^2(k_z^2+\frac{1}{4H^2})]-w^2(V_s^2+V_{AM}^2)(k_x^2+k_y^2)+V_s^2V_{AM}^2(k_z^2+\frac{1}{4H^2})(k_x^2+k_y^2)+$$
$$+V_s^2w_g^2(k_x^2+k_y^2)=0, \qquad (2)$$

where $w_g^2=\frac{(g-1)g}{gH}$ - square of Brunt-Vaisaala frequency, $k_x^2+k_y^2=k_\perp^2=k^2\sin q$, $k_z=k\cos q$, k - wave number.

3 Расчет дисперсионных кривых МГВ

The calculation of MGW dispersion curves according to dispersion equation (2) is made dimensionless quantities.

Dimensionless frequency and wave number taken, as in [McLellan and Winterberg, 1968], in the form of $W=\frac{w}{w_c}$ and $K=\frac{k}{k_c}$, where $w_c=\frac{H_0 g}{V_S^2\left(4pr_0\right)^{1/2}}$ and $k_c=\frac{w_c}{V_S}$.

Then the MGW equation in dimensionless variables takes form:

$$W^5+AW^4+BW^3+CW^2+DW+E=0, \qquad (3)$$

where

$$A=\frac{\sqrt{2}K\cos q}{g^{3/2}b^{1/2}\,\mathrm{Re}_m}+i(\frac{2K^2}{b\,\mathrm{Re}_m}-\frac{1}{\mathrm{Re}_m}),$$

$$B=\frac{g^2}{4}-\frac{2}{g^3b}K^2-(\frac{2}{gb}+1)K^2\sin^2 q-\frac{g^3b}{2}-K^2\cos^2 q+i\frac{\sqrt{2}g^{1/2}}{b^{1/2}}K\cos q,$$

$$C=-\frac{\sqrt{2}K^3}{g^{1/2}b\,\mathrm{Re}_m}\cos q-\frac{g^{5/2}b^{1/2}K}{4\sqrt{2}\,\mathrm{Re}_m}\cos q-i\frac{2K^4}{g^2b\,\mathrm{Re}_m}+i\frac{g^4b}{32\,\mathrm{Re}_m},$$

$$D=\frac{2K^4}{gb}\sin^2 q+\frac{g^2}{4}K^2-\frac{g^2K^2}{4}\cos^2 q-\frac{g^5b}{2}+\frac{(g-1)gbK^2}{2}\sin^2 q-i\frac{\sqrt{2}g^{1/2}}{b^{1/2}}K^3\cos^3 q-i\frac{g^{7/2}b^{1/2}}{4\sqrt{2}}K\cos q,$$

$$E=\frac{(g-1)g^{1/2}b^{1/2}}{\sqrt{2}\,\mathrm{Re}_m}K^3\sin^2 q\cos q+i\frac{(g-1)}{g\,\mathrm{Re}_m}K^4\sin^2 q-i\frac{(g-1)g^2b}{8\,\mathrm{Re}_m}K^2\sin^2 q,$$

$b=\frac{p_0}{H_0^2/8p}$ – relation of gazokinetic and magnetic pressure $\mathrm{Re}_m=\frac{4pV_AHs}{c^2}$, q – angle between $\mathbf{k}$ and $\mathbf{g}$, $\mathbf{H_0}$.

Note that the calculation of MGW dispersion curves according to equation (3) are valid only if the wave damping is small and propagating modes are considered. These conditions are satisfied for the height of 250 km, where we have accept the following ionospheric terms: $T=10^3$ K, $g=1.4$, $b=0.02$, $H=3\cdot10^6$ cm,

$\mathbf{H_0} = 0.5$ G, $\sigma = 10^{10}$ c^{-1} and $\mathrm{Re}_m = 3 \cdot 10^2$. Calculations of dispersion curves are executed for such parameters, which are close to reality.

In the weak absorption and executing limitation on the TID scales (the vertical wavelength λ_z is much smaller than $4\pi H$ $\left(k_z \gg \frac{1}{2H}\right)$) the dispersion MGW equation (2) becomes

$$\omega^4 - \omega^2\left(V_s^2 + V_A^2\right)\left(k_\perp^2 + k_z^2\right) + V_A^2 V_s^2 k_z^2\left(k_z^2 + k_\perp^2\right) + V_s^2 \omega_g^2 k_\perp^2 = 0 . \tag{4}$$

Figure 1 (a) presents a complete solution of the dispersion equation (3), and Fig. 1 (b) shows the dispersion curves for the short-cut equation (4) in dimensionless variables for MGW propagating fast "+" mode and slow "-" mode. Here solid line corresponds to longitudinal propagation (θ = 0 °), small dotted line - propagation at an angle (θ = 45 °), large dotted line - transverse propagation (θ = 90 °) of MGW. Dispersion curves are not shown in Fig. 1 (a) are outside the considered frequency range. Dependent (Figure 1 (b)) on the propagation angle for a fast "+" mode is absent.

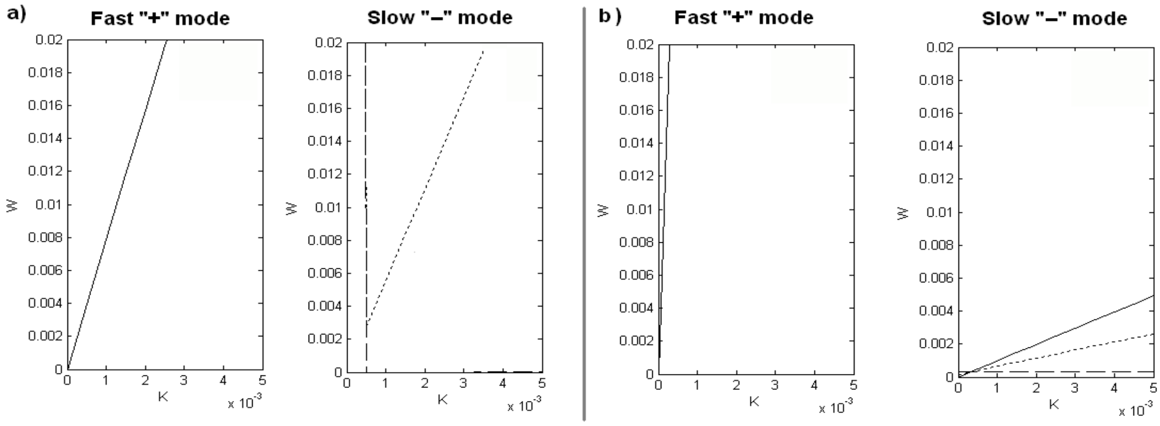

Fig. 1. The dispersion curves for MGW propagating fast "+" mode and slow "-" mode in dimensionless variables obtained for (a) of the complete dispersion equation (3) and (b) the short-cut equation (4): solid line corresponds to the longitudinal propagation (θ = 0 °), fine dotted line - the propagation at an angle (θ = 45 °), large dotted line - the transverse propagation (θ = 90 °) of MGW

4 Experimental data analysis

In our study it suggested that MGW excited during substorms by auroral source namely the eastward electrojet and propagate along the geomagnetic meridian to the mid-latitudes. Search of such MGW based on comparative spectral analysis of index AU fluctuations, variations of ionospheric layer F2 critical frequencies at vertical sounding stations and variations of geomagnetic field horizontal component on daily intervals in March-April 2006 was carried. Ionospheric and magnetic stations whose data selected for analysis are presented in Table 1.

Table 1.

Ionospheric and magnetic stations	Stations abbreviation	Geogr. latitude	Geogr. longitude	Geom. latitude	Geom. longitude	L (McIlwain parameter)
L Aquila	AQU	42.38	13.32	58.7	1.9	1.5
Furstenfeldbruck	FUR	48.17	11.28	64.2	1.6	1.9
HEL	HLP	54.61	18.82	69.5	4.0	2.5
Lvov	LVV	49.9	23.75	66.3	4.7	2.05
Juliusruh/Rugen	JR055	54.6	13.4	69.3	2.2	2.5
San Vito	VT139	40.6	17.8	57.0	2.7	1.47

Figure 2 shows examples of agreed dynamic spectra of index AU, ionospheric and magnetic disturbances for 4 and 5 April 2006. The coincidence of spectral features in dynamic spectra indicates the propagation of density and magnetic field disturbances from auroral source to middle latitudes.

Analysis of these spectra dynamics (Fig. 4) shows the practically absence of the time shift between them. This indicates the higher MGW velocities compared with classical internal gravity waves. The characteristic MGW frequencies estimated from the spectrograms are in the range $10^{-4} \div 4 \cdot 10^{-4}$ Hz. Such frequencies correspond to the values of the phase velocity "+" mode about 4000 m/s, "-" mode - about 3000 m/s. Figure 3 shows the calculated phase velocities of the MGW fast "+" mode and slow "-" mode in this wavelength range.

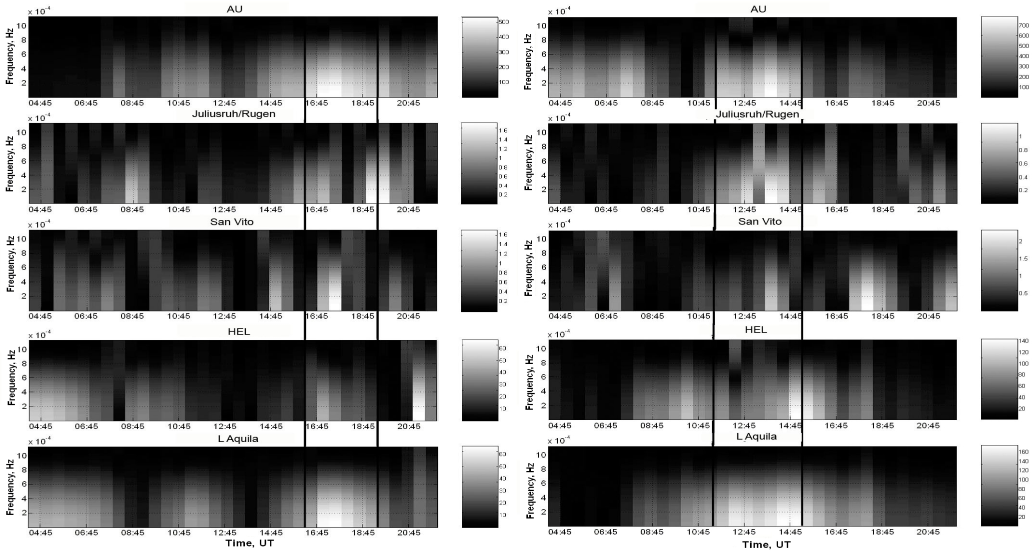

Fig. 2. Examples of dynamic spectra index AU, two ionospheric and two magnetic stations (from top to bottom) for April 4 (left) and April 5 (right) 2006

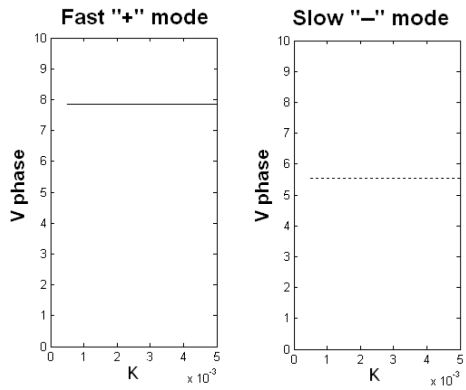

Fig. 3. The phase velocities of the MGW fast "+" mode and slow "-" mode: solid line corresponds to the longitudinal MGW propagation (θ = 0°), small dotted line - the propagation at an angle (θ = 45 °)

The experimental verification of the phase velocities values obtained by comparing of the geomagnetic field horizontal component dynamic spectra for the considered stations with minute resolution. It was found that the time shift of dynamic spectra in the region corresponding to MGW propagation is 5 - 15 min. In view of the distance between the magnetic stations and estimated velocities, a time like these corresponds to MGW propagation.

5. Summary

The MGW dispersion curves in ionospheric medium in the context of finite conductivity are analyzed. By comparing of f0F2 dynamic spectra and the geomagnetic field horizontal component detected the MGW propagation from auroral source to the middle latitudes. The MGW characteristic parameters: the frequency about $10^{-4} \div 4 \bullet 10^{-4}$ Hz, the phase velocities values of the "+" mode about 4000 m /s, "-" mode – about 3000 m / sec.

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References

Barkhatova O. M., Barkhatov N. A., Grigoriev G. I. Detection of magneto-gravity waves in the ionosphere by analysis of the maximum observable frequencies on oblique-sounding paths // Radiophysics and Quantum Electronics. V. 52. N. 10. P. 683-698. 2009.

Grigoriev G.I. Acoustic-gravity waves in Earth's atmosphere (review) // Radiophysics and Quantum Electronics, V. 42. N. 2. P. 3-25. 1999.

Sorokin V.M., Fedorovich G.V. Physics of slow MHD waves in the ionospheric plasma. Energoatomizdat, M. 1982. 136

McLellan A., Winterberg F. Magneto-gravity waves and heating of the Solar corona // Solar Physics 4, 401-408, 1968.