

## FULL-WAVE SOLUTION FOR A MONOCHROMATIC VLF WAVE PROPAGATING THROUGH THE IONOSPHERE

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Among the many problems in whistler study, wave propagation through the ionosphere is one of the most important, and the most difficult at the same time. Characteristics of propagation, such as reflection and transmission coefficients, are needed for interpretation of satellite and ground-based observations of VLF waves. Consequently wave penetration through the ionosphere has been in the focus of research since the beginning of whistler study: Budden (1985) (general full-wave analysis, including the problem of numerical swamping); Pitteway (1965) (analysis of differential equations governing the wave fields inside a horizontally stratified ionosphere in the case of oblique wave incidence from below; indication and discussion of the problem of numerical swamping); Helliwell (1965) (detailed consideration in the framework of ray theory revealing the most essential features of the phenomenon); Pitteway and Jespersen (1966) (a comprehensive numerical study of wave propagation through the ionosphere, including calculations of reflection and transmission coefficients); Hayakawa and Ohtsu (1972) (theoretical treatment of transmission and reflection of downgoing whistlers in the case of longitudinal propagation, using model plasma density and collision frequency profiles).

The difficulty in considering VLF wave passage through the ionosphere is, after all, due to fast variation of the lower ionosphere parameters as compared to typical VLF wave number. This makes irrelevant the consideration in the framework of geometrical optics, which, along with a smooth variation of parameters, is always based on a particular dispersion relation. Although the full-wave analysis in the framework of cold plasma approximation does not require slow variations of plasma parameters, and does not assume any particular wave mode, the fact that the wave of a given frequency belongs to different modes in various regions makes numerical solution of the field equations not simple. As is well known (see, e.g. Ginzburg and Rukhadze, 1972), in cold magnetized plasma, there are, in general, two wave modes related to a given frequency. Both modes, however, do not necessarily correspond to propagating waves. In particular, in the frequency range related to whistler waves, the other mode is evanescent, i.e. it has a negative value of  $N^2$  (the refractive index squared). It means that one of solutions of the relevant differential equations is exponentially growing, which makes a straightforward numerical approach to these equations despairing. This well known difficulty in the problem under discussion is usually identified as numerical swamping (see, e.g. Budden, 1985; Nagano et al., 1975, and earlier works mentioned above). Resolving the problem of numerical swamping becomes, in fact, a key point in numerical study of wave passage through the ionosphere. As it is typical of work based on numerical simulations, its essential part remains virtually hidden. Then, every researcher, in order to get quantitative characteristics of the process, such as transmission and reflection coefficients, needs to go through the whole problem. That is why the number of publications dealing with VLF wave transmission through the ionosphere does not run short.

In this work we develop a new approach to the problem, such that its intrinsic difficulty is resolved analytically, while numerical calculations are reduced to stable equations solvable with the help of a routine programme. Another goal of the work is to present all equations and related formulae in an undisguised form in order that the problem may be solved in a straightforward way once the ionospheric plasma parameters are given.

We consider the problem of wave propagation in the ionosphere in case of wave incidence on the ionosphere from above when the angle of incidence is small enough.

We used the model of plane medium in which all ionospheric plasma parameters depend only on height  $h$  and do not depend on horizontal coordinates  $x$  and  $y$  ( $y$  axis is directed westward,  $x$  axis completes the right-hand system  $\{x, y, h\}$ ). The wave field was assumed to be monochromatic, harmonic in  $x$  and independent of  $y$  (all variables are dimensionless):  $\mathbf{E} = \text{Re} \left\{ \mathbf{E}(h) \exp(i k x - i \Omega t) \right\}$ , and the similar formula is for the magnetic field.

Horizontal component of the wave vector  $\mathbf{k}$  for a given wave frequency is defined by angle incidence of the wave. Having assumed such field structure one can easily obtain from general Maxwell's equations following equation for the field amplitudes  $\mathbf{E}(h)$ ,  $\mathbf{B}(h)$ :

$$\begin{aligned} \frac{d E_y}{d h} &= -i \Omega B_x; & \frac{d E_x}{d h} &= i \Omega B_y + i k E_h; & k E_y &= \Omega B_h; \\ \frac{d B_y}{d h} &= i \Omega \left\{ e_{-1} E_x + i e_2 \cos a E_y + e_{-31} E_h \right\}; & & & & (1) \\ \frac{d B_x}{d h} &= i k B_h + i \Omega \left\{ -e_1 E_y + i e_2 \cos a E_x - i e_2 \sin a E_h \right\}; \\ k B_y &= -\Omega \left\{ e_{-31} E_x - i e_2 \sin a E_y + e_{-3} E_h \right\}. \end{aligned}$$

There all  $\mathbf{e}$  with indices are the components of dielectric tensor in coordinate system  $\{x, y, h\}$  ( $a$  is an angle between  $x$  axis and external magnetic field):

$$\begin{aligned} \overline{e}_1 &= e_1 \cos^2 a + e_3 \sin^2 a ; \overline{e}_{31} = (e_3 - e_1) \sin a \cos a \\ \overline{e}_3 &= e_3 \cos^2 a + e_1 \sin^2 a ; \\ e_1 &= 1 + \frac{w_{pe}^2 (w + in_e)}{w (w_{ce}^2 + n_e^2)} - \frac{1}{M_{eff}} \frac{w_{pe}^2}{w (w + in_i)} ; \\ e_2 &= - \frac{w_{pe}^2 w_{ce}}{w (w_{ce}^2 + n_e^2)} ; e_3 = 1 - \frac{w_{pe}^2}{w (w + in_e)} ; \frac{1}{M_{eff}} = \frac{m_e}{n_e} \sum_{ions} \frac{n_i}{m_i} ; \end{aligned}$$

We assume the Earth to be perfectly conductive, hence tangential components of the electric field equal to zero at the surface, at the upper boundary of height (which was set equal to 600 km) the field is the superposition of incident wave with given amplitude and reflected wave.

System (1) consists of four ordinary differential equations and but for numerical swamping it could be integrated numerically in a straightforward way. So we need to find a way to regularize this instability. It can be noticed that for enough high heights in ionosphere following inequalities are fulfilled:

$$|e_3| \gg |e_2| \gg |e_1| ,$$

which allows us to use some kind of method of successive approximations. Now it becomes more convenient to use another coordinate system  $(x, y, z)$  where  $z$  axis is along external magnetic field. Passing on to these coordinates we can easily obtain following equations for the new components of the electromagnetic field:

$$\begin{aligned} \frac{d^2 A_1}{dh^2} + h A_1 &= -i l \frac{dA_2}{dh} + g A_2 ; \quad (2) \\ \frac{d^2 A_2}{dh^2} - m A_2 &= -i l \frac{dA_1}{dh} + g A_1 ; E_z \approx 0 ; \\ A_1 &= (E_x - i E_y | \sec a |) e^{i l h/2} ; A_2 = (E_x + i E_y | \sec a |) e^{i l h/2} ; \\ h, m, g &= h, m, g (e_1, e_2) ; l = k \tan a . \end{aligned}$$

Having assumed that the angle of incidence is small enough, so that  $|e_2| \Omega^2 \gg k^2$ , we can expect that for enough high heights (for  $h > 80-90$  km) coefficients in (2) will be related as following:

$$h : m \gg m \text{ a x } \{l^2, |g|\} ,$$

so we are able to perform the expansion of system (2) in this small parameters.

Zero order equations will be

$$\frac{d^2 A_1}{dh^2} + h_0 A_1 = 0 ;$$

$$A_2 = 0 ;$$

and the first order:

$$\frac{d^2 A_1}{dh^2} + h A_1 = 0 ; \quad (3)$$

$$A_2 = - \frac{1}{2 h_0} \left( -i l \frac{dA_1^{(0)}}{dh} + g A_1^{(0)} \right)$$

Zero order equations and the first equation in (3) do not need anything besides aforementioned inequalities, but the second relation in (3) is correct only in assumption of slow variation of plasma parameters. These equations are stable and can be easily integrated.

For low altitudes exact system (1) is provided to be stable enough, hence one can integrate it numerically. Moreover the regions, where exact system (1) and approximate system (3) are applicable, overlap, consequently it is possible to join the solutions of these systems and finally we can obtain unique solution of the problem in the whole range of altitudes.

Using the developed method we perform calculations of electromagnetic field and reflection coefficient  $R$  defined by formula below for a set of ionospheric parameters obtained from IRI model (see Fig. 1).

$$R = \frac{S_{ref}}{S_{inc}} \approx \left| \frac{A_{1ref}}{A_{1inc}} \right|^2,$$

where  $S_{ref}$  and  $S_{inc}$  are time-averaged energy fluxes at the upper boundary of height in reflected and incident wave correspondingly. The outputs of calculations are shown in Fig. 2. In these figures, the values of  $R$  corresponding to relatively large perpendicular wave vectors, thus, large angles of incidence, for which waves do not propagate in the atmosphere, are marked by points; values of  $R$  for the cases when waves reach the ground are marked by circles. Graphs related to six angles of incidence are plotted. Upper graphs that correspond to small angles of incidence and, thus, propagation in the earth-ionosphere cavity, show a quasiperiodic behaviour of the reflection coefficient as a function of wave frequency. With increasing angle of incidence, waves in the atmosphere become evanescent, and the quasiperiodic feature of  $R$  ceases. This suggests that this feature is due to resonance properties of the earth-ionosphere cavity. The quasiperiodic dependence of the reflection coefficient on frequency has earlier been pointed out by Hayakawa and Ohtsu (1972). To understand the quasiperiodic dependence of the reflection coefficient on wave frequency, we first remember that at the ground, the total energy flux is equal to zero, so that the reflected energy flux is equal (in magnitude) to the incident one. Thus, a deviation of the reflection coefficient from unity at  $h = 600$  km is only due to energy absorption in the ionosphere. The peculiarity of this absorption consists in that maximum absorption takes place in a small region  $\sim 30$  km at the height  $H_{abs}$  about 90 km. Although at these heights the wave field structure is quite complicated and differs significantly from sinusoidal one typical of atmospheric region, the magnitude of the electric field that determines the wave field absorption still has a quasiperiodic structure with a characteristic length equal to (in dimensional units)

$$l = \frac{p}{k} \sim \frac{p}{w} c.$$

The wave field absorption will then have a maximum and, consequently, the reflection coefficient will have a minimum if  $H_{abs} = (n+1/2) l$ , where  $n = 0, 1, 2, \dots$ , thus,

$$w \sim \left( n + \frac{1}{2} \right) \frac{p}{H_{abs}} c.$$

This gives a period of variation of the reflection coefficient over frequency:  $\Delta w : pc/H_{abs} \sim 10^4 \text{ rad/s}$ , or  $\Delta F \sim 1.7 \text{ kHz}$ . To verify this interpretation, we calculated the reflection coefficients with the ground level artificially shifted towards the absorption region, which indeed resulted in the corresponding increase of the period  $\Delta F$  (see Fig. 3).

In this work, we have presented a new approach to full-wave description of VLF wave penetration through the ionosphere. In this approach, the problem of numerical swamping, which constitutes the main difficulty of previous considerations, is resolved analytically, making numerical calculations plain and straightforward. The developed approach is based on the method of successive approximations which, in its present form, is applicable only to the case of wave incidence on the ionosphere at small angles. Investigation of the problem for arbitrary angles of incidence will be the subject of further work. The results of the present study imply a frequency modulation in wide-band VLF measurements of whistler mode waves reflected from the lower ionosphere. The mechanism of this modulation explained in the present work also hints at a possible quasiperiodic frequency dependence of VLF spectral intensity observed on the ground.

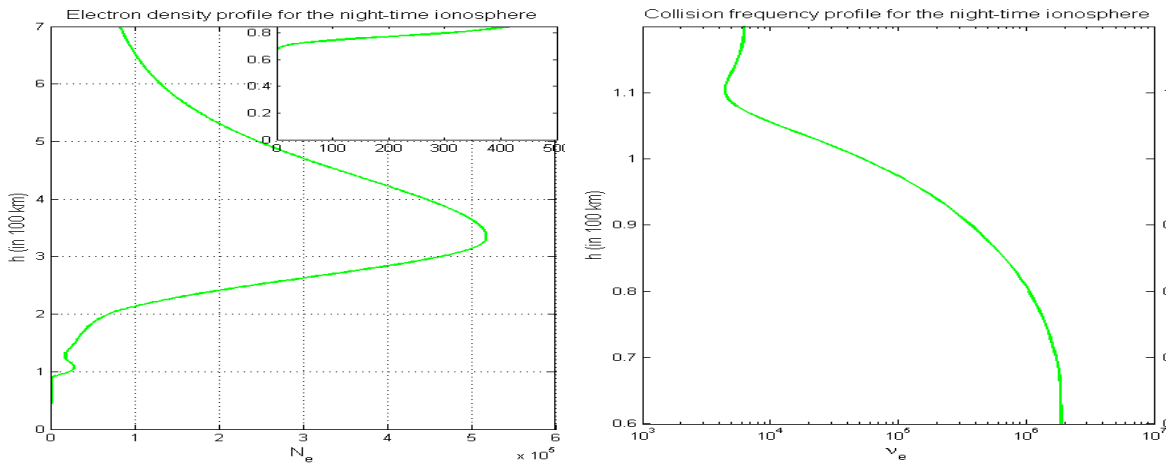
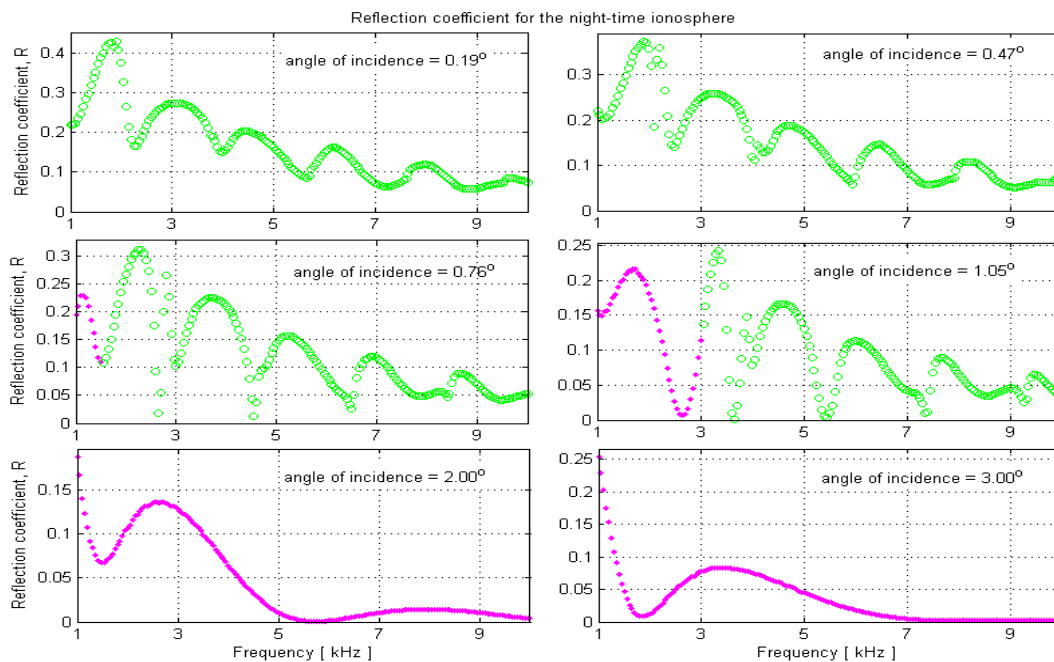
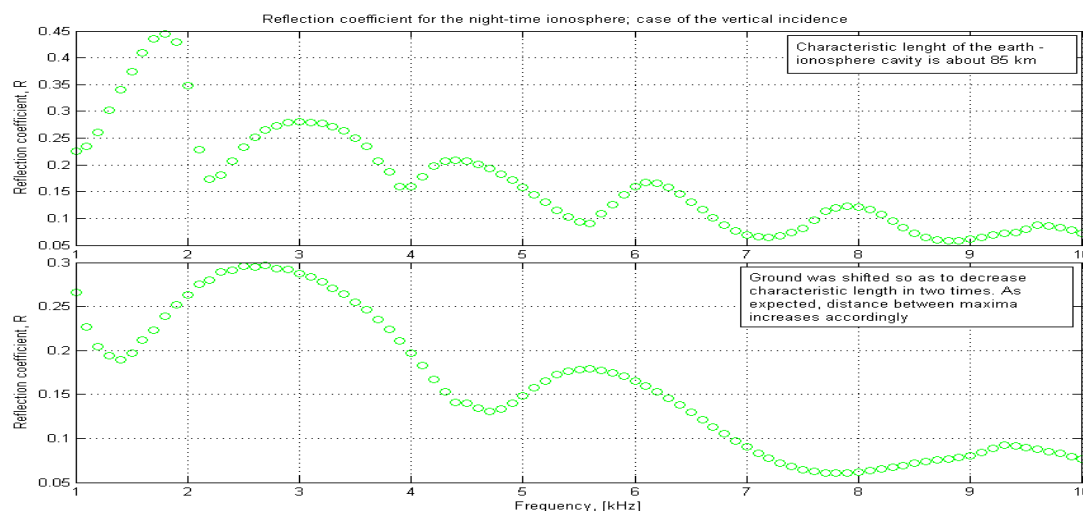


Fig. 1 Electron density and collision frequency profiles.



**Fig. 2** Reflection coefficient for the night-time ionosphere.



**Fig. 3** Demonstration of the resonance properties of the Earth-lower ionosphere cavity.

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