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INFLUENCE OF INTERMITTENCY ON PARTICLE ACCELERATION

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Abstract. In this paper we constructed an analytical model reproducing the main characteristic of intermittent turbulent electromagnetic field. The comparison between the model and experimental data obtained in the Earth's magnetotail confirms that such model is capable to describe satisfactorily the main observational features of magnetotail turbulence. We investigate the dependence of particle acceleration on the level of intermittency. The obtained results indicate that the efficiency of particles energy gain increases with the growth of this parameter. Moreover, the efficiency of the spatial transport grows with intermittency level slower than efficiency of acceleration. As a result particles can gain more energy travelling the same distance in the intermittent turbulence. This result allows to explain charged particles acceleration up to very high energies in spatially localized areas of turbulent electromagnetic fields.

Introduction

The appearance of populations of high-energy particles in the Earth's magnetosphere is a common phenomenon [*Vasyliunas*, 1968; *Sarris et al.*, 1976; Christon et al., 1989]. Along with the acceleration of particles in the vicinity of X-lines [*Hoshino,* 2005; *Drake et al.,* 2006] and quasi-adiabatic acceleration [*Lyons and Speiser*, 1982] it is seems that another important mechanism of the formation of these populations is the turbulent acceleration [*Zelenyi et al.,* 2008; *Perri et al.* 2009; *Ono et al.,* 2009]. However, in the framework of existing models it is difficult to explain the acceleration of particles up to high energies in relatively small localized regions of space. In the model of electromagnetic clouds [*Perri et al.,* 2009] the spatial diffusion is coupled with the diffusion in velocity space and the spatial one appears to be more reped. Similar result was obtained for the model of turbulent electromagnetic field constructed by an ensemble of plane travelling waves [*Zelenyi et al.,* 2008]. Thus, if the turbulent electromagnetic field is localized in the small spatial region, then the particles can not gain sufficiently high energy before leaving this area. Consequently, the observation of particles with sufficiently high energies indicates that the turbulence might have quite different properties than those incorporated in existing models. On the other hand many satellite observations of the magnetic field in the Earth's magnetosphere indicate that it has an intermittent nature [*Dudok de Wit and Krasnosel'skikh,* 1996; *Vörös et al.,* 2004; *Petrukovich,* 2005]. This peculiarity of turbulence is associated with the presence of certain regions with significantly different statistical properties in one-dimensional time-series of magnetic field $B(t_n)$. In this paper will investigate the influence of the intermittent electromagnetic turbulence on transport of charched particles both in velocity (acceleration) and configurational space. We show that efficiency of particle acceleration grows with the level of intermittency faster than efficiently of particle spatial transport. This result allows explaining the acceleration of particles up to high energies in the spatial localised regions with turbulence in the Earth's magnetosphere.

Model

In this paper we use simple 2D model of turbulent electromagnetic field: particles move in the neutral plane of the Earth magnetotail. Only single component of magnetic field $B_z(x,y,t)$ is taken into account(here the GSM coordinate system is used). Particles move in the plane *(x,y)* and are accelerated by inductive electric fields E_x and E_y. The intermittency of field B_z can be characterized by means of the structure function method [*Frisch* 1995]:

$$S_p = \sum_n \left| B_z\left(t_n + \Delta\right) - B_z\left(t_n\right)\right|^p \qquad (1)$$

Let us assume that we draw some straight line in the plane *(x, y)* and start to measure magnetic field along it. Then using the dependence $S_p \sim \Delta^{z_p}$ one can plot the power law exponent z_p as a function of *p*. The difference between function z_p and linear function $z_p \sim C_0 p$ can be used to quantity degree of intermittency. Here we should notice that for the classic Kolmogorov turbulence $z_p \sim C_0 p$ and C_0 corresponds to the spatial dimension of elements forming the turbulence [*Frisch,* 1995].

In our model field $B_z(x,y, t)$ is assumed to consist from the superposition of magnetic clouds [*Perri et al.,* 2009] and an ensemble of plane magnetic waves [*Zelenyi et al.,* 2008].The magnetic cloud model is based on the assumption of the existence of the local structures with the intense magnetic fields. Pulsations of these structures provide the inductive electric field. For geometry under consideration each magnetic cloud in the plane *(x, y)* is defined by two components of vector potential:

$$A_{x,y} = A_0 \exp\left\{-k_x^2 \left(x - x_0 - a \sin wt\right)^2 - k_y^2 \left(y - y_0 - a \sin wt\right)^2\right\} \tag{2}$$

Magnetic and electric fields can be found from Maxwell equations: $B_{z(cloud)} = \mathbf{e}_z \mathrm{rot}\mathbf{A}$, $\mathbf{E} = c^{-1}\partial\mathbf{A}/\partial t$. Here $\mathbf{k} = \{k_x, k_y\}$ is the vector of reversed spatial scale of a cloud, $\mathbf{r}_0 = \{x_0, y_0\}$ is the vector of cloud position, a and ω are magnitude and frequency of cloud oscillations. We used a special distribution of amplitudes of magnetic clouds A_0 and their positions to obtain the model with the structure function different from the Kolmogorov one well. It is known that some one-dimensional maps of an interval at the real axis onto itself possess the property of temporal intermittency [*Manneville, 1980*]. To incorporate the one-dimensional map with intermittency into the model of magnetic clouds we use the following method. We divided the whole plane (x, y) into uniform cells with the size $(\Delta l, \Delta l)$, then we place a single cloud with an amplitude $A_0 = f_n^m$ into each of these cells, where the indices n and m define the numbering of the cells along the x and y axis respectively. The coordinates of the clouds centres are given by $\mathbf{r}_n^m = \{n\Delta l + \Delta l/2, m\Delta l + \Delta l/2\}$. The amplitudes f_n^m along the direction x are defined by the map $f_{n+1} = F(f_n^m)$ where the map F is intermittent. The amplitudes of the maps are coupled by the discrete diffusion relation $D(f_n^{m-1} - 2f_n^m + f_n^{m+1})$. Due to the fact that diffusion tends to make values neighbour in a space equal, the chaotic bursts and laminar phases (regions filled by f_n^m with similar values) will extend in both directions and will form clusters. The scale of the grid Δl is chosen so that the inequality $\Delta l/|\mathbf{k}_c| < 1$ is satisfied (which means that each cloud is confined within its cell – see scheme in Fig. 1).

We choose the map of the interval [0, 1] with the complex intermittent dynamics known as the Pomeau-Manneville map [*Manneville, 1980*]: $F(f_n) = f_n + f_n^b$ (mod)1 where $f_n \in [0, 1]$, $b > 1$. It is known that this map possesses the three main dynamic regimes depending on the parameter b. When $1 \le b < 3/2$ the dynamics is normal: the fluctuations of a random variable generated by the map are distributed by Gaussian law. When $3/2 < b < 2$ the dynamics is transitional-anomalous (non-gaussian) and when $b > 2$ the dynamics becomes strictly non-gaussian described by Levy statistics with the index $1/(b - 1)$. The latter regime corresponds to the map with intermittency.

Figure 1. Scheme of magnetic clouds positions

The given map generates one-dimensional series of intermittent turbulent bursts which are always positive. For our magnetic field model it is more natural for bursts to have both positive and negative values. This can be achieved by modifying the map $F(f_n)$ extending it to the negative values as follows,

$$f_{n+1} = F(f_n) = \mathrm{sgn}(f_n)\left[(|f_n| + |f_n|^b + 1) \,(\mathrm{mod}\,2) - 1\right], \quad f_n \in [-1,1], \, b > 1 \tag{4}$$

The second component of our model is the ensemble of standing magnetostatic structures [*Veltri et al., 1998; Zelenyi et al., 2008*]:

$$B_{wave} = B_0^w \sum_n \left(1 + (\mathbf{k}_n \mathbf{l})\right)^{-a} \cos\left(\mathbf{k}_n \mathbf{r} + j_{0n}\right) \tag{3}$$

Here $\mathbf{l}$ is the correlation vector, φ_{0n} - initial phase has a random values for each wave, B_0^w is the magnitude of waves.

Comparison of model results with observations

To compare our model with typical observations of turbulent magnetic field in the Earth magnetotail we construct the model magnetic field along the straight line in plane $(x; y)$. Then for the obtained 1D set the structure functions are calculated and power law exponent z_p is plotted as a function of order p. Fig. 2(a) shows that model parameter b controls the dependence z_p on p: curvature of z_p increases with the increase of b.

Below we compare the model data with observation of turbulence in the Earth magnetotail [*Petrukovich, 2005*]. Typical example of spacecraft (Interball-tail) data is shown in Fig. 2(b). Fig. 2(c) presents a model magnetic field. The model does not include a whole spectrum of the observed magnetic field (here we take into account only two orders in the spectrum), but even in such rather simple approximation the power law exponents from the model and observations have a similar behaviour (Fig. 2(d)). Therefore, one can conclude that the model constructed here is capable to reproduce main features of observation with good quality.
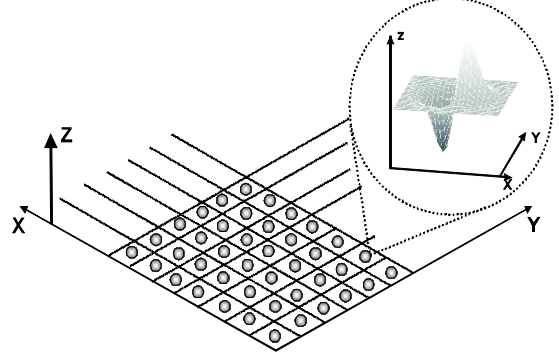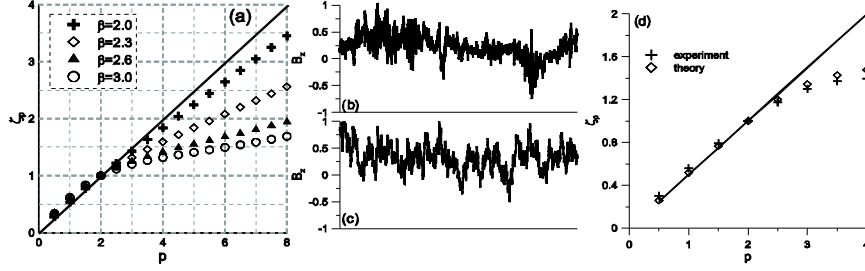
Figure 2. Panel (a): Power law exponent of structure functions as a function of its order for different values of *b*. Panel (b): observation of magnetic field (from [*Petrukovich*, 2005], fig. 1.3). Panel (c): model magnetic field. Panel (d): power law exponents of model and observation

Particle acceleration

In this section the influence of intermittency of electromagnetic turbulence on particle transport and acceleration will discussed. We use the modelling domain with the spatial scale $L = 400/|\mathbf{k}_{max}|^{-1}$. Wavenumbers of magnetic field (4) are taken from the interval [0.02, 1] $/|\mathbf{k}_{max}|$ and their distribution over direction of propagation is uniform (twenty waves with angle of propagation from 0 to 2π are set for each value of wavenumber magnitude).

The energy of magnetic component of waves ($\sim(B_0^{wave})^2$) is equal to 4/5 of the total magnetic field energy. The number of magnetic clouds in modelling domain is $N_c = 40000$, their spatial scale is $/k_c|^{-1} = 0.5/|\mathbf{k}_{max}|^{-1}$ (spatial scales of clouds are isotropic in plane (*x*, *y*)). Magnitude of clouds oscillations is $a = 0.2/|\mathbf{k}_{max}|^{-1}$ and frequency of oscillations *w* will be normalized to eB_0^w/mc.

We use magnetic energy normalization for magnetic field of clouds to provide the further comparison of our results with various turbulence parameters:

$$W = \frac{1}{4L^2} \int_{-L}^{L} \int_{-L}^{L} \left| B_{z(cloud)} \right|^2 dxdy = \text{const} \tag{5}$$

Particle spatial coordinates are normalized on $2/|\mathbf{k}_{max}|^{-1}$. Time is normalized on mc/eB_0^{wave}. Particle velocities are normalized on $u = 2/|\mathbf{k}_{max}|^{-1}mc/eB_0^{wave}$ (and energy $e \sim v^2/u^2$).

To study the influence of intermittency of electromagnetic turbulence on particle transport and acceleration we use the test particle method (number of particles is $N_p = 10^6$ and periodical boundary conditions are used). Initially ensemble of particle has a maxwellian energy distribution with temperature $\sim u^2$. Average energy of particle ensemble as a function of time for different values of *b* parameter (different intermittency levels) is presented in Fig. 3. With the increase of intermittency level the energy gained by particles during equal time intervals is growing. The ratio between energy gained by particles in the system with $b = 2$ (almost non-intermittent turbulence) and in the system with $b = 3.5$ (strong intermittent turbulence) is about two. This ratio is also dependent on the magnetic energy of clouds.
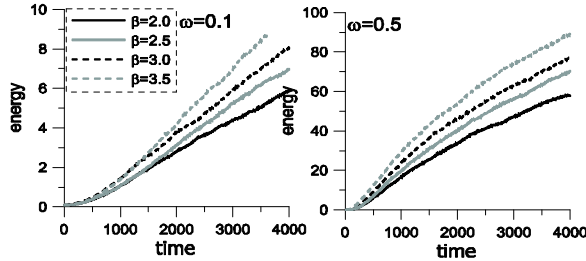

Figure 3. Average energy of particle ensemble as a function of time for four values of parameter

In Fig. 4 one can see the average energy of particle ensemble as a function of the average spatial displacement. The model with the strong intermittent turbulence ($b = 3$, $b = 3.5$) accelerates particle more effectively than almost non-intermittent model ($b = 2$) while the level of spatial transport remain the same. This effect allows to accelerate the particles in the spatially localized region 'filled' by the intermittent turbulence.
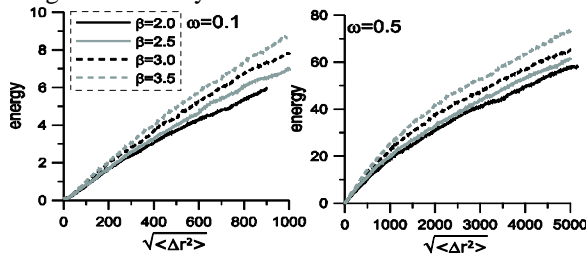

Figure 4. Energy, gained by particles as a function of spatial transport for several models with different value of parameter

Conclusions

In this work the model of turbulent electromagnetic field with intermittency has been developed. It has been shown that the degree of intermittency can be varied and set as a free parameter of the model. Moreover the possibility of our model to approximate qualitatively the experimental observation has been demonstrated. Numerical calculations show that more efficient particle acceleration occurs in a more intermittent field. The effect of the influence of intermittency on the ratio between spatial transport and particle acceleration has been briefly discussed: in the intermittent field energy gained by particle turns out to be larger than in the non-intermittent field. The obtained results can explain the particle acceleration up to high energies even in the rather localized regions of the electromagnetic field of the turbulent Earth's magnetotail while the level of spatial transport remain the same.

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