"Physics of Auroral Phenomena", Proc. XXXI Annual Seminar, Apatity, pp. 171-174, 2008

Polar Geophysical Institute

THE THEORY FOR SCATTERING OF RADIOWAVES FROM REFRACTIVE INDEX FLUCTUATIONS IN THE POLAR IONOSPHERE

V.D. Tereshchenko (*Polar Geophysical Institute*, *Murmansk*, *183010*, *Russia*; *E-mail*: *vladter@pgi.ru*)

Abstract. The theory of single scattering of high-frequency electromagnetic waves on refractive index fluctuations in ionospheric plasma is examined. In approximation of weak scattering the general formulas for differential and total cross-sections of Fresnel and Thermal Scatter of radiowaves in the polar ionosphere are received most. The possibility of using them to determine of parameters of D-region of the polar ionosphere by methods of incoherent scattering and partial reflection of radiowaves is discussed.

Introduction

When electromagnetic waves propagate in the ionosphere with random irregularities (fluctuations of dielectric permeability) there is a scattering of these waves. The phenomenon of scattering of waves plays the important role in the field of diagnostics of space and laboratory plasma [Sheffild, 1975]. Among radiophysical methods of research of lower ionosphere, using the mechanism of volumetric scattering, the widest applications were received with methods of incoherent scattering and partial reflection of radiowaves [Holt, 1983]. Bases of the theory of these methods have been incorporated by Booker [1959] with reference to a range of VHF on which influence of the geomagnetic field can be neglected. However for waves of average frequency use of approximation of isotropic plasma can lead to material mistakes in a valuing of the scattered power. Attempt of the account of effects of a geomagnetic field has been undertaken in paper [Flood, 1968], but calculations of an absent-minded field were not strict enough. This paper is an attempt to unify the scattering and reflection theories for magnetic plasma with the purpose of them use for studying the lower ionosphere.

The radar equation

Starting point of the theory of methods of weak volumetric scattering is the radar equation:

$$P_s = \frac{P_t c \tau \sigma \lambda^2}{128 \pi^3 h^2} \exp\left[-4 \int_0^h \kappa \, dh \right] \int_\vartheta \int_\varphi G^2(\vartheta, \varphi) \sin\vartheta \, d\varphi \, d\vartheta , \tag{1}$$

where P_s is the received power of the reflected signal; P_t is the peak power of the transmitter; c is the speed of light in vacuum; τ is the pulse duration; σ is the effective area of scattering per unit of pulse volume; λ is the transmitter wavelength; $G(\vartheta, \varphi)$ is the antenna gain in comparison with isotropic; κ is the absorption coefficient of the radio wave in the ionosphere; h is the virtual height of reflection; ϑ and φ are the polar angle and azimuth in spherical system of coordinates which polar axis coincides with a direction of a maximum of radiation. The exponential factor in the equation (1) considers into account attenuation of a wave in the ionosphere at $2h$ distance.

The differential and total cross–section for backscatter

Calculation of effective section of scattering σ is made on the basis of a perturbation theory which is used in two forms: a method of small perturbations and the theory of the single scattered field [Brunelli et al., 1979]. Following the plan of calculation suggested in [Brunelli et al., 1979], at first we find the differential section of scattering $d\sigma$ determined as the power scattered through 180° per unit scattering volume per unit solid angle per unit frequency range and per unit incident power.

For small fluctuations of plasma the operator of dielectric permeability $\hat{\boldsymbol{\varepsilon}}$ it is possible to present as:

$$\hat{\boldsymbol{\varepsilon}} = \hat{\boldsymbol{\varepsilon}}(\bar{s}_k) + \frac{\partial}{\partial s_k} \hat{\boldsymbol{\varepsilon}}(s_k)_{|\delta s_k = 0} \, \delta s_k , \tag{2}$$

where the electrodynamics parameters of the medium are designated in symbolic form s_k, for example, s_0 is the density, s_1 is the magnetic field, and etc; $\bar{s}_k$ is the mean value; δs_k is the deviation from the mean.

The mechanism for the appearance of scattered waves in mediums with random irregularities reduces to occurrence in them of induced charges and current under the influence of the field of the so-called incident wave. The fluctuation additive of these quantities can become a source of radiation of new waves with frequencies and wave vectors, (i.e., propagation direction) differing from the frequency and wave vector of the initial wave. The so-called scattering process occurs.

Assuming that the scattering is weak, the expression for the differential cross–section $d\sigma$ at the longitudinal (relative to the geomagnetic field) propagation of the ordinary or extraordinary radio-wave can be written in the following form:

$$d\sigma_{1,2} = \frac{r_e^2}{2\pi} \xi(\omega_0, \omega_1, \bar{s}_i, \bar{s}_j) \left\langle \delta s_i \, \delta s_j \right\rangle_{\mathbf{q}\omega},$$

$$\xi_{1,2} = N_e^2 \frac{\omega_1^4}{\Omega_e^4} \frac{n_{2,1}(\omega_1)}{n(\omega_0)} \frac{\partial n^{2*}}{\partial \bar{s}_i} \frac{\partial n^2}{\partial \bar{s}_j},$$

(3)

where $r_e = e^2 / m_e c^2 = 2{,}8 \cdot 10^{-13}$см is the classical electron radius; e, m_e and N_e are the electron charge, mass, and concentration; $\omega = \omega_1 - \omega_0$ is the frequency Doppler shift due to random thermal motion of the charges, where ω_0 and ω_1 are the angular frequencies of the incident and scattered waves; $\mathbf{q} = \mathbf{k} - \mathbf{k}_0$ is the scattering vector, where $\mathbf{k}_0$ and $\mathbf{k}$ are the wave vectors of the incident and scattered waves; Ω_e is the electron plasma frequency; $n_{1,2}$ is the refractive index of the ordinary (n_1) or extra ordinary (n_2) wave; $\left\langle \delta s_i \, \delta s_j \right\rangle_{\mathbf{q}\omega}$ is the space-time spectrum of the statistical fluctuations of plasma parameters. The angular brackets denote the expected value or ensemble average. The asterisk means complex conjugation.

The formula (3) is valid for any frequency shift at scattering, it is supposed only, those frequencies ω_0 and ω_1 significantly surpass Ω_e. In the most interesting case of Rayleigh scattering, i.e. scattering with small shift of frequency $(\omega << \omega_0)$, factor ξ is converted in function, which is not dependent on shift of frequency. In this case the differential section of scattering can be integrated on frequencies, using the following relation:

$$\int_{-\infty}^{\infty} \left\langle J_i \, J_j \right\rangle_{\mathbf{k}\omega} d\omega = 2\pi \left\langle J_i \, J_j \right\rangle_{\mathbf{k}},$$

(4)

where $\left\langle J_i \, J_j \right\rangle_{\mathbf{k}}$ is the spectral distribution of spatial correlation function.

In result the integrated scattering coefficient will be equal to

$$\sigma_{1,2} = \frac{\omega_0^4}{4\pi^2 c^4} n_{2,1} \, n^* \left\langle \left| \delta n \right|^2 \right\rangle_{\mathbf{q}}.$$

5)

This expression defines the power re-emitted by a unit volume of plasma in a unit solid angle per unit of energy flux density of the incident wave, i.e., the total scattering cross-section.

If fluctuations of the refractive index are determined, basically, fluctuations of the electron density δN_e, then (5) will become

$$\sigma_{1,2} = \frac{\omega_0^4}{4\pi^2 c^4} n_{2,1} \, n^* \left| \frac{\partial n}{\partial N_e} \right|^2 \left\langle \delta N_e^2 \right\rangle_{\mathbf{q}}$$

(6)

One of the main features of electromagnetic wave scattering in irregularities is the selective nature of the scattering. To demonstrate, the intensity of the scattered field is proportional to the spatial spectrum of the electron density fluctuations, more precisely, to only one of its harmonics with the wave number q. Furthermore, the wavelength of this spatial harmonic satisfies the Bragg-Wolf condition

$$\lambda_q = \frac{2\pi}{|\mathbf{q}|} = \frac{\lambda}{2\sin\frac{\theta}{2}},$$

(7)

where λ is the wavelength of the incident emission, λ_q is the wavelength of that harmonic at which scattering occurs, and θ is the angle between $\mathbf{k}_0$ and $\mathbf{k}$. When the relation (7) is satisfied waves scattered in direction $\mathbf{k}$ by irregularities spaced λ_q apart will be added in phase.

Calculation of the ionospheric parameters

The radiowaves which frequency considerably surpasses the maximal plasma frequency of the ionosphere are used in the incoherent scattering method, and the basic source of the scattered field are thermal fluctuations in plasma, which theory is well enough developed [Brunelli et al., 1979; Suni et al., 1989]. According to these papers the total scattering cross-section in the lower ionosphere (the D region), consisting of electrons, and positive and negative ions, is defined by the expression.

$$\sigma = \frac{r_e^2 N_e}{\left(1+q^2 D_e^2\right)} \left\{ q^2 D_e^2 + \frac{1+\lambda^-}{2\left(1+\lambda^-\right)+q^2 D_e^2} \left[1 + \frac{\lambda^-}{1+\lambda^-} \right] \right\},$$

(8)

where $q = \left| 2\mathbf{k}_0 \right|$; $D_e = \sqrt{T / 4\pi N e^2}$ is the electron Debye length; $\lambda^- = N_i^- / N_e$; T_e is the electron temperature (in energy units); N_i^- is the concentration of negative ions.

It follows from (8) that the total power of the scattered emission increases as the concentration of negative ions increases. Numerical estimates made in [Ivanov and Tereschenko, 1983] for a probe frequency of 224 MHz and parameters of the medium corresponding to actual conditions in the ionosphere at heights of 70-80 km show that the total scattering cross-section may increase by 50% due to negative ions at $\lambda^- = 1$, and by 80% at $\lambda^- = 10$. The increase in the scattered power occurs in the ion component of the spectrum and is caused by an increase in the effective number of electrons involved in screening of other particles. If the radar wavelength is significantly greater than double the Debye circumference of the electrons ($q D_e << 1$), then the scattering process in such a medium can also be explained using the physical principles forming the basis for the test particles method [Rosenbluth and Rostoker, 1962]. According to this method, any charged particle, viewed as a probe charge, polarizes the plasma and is covered by a screening cloud appearing due to the excess of charges with the opposite sign and the decrease in the charge with the same sign as the particle. Since polarization of the plasma takes time, only those particles with velocities of the order of the velocity of the test charge contribute to the screening. To calculate the scattering characteristics, it is sufficient to add the contributions from all test particles.

An electron moving in the plasma is covered by a screening cloud by means of displacement of an equivalent electron, and scattering does not occur on it. Both electrons and positive and negative ions react to the ions. The structure of the screening cloud is obtained from the requirement that the total charge, including the test ion, should be equal to zero. Screening of a positive ion occurs due to part of the electron and the negative ion, which appear in the Debye sphere under the influence of attraction, and losses of part of the positive ion caused by charge repulsion. The screening cloud around negative ions containing a surplus of positive charges and a deficit of negative charges is formed in a similar manner. Thus, a positive ion behaves like $1/2\left(1+\lambda^-\right)$ of an electron relative to the incident wave, while a negative ion behaves like $1/2\left(1+\lambda^-\right)$ of an electron "hole". Since the electron and the "hole" have identical masses, they react to the probe wave in a similar manner.

The first component in (8) describes scattering on the screened electrons, the second – on the screened positive ions, and the third – on the screened negative ions when the charged particles of plasma are considered as test charges.

Consequently, the appearance of negative ions may lead to a change in the functional relation between the total scattered power and the electron concentration in the ionosphere. The increase in the total scattered power is caused by damping of the Coulomb interaction between electrons and positive ions due to partial screening of the latter by negative ions. From the equations (1) and (8) it is visible, that if to measure the reflected power basically it is possible to determine the high-altitude structure N_e or N_i^-. Thus the total section of scattering will be approximately twice more, than in absence of negative ions.

The radiowaves which frequency is below local plasma frequency are used in the partial reflection method. Most experiments have been made in the frequency range 2-6 MHz. In this paper the terminology partial reflection is used to contrast with total reflection. Although some of these partial reflections are probably indeed due to Fresnel reflection from coherent gradients in electron density, others may be caused by scattering from turbulent irregularities. The mechanism of this reflection is known not up to the end.

Fresnel reflection results from the presence of irregularities in refractive index transverse to the radio wave propagation direction which are thin compared to the radar wavelength. For vertical incidence, true Fresnel reflection requires that the horizontal extent of the irregularity be greater than one Fresnel zone, which is $(\lambda \cdot h)^{1/2}$, and that the vertical extent of the irregularity be less than about $\lambda/4$. In practice, the minimum horizontal extent of the irregularity needs only be greater than that of the radar beam. Fresnel scatter results when the scattering medium is coherent in the two dimensions transverse to the probing wave, and random in the direction parallel to the radiowave vector.

To obtain the concentration as a function of a height apply radiation of two wave modes of circular polarization as alternating pulses and separate reception of signals. The ratio of the powers of the received ordinary and extraordinary waves is given by expression

$$P_{S2} / P_{S1} = (\sigma_2 / \sigma_1)\exp\left[-4\int_0^h (\kappa_2 - \kappa_1)\mathrm{d}h \right], \quad (9)$$

where $\sigma_2 / \sigma_1 = (n_2^* / n_1^*)^2 \left| R_2 / R_1 \right|^2$; $R_{1,2} = \delta n / 2n$ are the reflection coefficients for scattered waves. The exponential term gives differential absorption of the magneto-ionic components scattered from a height h and depends on the electron density and collision frequency below this height. The ratio P_{S2} / P_{S1}, determined by the wave frequency, the gyrofrequency, and electron collision frequency, but is independent of the actual electron density or the size and shape of the irrequlарties [Belrose, Burke, 1964]. When P_{S2} / P_{S1} is observed, as a function of height, and collision frequency is known, the electron density may be determined by differentiation of above equation.

Conclusion

A derivation is given of the general formulas for differential and total scattering cross-section in the lower polar ionosphere (the D region), and the application of the results are discussed.

The study was supported by the RFBR grant № 07-05-00012.

References

Belrose, J.S., Burke, M.J. Study of the lower ionosphere using partial reflection. 1. Experimental technique and methods of analysis, *J. Geophys. Res.*, 69(13), 2799-2818, 1964.

Booker, H.G. Radio scattering in the lower ionosphere, *J.Geophys. Res.*, 12(12), 2164-2177, 1959.

Bryunelli, B.E., Kochkin, M.I., Presnyakov, I.N., Tereshchenko, E.D., Tereshchenko, V.D. *The incoherent radiowaves scattering method*, 188 pp., Nauka, Leningrad, 1979.

Flood, W.A. Revised theory for partial reflection D-region measurements, *J.Geophys. Res.*, 73(17), 5585-5597, 1968.

Holt, O. Methods of observation of D region, in *Exploration the polar upper atmosphere.* 134-146, D. Reidel Publishing Company, Dordrecht, Holland, 1981.

Ivanov, A.A., Tereshchenko, V.D. Influence of negative ions on the characteristics of incoherent scattering of radio waves, *Geomagnetizm i aehronomiya*, 23(5), 759-763, 1983

Rosenbluth, M.N., Rostoker, N. Scattering of electromagnetic waves by a nonequilibrium plasma, *Phys. Fluids*, 5(7), 776-788, 1962.

Suni, A.L., Tereshchenko, V.D., Tereshchenko, E.D., Khudukon B.Z. *Incoherent Radio wave scattering in the high-latitude ionosphere*, 184 pp., KSCRAS, Apatity, 1989.

Sheffield, J. *Plasma scattering of electromagnetic radiation*, 305 pp., Academic Press, New York, London 1975.