"Physics of Auroral Phenomena", Proc. XXXI Annual Seminar, Apatity, pp. 21- 24, 2008

Polar Geophysical Institute

RELATIVE ORDER IN THE CELLULAR AUTOMATA MODEL OF THE MAGNETOSPHERIC-IONOSPHERIC SYSTEM

Boris V. Kozelov [1,2]

[1] *Polar Geophysical Institute, Murmansk region, Apatity, 184209 Russia*
[2] *Department of Physics and Technology, University of Tromsø, Tromsø, Norway*

Abstract. The cellular automata model (CAM) was proposed in [Kozelov and Kozelova, 2003] to describe the substorm activity of the magnetospheric-ionospheric system. The state of each cell in the model is described by two numbers that correspond to the energy content in a region of the current sheet in the magnetospheric tail and to the conductivity of the ionospheric domain that is magnetically connected with this region. The driving force of the system is supposed to be provided by the solar wind that is convected along the two boundaries of the system. The energy flux inside is ensured by the penetration of the energy from the solar wind into the array of cells with a finite velocity. The third boundary is closed and the fourth boundary is open, thereby modeling the flux far away from the tail. The energy dissipation in the system is quite similar to other CAM models; however the second number attributed to each cell mimics the ionospheric conductivity that can allow for a part of the energy to be shed on field-aligned currents. The feedback between "ionosphere" and "magnetospheric tail" is provided by the change in a part of the energy, which is redistributed in the tail when the threshold is surpassed. The control parameter of the model is the south component of the interplanetary magnetic field (B_S IMF). It is known that the dynamics of the system undergoes several bifurcations, when the control parameter varies [Kozelov and Kozelova, 2003].

Here we analyze the relative order of the system states as a function of time and the control parameter. An approach based on the S-theorem by Yu.L. Klimontovich has been used. The considered characteristic is an analogy of entropy which has been extended to non- equilibrium states.

We conclude that: 1) for fixed control parameter the order in the system increases as a new transient develops; 2) the strongly driven system is more ordered.

1. Introduction

Ubiquitous signatures of complex dynamic behavior of space plasma are often referred to as 'turbulence', meaning strong coupled fluctuations at wide range of scales [Borovsky et al., 1997, Borovsky and Funsten, 2003]. However even for classical turbulent flows theories are incomplete [Frisch, 1995]. The self-organized criticality (SOC) paradigm was suggested to describe the plasma complexity [Consolini, 1997; Angelopoulos et al., 1999; Chang et al., 2004]. As yet, there is no known direct theoretical correspondence between SOC and turbulence, but there are manifestations of both SOC and turbulence in the same data sets [Kozelov et al., 2004; Kozelov and Rypdal, 2007; Uritsky et al., 2006, Rypdal et al., 2008]. Therefore, it would be important to demonstrate that the dynamics during these events represents organization, and not disorganization. The problem is that the turbulent systems (like the magnetosphere-ionosphere system) are open and non-equilibrium, and thus classical thermodynamics is not directly applicable. The great diversity in statistical distributions observed in complex systems is anomalous from the viewpoint of traditional statistical mechanics based on the Boltzmann-Gibbs-Shannon entropy. Consequently, some generalization is needed.

In this contribution we use an approach based on the S-theorem by Yu. L. Klimontovich [1996]. This approach allows us to compare the order which characterize the current (non equilibrium) state of the system with experimental data. The considered characteristic is an analogy of entropy extended to non equilibrium states. The main idea of the approach is that the order of two different states of an open system should be compared for the same average energy of the system. One of the states should be selected as a state of 'physical chaos' and the distribution characterizing the state should be 'heated' to the same average energy as the second state. Then, the entropy of the states can be compared.

Previously the approach was applied to the auroral structure observed at the Barentsburg observatory (Svalbard) during substorm transients [Kozelov and Rypdal, 2007].

2. Formalism of the criterion

The criterion of relative order for the states of a system was developed in the works of Yu. L. Klimontovich [1995-1998]. The brief overview of the formalism was presented in [Kozelov and Rypdal, 2007].

A system state be described by the distribution function $f(x, a)$, where x is an intrinsic parameter of the distribution and a is a governing parameter. Let us assume that the state with a=a_0 is a chaotic state and we want to compare the order of two states with a_0 and a_0+Δa for Δa >0. The two distributions correspond to the states:

$$f_0 = f(x, a_0), \ \ f = f(x, a_0+\Delta a),$$

where $\int f_0 dx = \int f\, dx = 1$. (1)

From the distribution f_0 we can find the function $H_{eff} = -\ln f_0$, which will play the role of an effective Hamiltonian. In the most common cases the average value of the effective energy obtained by the distributions (1) depend on Δa. Let us consider re-normalization to the same value $\langle H_{eff}\rangle$ by a re-normalized function $\tilde{f}_0$. The function we consider as a canonical distribution:

$$\tilde{f}_0(x, a_0, \Delta a) = \exp[D^{-1}(\Delta a)(F(D) - H_{eff}(x, a_0))] \quad (2)$$

$$\int \tilde{f}_0 dx = 1$$

The dependence of the effective free energy $F(D)$ on the temperature D is determined from the normalization condition for $\tilde{f}_0$. The dependence of the effective temperature D of the governing parameter Δa can be found from the equation

$$\int H_{eff}(x, a_0)\tilde{f}_0(x, a_0, \Delta a)\, dx = \\ = \int H_{eff}(x, a_0) f(x, a_0 + \Delta a)\, dx, \quad (3)$$

which is a condition of constant average value of effective Hamiltonian. The solution of the equation gives us the function

$$D(\Delta a); \qquad D(\Delta a)|_{\Delta a=0} = 1, \quad \Delta a \geq 0 \qquad (4)$$

If $D(\Delta a)>1$ for $\Delta a \neq 0$ then the state with $a_0+\Delta a$ is more ordered than the state with $a = a_0$, which we select as a state of physical chaos. However, to test this assumption the effective temperatures for processes in forward (from a_0 to $a_0+\Delta a$) and backward (from $a_0+\Delta a$ to a_0) directions should be compared. If the effective temperature for the backward process is less then 1, then the selection of the chaos state is valid. If the effective temperature in both cases is >1 then the self-organization evolves in the direction for which higher value of the temperature is obtained.

Then, the difference of entropies can be found by distributions $\tilde{f}_0$ and f as:

$$L_S = \tilde{S}_0 - S = \int f \ln(f / \tilde{f}_0)\, dX \geq 0 \qquad (5)$$

This value is used as a numerical characteristic to compare the relative order of the system states.

3. Model

The cellular automata model (CAM) was proposed in [Kozelov and Kozelova, 2003] to describe the substorm activity of the magnetospheric-ionospheric system. The state of each cell in the model is described by two numbers that correspond to the energy content in a region of the current sheet in the magnetospheric tail and to the conductivity of the ionospheric domain that is magnetically connected with this region. The driving force of the system is supposed to be provided by the solar wind that is convected along the two boundaries of the system. The energy flux inside is ensured by the penetration of the energy from the solar wind into the array of cells with a finite velocity. The third boundary is closed and the fourth boundary is open, thereby modeling the flux far away from the tail.

The energy dissipation in the system is quite similar to other CAM models; however the second number attributed to each cell mimics the ionospheric conductivity that can allow for a part of the energy to be shed on field-aligned currents.

The feedback between "ionosphere" and "magnetospheric tail" is provided by the change in a part of the energy, which is redistributed in the tail when the threshold is surpassed. The control parameter of the model is assumed to be the south component of the interplanetary magnetic field (B_S IMF).

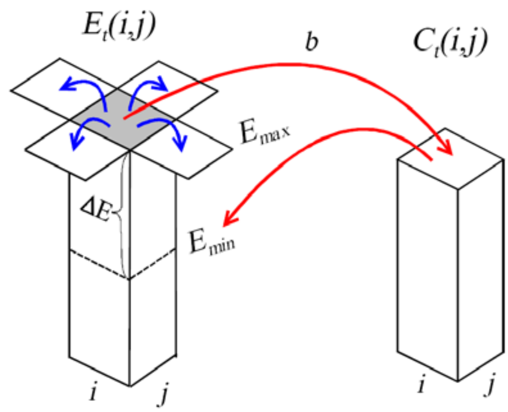

Fig. 1. The feedback between "ionosphere" and "magnetospheric tail" cells.

The model reproduce some morphological features of the auroral substorm dynamics. It is known that the dynamics of the system undergoes several bifurcations, when the control parameter varies from -0.6 to -6.0 nT [Kozelov and Kozelova, 2003].

4. Temporal evolution for fixed control parameter

Firstly, we consider the model dynamics with constant control parameter. Top panel of Fig.3 presents a model keogram for "ionospheric" activity with the control parameter B_S=3 nT. We discuss 100 min of the system dynamics observed after more than 1000 min of constant B_S. The horizontal axis is a time. The vertical axis is a distance from ionospheric projection of the inner edge of the plasma sheet. The whiter regions are corresponded to stronger activity (dissipation). The pseudo-periodical generation of transients and their 'poleward' motion are seen well. Parameters of this pseudo-periodic generation regime was discussed in [Kozelov and Kozelova, 2003]. It was shown that the starting point of transients shifts 'equatorward', and the period of generation decreases with B_S increase.

To characterize the system state at each moment t we choose the PDF of the cell energy in $E_t(i,j)$ array. The state for t=56 was used as the most chaotic one. Test of the effective temperature needed to justify the average energy supports this choice, see second panel of Fig.2.

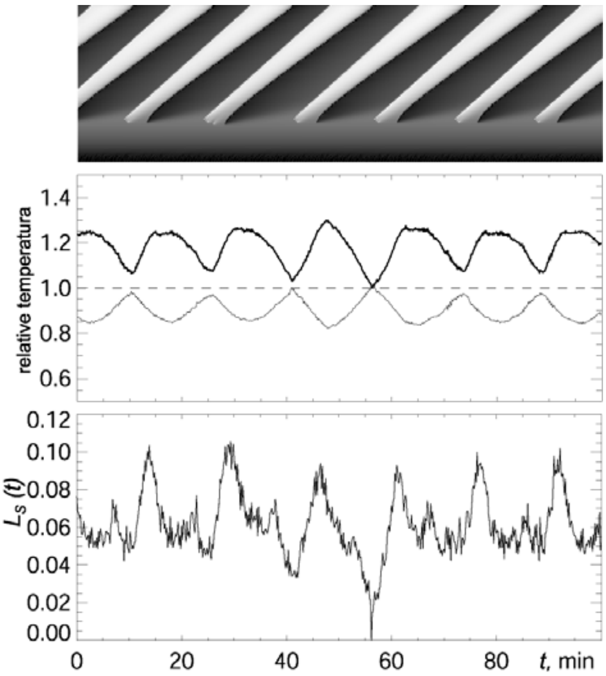

Fig.2. Top panel: model analogy of the N-S keogram, simulation for B_S=3 nT. Middle panel: evolution of effective temperature calculated for direct (black line) and backward (grey line) processes, dotted line - temperature of the state of 'physical chaos'. Bottom panel: evolution of the relative entropy.

The difference of entropies calculated by (5) is shown in bottom panel of Fig.2. One can see that the development of a new transient corresponds to increase of the entropy difference. This means that the relative order in the system increases. Then, the developed transient is only moving out of system and the relative order come back to smaller values.

5. Dependence on external control parameter B_S

Let we compare the system states at different (but time constant) control parameter. The states are characterized by distribution of the total (integrated over *i* and *j*) energy in $E_t(i,j)$ array. For each B_s value the distribution has been obtained by long enough simulation the model dynamics. The distributions are shown in Fig.3 (top panel). With B_s value increase one can see not only an expansion to larger energy values but and changes of the distribution shape.

We choose the system state at smallest value, B_s=1 nT, as a state of physical chaos. Test of the effective temperature described in Section 2 supports this choice, see second panel of Fig.3.

The difference of entropies calculated by (5) is shown in bottom panel of Fig.3. In general, the entropy difference increases with the control parameter increase. From B_s=1 nT to B_s=4 nT this dependence is monotonous. This range of the control parameter corresponds to pseudo-periodic generation regime [Kozelov and Kozelova, 2003]. As it was mentioned in [Kozelov and Kozelova, 2003], for B_s>4 nT the generation regime looks like a more chaotic one. This assumption is supported by decrease of effective entropy at B_s =4.5 nT and B_s =5 nT. This means that this generation regime is more effective way of entropy production in comparison with the pseudo-periodic one. However for B_s>4 nT the entropy difference also increases with B_s. By analogy with other bifurcated systems we can assume that the model system has more complex dependence on control parameters B_s>4 nT if less B_s step is considered.

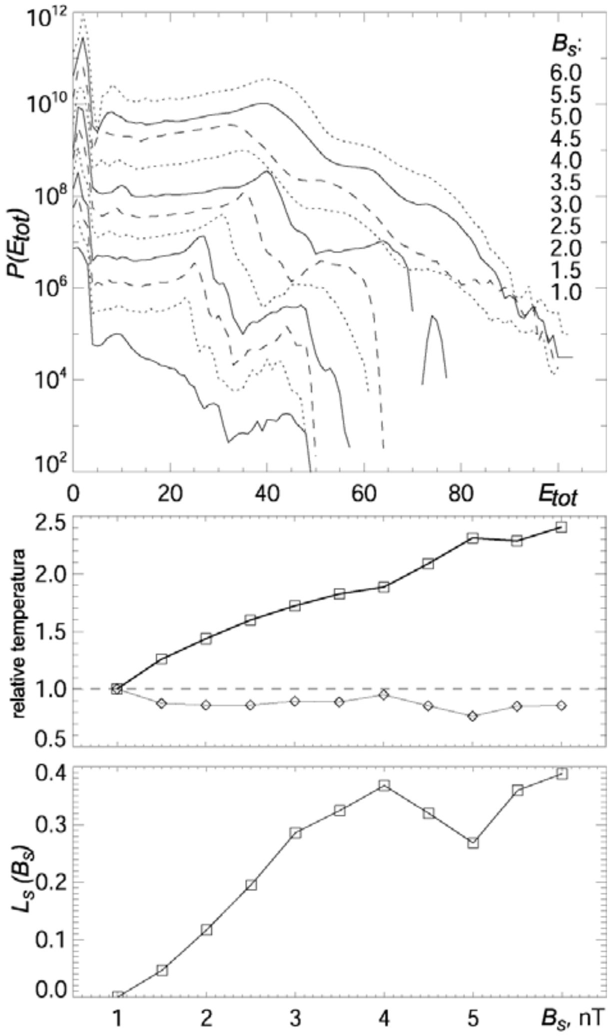

Fig.3. Top panel: distribution of total redistributed energy. Middle panel: evolution of effective temperature calculated for direct (black line) and backward (grey line) processes, dotted line - temperature of the state of 'physical chaos'. Bottom panel: evolution of the entropy difference.

6. Discussion and conclusions

Recently the Klimontovich' approach was applied to the auroral structure observed at the Barentsburg observatory (Svalbard) during substorm transients [Kozelov and Rypdal, 2007]. That study was focused on the dynamics of the magnetosphere-ionosphere system during the observed disturbance; and the time *t* was considered as a governing parameter. The relative order at different spatial scales was also discussed. The obtained increase of the order of the aurora during substorm activation is in qualitative agreement with Klimontovich' conclusion that the turbulent state is more ordered than a laminar state for flows.

Here the approach based on the S-theorem by Yu. L. Klimontovich has been applied to dynamics of the cellular automata model of the magnetospheric-ionospheric substorm activity. The relative order of the model system states has been considered depending on time and on external driving parameter. According to the criterion, for fixed control parameter the order of the system state increases as a new transient develops. The similar feature was obtained by the same approach to the aurora data [Kozelov and Rypdal, 2007].

The dynamical steady states observed at different control parameter have been compared also. It was found that in general the strongly driven system is more ordered. However further studies are needed to obtain relative entropy dependence on control parameter in details. We can suppose that the entropy production rate should also be taking into account in addition to relative entropy.

Acknowledgements. The work was supported by the Norwegian Science Council (grant 171076/V30) and by the Presidium of the Russian Academy of Sciences (RAS) through the basic research program "Solar activity and physical processes in the Sun-Earth system" and by the Division Physical Sciences of RAS through the program "Plasma processes in the solar system".

References

Angelopoulos, V., Mukai, T., and Kokubun, S.: Evidence for intermittency in Earth's plasma sheet and implications for selforganized criticality, Phys. Plasmas, 6, 4161–4168, 1999.

Borovsky, J. E., Elphic, R. C., Funsten, H. O., and Thomsen, M. F.: The Earth's plasma sheet as a laboratory for flow turbulence in high-b MHD, J. Plasma Physics, 57(1), 1–34, 1997.

Borovsky, J. E. and Funsten, H. O.: MHD turbulence in the Earth's plasma sheet: Dynamics, dissipation and driving, J. Geophys. Res., 108, 1284, doi:10.1029/ 2002JA009625, 2003.

Chang, T., S. W. Y. Tam, and C. C. Wu: Complexity induced anisotropic bimodal intermittent turbulence in space plasmas, Phys. Plasma, 11, 1287–1299, 2004.

Consolini G. Sandpile cellular automata and magnetospheric dynamics, Proc. of Cosmic Physics in the Year 2000, 58, edited by Aiello et al., SIF, Bologna, Italy, 123, 359–389, 1997.

Frisch, U.: Turbulence: The Legacy of A. N. Kolmogorov, Cambridge University Press, New York, 1995.

Klimontovich Yu.L.: A criterion of relative degree of order for open systems, USP. FIZ. NAUK, V.166, P.1231, 1996, in Russian.

Klimontovich Yu.L.: Statistical theory of open systems. V.1. Moscow: "Yanus", 1995, in Russian.

Klimontovich Yu.L.: Information concerning the states of open system, Physica Scripta, **58** 549-555 1998, doi:10.1088/0031-8949/58/6/002

Klimontovich Yu. L.: Criteria of Self-organization, Chaos, Solitons & Fractals, 5(10), 1985-2002, 1995.

Klimontovich Yu. L.: Relative ordering criteria in open systems, Physics-Uspekhi, 39 (11), 1169 - 1179, 1996.

Klimontovich Yu. L.: A criterion of relative degree of chaos or order for open systems, BioSystems, 42, 85-102, 1997.

Kozelov B.V. and T.V.Kozelova, Cellular automata model of magnetospheric-ionospheric coupling, Ann.Geophys., V.21, P.1931, 2003

Kozelov B. V., Uritsky V. M., and A. J. Klimas: Power law probability distributions of multiscale auroral dynamics from ground-based TV observations. Geophys. Res. Lett., V.31, L20804, 2004

Kozelov B. V. and K. Rypdal: Spatial scaling of optical fluctuations during substorm-time aurora. Ann. Geophys., 25, 915–927, 2007.

Kozelov B. V. and K. Rypdal: Relative order of auroral structure during substorm activation. Proceedings of 30th Annual Seminar "Physics of Auroral Phenomena", Apatity, 27 February-2 March 2007, ISBN 5-91137-032-8, P.38-41, 2007.

Rypdal K., B. Kozelov, S. Ratynskaia, B. Klumov, C. Knapek and M. Rypdal, Scale-free vortex cascade emerging from random forcing in a strongly coupled system, New J. Phys. V.10, 093018 (18pp), doi: 10.1088/1367-2630/10/9/093018, 2008.

Uritsky, V. M., Paczuski, M., Davila, J. M., and Jones, S. I.: Coexistence of Self-Organized Criticality and Intermittent Turbulence in the Solar corona, arXiv:astro-ph/0610130 v1, 4 October 2006..