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A METHOD OF DETECTING SOLITONS AMONG GEOPHYSICAL SIGNALS

N.G. Mazur[1], V.A. Pilipenko[2], and K-H. Glassmeier[3]

[1]*Institute of the Physics of the Earth, Moscow (n.g.mazur@mtu-net.ru)*
[2]*Space Research Institute, Moscow (pilipenko_va @mail.ru)*
[3]*Institut für Geophysik und extraterrestrische Physik, Germany (kh.glassmeier@tu-bs.de)*

Abstract. A method of detecting solitons and determine their parameters based on the scattering problem solution for the relevant nonlinear equation is developed. As an example the Derivative Nonlinear Schrödinger (DNLS) equation has been considered. The integral reflection coefficient, which should rapidly drop when a signal is close to N-soliton profile, has been used as a soliton detector. Application of this technique to numerically simulated signals shows that it is more efficient than standard Fourier transform and can be used as a practical tool for the analysis of outputs from nonlinear systems.

Introduction: Solitons in geophysical media

Nonlinear waves and solitons are commonly observed in various geophysical media: the interplanetary space [*Ovenden et al.,* 1983], near-Earth plasma [*Patel and Dasgupta,* 1987; *Baumgärtel,* 1999], atmosphere [*Shen,* 1966; *Pelinovsky and Romanova,* 1977; *Petviashvili and Pokhotelov,* 1992], Earth's crust [*Lund,* 1983]. Solitons are the basic structural elements of developed turbulence, because a disturbance with finite amplitude in a nonlinear medium commonly evolves to the soliton state. The modern theory predicts and has mathematical tools to describe N-soliton structures and soliton turbulence gas [*Gurevich et al.,* 2000; *Mazur et al.,* 2002]. The detection of soliton component and determination of its properties demands elaboration of special nonlinear methods of signal analysis. Standard methods of spectral analysis based on the Fourier transform (FT) fit well the detection of linear waves, but they are not very effective for the examination of highly structured space plasma turbulence.

The simplest approach is based on the determination of the statistical relationships between amplitudes, duration, velocity, etc. of the observed signal ensemble. Then, the comparison with the theoretically predicted relationships for a given soliton class may be used as a simple observational test for its identification [*Guglielmi et al.,* 1978].

However, the above simple statistical method of the soliton identification requires an analysis of substantial number of signals under the same external conditions. The method described in this paper can be applied to a single event. The proposed method is based on the idea of *Hada et al.* [1993] who suggested to apply the scattering transform (ST) to a complex time series of analyzed data instead of FT. We have built an effective numerical algorithm to implement the soliton transform. Below we give a short description of this algorithm, comprising calculations of discrete data of the scattering problem (otherwise, soliton parameters) and variation of spatial scale.

Derivative nonlinear Schrödinger equation

The derivative nonlinear Schrödinger (DNLS) equation

$$b_t + ib_{xx} + (|b|^2 b)_x = 0 \qquad (1)$$

may describe the nonlinear circularly polarized Alfvén wave $b = b_x + ib_z$, propagating along x-axis. Multi-soliton solution $b_N(x,t)$ of the equation (1) can be derived via elementary functions, although even under $N = 2$ the relevant formula is too cumbersome. One-soliton solution has the form:

$$b_{sol}(x,t) = a(\xi)\exp[i(\varphi_0 + \varphi(\xi) - 4|\lambda|^2 t)],$$

where

$$\xi = x - x_0 - 4\lambda_r(t - t_0),\ a^2 = \frac{8\lambda_i^2}{|\lambda|\cosh(4\lambda_i\xi) - \lambda_r},$$

$$\varphi = -2\lambda_r\xi + 3\arctan\left[\frac{\lambda_i}{|\lambda| - \lambda_r}\tanh(2\lambda_i\xi)\right],$$

i.e. one-soliton solution is determined by two independent real parameters λ_r, λ_i.

When the complex eigenvalue $\lambda = \lambda_r + i\lambda_i$ has been determined all the physical parameters of searched soliton can be found. The found eigenvalues enable one to determine with explicit formulas the "physical" parameters of solitons, such as amplitude A, non-linear component of velocity V, characteristic length L and duration T:

$$A^2 = 8(|\lambda| + \lambda_r),\ V = 4\lambda_r,$$

$$L = (4\lambda_i)^{-1},\ T = (16|\lambda_r|\lambda_i)^{-1}.$$

Integral reflection coefficient as a soliton detector

We demonstrate the proposed method using as an example DNLS solitons. The exact solution of the DNLS equation (1) may be reduced to the solution of linear problem with a well-elaborated algorithm. This algorithm is based on the solution of the direct and inverse scattering problems for the auxiliary linear system of *Kaup and Newell* [1978]:

$$\partial_x v_1 = -i\lambda v_1 + \sqrt{\lambda} b v_2 \ , \qquad \partial_x v_2 = i\lambda v_2 + \sqrt{\lambda} b^* v_1 \ . \qquad (2)$$

Discrete data of scattering are a finite set of complex eigenvalues, located in an upper half-plane, together with a set of complex phases of solitons. The method of their calculation is relied upon the fact that the discrete eigenvalues are zeros of the diagonal element a of the scattering matrix, considered as a function of complex spectral parameter λ. The following two-stage algorithm for the effective numerical calculations has been elaborated. At the first stage, the points on the real axis where $\mathrm{Re}\, a(\lambda)$ tends to zero are to be found. Then, starting from the points obtained, the contours of zero level of $\mathrm{Re}\, a(\lambda)$ are calculated in the upper half-plane. Calculations continue until $\mathrm{Im}\, a(\lambda)$ along such a contour tends to zero as well.

As a detector of DNLS solitons the integral reflection coefficient is used, that drastically decreases (theoretically — to zero) when an analyzed signal is close to the N-soliton profile. The integral reflection coefficient R is determined as an integral of absolute value of the reflection coefficient $r(\lambda)$ over a real axis, that is

$$R = \int_{-\infty}^{\infty} |r(\lambda)| d\lambda .$$

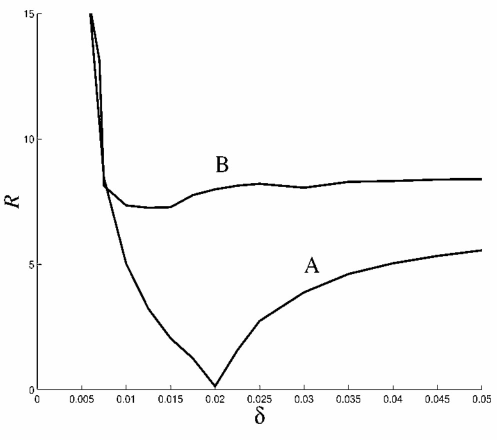

Fig. 1. Integrated reflection coefficient $R(\delta)$ for the exact one-soliton profile (1) (A) and for its imitation with the localized wave packet (2) (B).

For *N*-soliton spatial profile, which is a reflectionless potential, this value is exactly zero, whereas it is positive for any other distributions. Any changes in the spatial scale (that is changes of step δ over x in a numerical values of the profile under study) results in a change of the value R. For an exact *N*-soliton profile, these variations of scale yield a deep zero minimum of the function $R(\delta)$ for a correct scale δ. Thus, the integral reflection coefficient is very sensitive to a change of the linear scale. Therefore, if the analysis of an experimentally detected spatial profile with the use of the scale variation method gives a dependence $R(\delta)$ with an evident minimum, this may imply an occurrence of a substantial soliton component in this disturbance. As a by-product of this method, a correct value of the scale $\delta = \delta_{\min}$ is determined. Using the found value of δ one can calculate the discrete data of the scattering problem and retrieve a pure soliton part of the disturbance under study, using the known formulas for the N-soliton solution.

Discrimination of solitons and "linear" wave packets

Here we examine how well the proposed method of time series analysis can discriminate between an actual soliton and similar to it isolated wave packet. As a test we use the soliton $b_{sol}(x;\lambda)$ with eigenvalue λ=0.3+0.5i . The result of calculation (curve A in Fig.1) shows that the dependence *R*(δ) has a deep minimum, reaching zero under the same sampling step $\delta = \delta_S$ as the raw function $b_{sol}(x;\lambda)$ was determined. In due course, the linear wave packet may be described by the imitating function of the following form

$$b_{imit}(x) = \exp[-\gamma(x - x_0)^2][\cos(\theta - \theta_1) + i\sin(\theta - \theta_2)], \quad (3)$$

where $\theta = k(x - x_0)$. This function depends on many parameters, which enabled us to choose a function $b_{imit}(x)$ having nearly the same waveform as the exemplary solution $b_{sol}(x;\lambda)$. The waveforms of "soliton" $b_{sol}(x;\lambda)$ and "linear" $b_{imit}(x)$ functions are shown in Fig.2.

The calculation of the integral reflection coefficient *R*(δ) for the function $b_{imit}(x)$ has provided the following result (Fig.1). The plot *R*(δ) for the soliton-imitating signal (3) (curve B) is essentially different from the corresponding plot for actual soliton (curve A). Instead of deep near-zero minimum at $\delta = \delta_S$, the coefficient *R*(δ) has at all δ of the order of δ_S a plateau at rather high level. Thus, the proposed characteristics $R(\delta)$ has turned out to be very sensitive to a signal deviation from a soliton waveform.

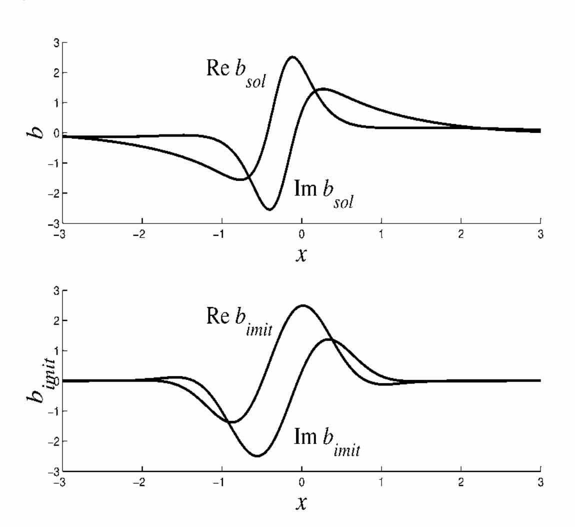

Fig. 2. Comparison of soliton $b_{sol}(x;\lambda)$ and "linear" signal $b_{imit}(x)$, which have been used for the calculation of integral reflection coeffi-

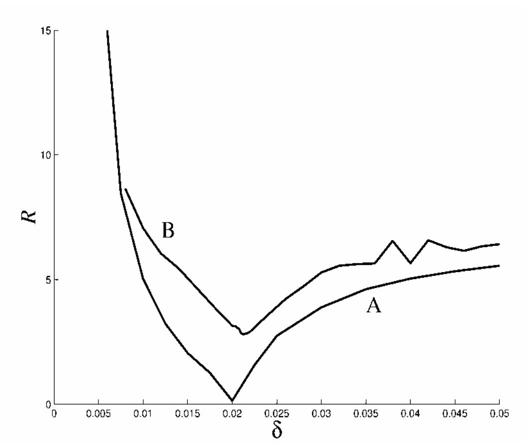

Fig. 3. Comparison of the integral reflection coefficient R(δ) estimated for the "noisy" soliton (B) and "pure" soliton (A).

The high-frequency noise influence on the soliton detector

To validate a robustness of the proposed technique to the possible occurrence of high-frequency noise in data, this method has been applied to the testing signal $b(x) = b_{sol}(x) + b_{pert}(x)$, consisting of a soliton $b_{sol}(x)$ with parameter λ=0.3+0.5i, and high-frequency interference signal $b_{pert}(x) = \alpha e^{2i\pi x}$ with amplitude α=0.3. The estimated reflection coefficient $R(\delta)$ for this "noisy" soliton is shown in Fig.3 (curve B). The comparison with pure soliton (curve A) shows that despite the noise occurrence the coefficient $R(\delta)$ still has a clear minimum in the vicinity of $\delta = \delta_S$.

The calculation of eigenvalue λ for a "noisy" soliton has been made for the sampling scale δ_S with the use of the algorithm described above. Fig.4 shows the zero-level contours $\mathrm{Re}\, s_1(\lambda) = 0$ and $\mathrm{Im}\, s_1(\lambda) = 0$; their intersection is the searched complex eigenvalue λ. For a relatively weak noise amplitude α =0.3 a perturbed eigenvalue does not shift far from a nominal eigenvalue for pure soliton $b_{sol}(x)$ (marked by a cross).

Possible applications and further studies

A similar approach after a minor modification can be applied for the detection of solitons described by other integrable nonlinear equations. The method developed here can be applied, in principle, to any equation with soliton-like solutions.

The application of the proposed technique to modeling signals shows its superiority over the standard FT. Complicated FT spectra of a soliton ensemble are substantially simplified after the use of the soliton transform, based on ST. A wave envelope that seems complicated to the FT may be a superposition of just a few solitons.

The suggested method is not limited by analysis of one-soliton profile b(x), but may be applied to the analysis of more complicated events with several interacting solitons. The technique effectively discriminates a multisoliton solution (N=1-5) from a non-soliton isolated disturbance (Gaussian packets).

Conclusion

Method to detect solitons and determine their parameters and propagation velocities, based on the solution of the scattering problem has been considered. The method developed here can be applied, in principle, to any equation with soliton-like solutions. We elaborated this technique for DNLS because this equation describes a wide range of non-linear phenomena in space plasma. As an example, we have constructed the algorithm of numerical solution of direct scattering problem for the linear system (2), associated in the inverse scattering techniques with the DNSL equation (1). The integral reflection coefficient has been used as a detector of the DNLS solitons, which steeply drops (theoretically — to zero) when an analyzed signal is close to the N-soliton waveform.

Application of this technique to numerically simulated signals showed that it is more efficient than standard Fourier transform and can be developed into a practical tool for the analysis of outputs from nonlinear systems. The application of the proposed technique to modeling signals shows its superiority over the standard FT. The technique effectively discriminates N-soliton solution (N=1-5) from non-soliton isolated disturbances (Gaussian packets). A wave envelope that seems complicated to the FT may be a superposition of just a few solitons, easily retrieved with the proposed method. This approach seems promising for the analysis of nonlinear signals in space physics, often detected in the solar wind, magnetosheath, auroral region, etc. This method enables one to determine using a single-point observations the basic parameters of soliton component of a disturbance, such as velocity, amplitude, duration, etc.

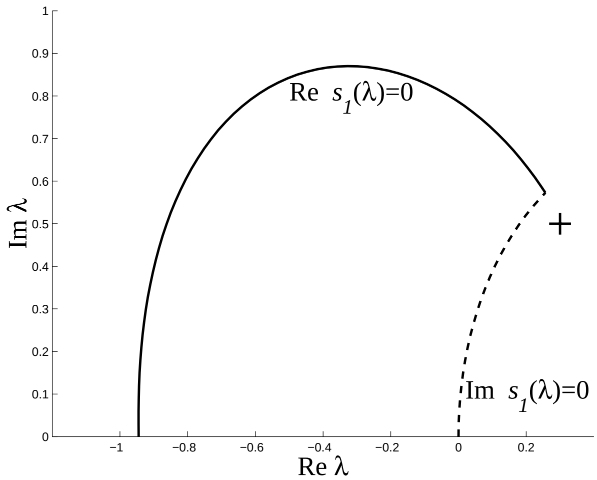

Fig. 4. An example of the eigenvalue λ finding for one-soliton profile, perturbed by high-frequency harmonic interference. The value λ for the exact one-soliton profile is marked with a cross.

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References

Baumgärtel K., Soliton approach to magnetic holes. J. Geophys. Res., 104, NA12, 28295-28308, 1999.

Hada T., R.J. Hamilton, C.F. Kennel. The soliton transform and a possible application to nonlinear Alfven waves in space, Geophys. Res. Lett., 20, 779-782, 1993.

Kaup D.J., A.J. Newell. An exact solution for a derivative nonlinear Schrödinger equation. J. Math. Phys., 19, 798-801, 1978.

Lund F. Interpretation of the precursor to the 1960 Great Chilean Earthquake as a seismic solitary wave, Pure Appl. Geophys., 121, 1, 17-26, 1983.

Ovenden C.R., Shah N.A., Schwartz S.J., Alfven solitons in solar wind, J. Geophys. Res., 88, NA8, 6095-6101, 1983.

Patel V.L., Dasgupta B. Theory and observations of Alfven solitons in the finite beta magnetospheric plasma, Physica, D27, N3, 387-398, 1987.

Petviashvili V.I., Pokhotelov O.A., Solitary Waves in Plasmas and in the Atmosphere, London, Gordon and Breach Science Publishers, 1992.

Shen M.C., Solitary waves in an atmosphere with arbitrary winds and density profiles, Phys. Fluids, 9, N10, 1966.

Guglielmi A.V., Bondarenko N.M., Repin V.N. Solitary waves in near-Earth environment, Doklady AN SSSR, 240, № 1, 47-50, 1978.

Gurevich A.V., Mazur N.G., Zybin K.P., Statistical limit in an completely integrable system under deterministic initial conditions, ZhETF, 117, N4, 797-817, 2000.

Mazur N.G., Georgdzaev V.V., Gurevich A.V., Zybin K.P., Statistical limit in the nonlinear Schrödinger equation solution under deterministic initial conditions, ZhETF, 121, N4, 971-990, 2002.

Pelinovsky E.N., Romanova N.N., Nonlinear stationary waves in the atmosphere, Izvestija AN SSSR (Physics of atmosphere and ocean), 13, №11, 1977.