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RESONANCE MHD-OSCILLATIONS IN A DIPOLE MAGNETOSPHERE WITH ROTATING PLASMA

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Abstract

A theory of Alfven resonance is developed for a magnetospheric model with a dipole magnetic field and rotating plasma. An equation is derived that describes a spatial structure of Alfven oscillations driven by a monochromatic fast magnetosonic wave. The structure of the disturbed magnetic field of resonant Alfven waves excited at different magnetic shells is investigated. It is shown that the phase of the azimuthal magnetic component changes non-monotonically near the resonant shell. This effect occurs only for the fundamental harmonic of standing Alfven waves. In the neighborhood of the magnetopause, where the velocity of plasma rotation rises abruptly, the scale of the phase change is comparable to the resonant peak width.

Introduction

A theory of Alfven resonance in the magnetospheric physics was formulated at first for simple magnetized plasma configurations in [1,2]. They showed that resonant interaction between Alfven waves and fast magnetosonic waves occurs in plasma that is inhomogeneous on one of the transverse (with respect to the magnetic field) coordinates.

For two-dimensionally inhomogeneous plasma in the model with dipole magnetic field the problem of Alfven resonance was developed in [3-5]. A fast magnetosonic wave was demonstrated to be able to excite the Alfven oscillation at the resonant magnetic shell. The position of the shell depends on the driving frequency and is determined by the equality of the Alfven eigenfrequency and frequency of a fast magnetosonic wave. Along field lines resonant oscillations are standing waves between magnetoconjugate ionospheres. Across the magnetic shells the amplitude distribution of Alfven oscillations is a typical resonant one.

It was shown in [6] that fast magnetosonic waves can also excite another branch of MHD waves - slow magnetosonic oscillations. Frequencies of slow magnetosonic and Alfven waves differ considerably. That is why effective interaction between these branches is impossible. Slow magnetosonic waves cannot be observed on the ground or close to the ionosphere because their magnitude steeply decreases along the field lines from the equatorial plane to the ionosphere. Moreover, in the real magnetosphere slow magnetosonic oscillations are overdamped.

In this paper we investigate the spatial structure of resonant Alfven waves excited by a monochromatic fast magnetosonic wave in a model dipole magnetosphere with moving plasma.

Environment model and main equation

In order to take into account the plasma motion, we use a self-consistent model of the magnetosphere with a dipole magnetic field and plasma rotating in the azimuthal direction [7]. We introduce an orthogonal curvilinear coordinate system attached to the magnetic field lines. The x^3 coordinate is directed along field lines, the x^1 coordinate is orthogonal to magnetic shells, and the x^2 coordinate points in the azimuthal direction so as to complete a right-handed system (Fig. 1).

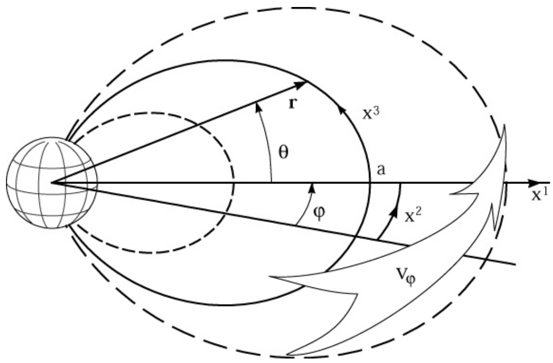

Figure 1. Curved orthogonal coordinates (x^1, x^2, x^3), related to magnetic filed lines and non-orthogonal coordinates (a, φ, θ) used in numerical study.

To examine the structure of resonant Alfven oscillations we will use the system of ideal MHD equations:

$$\frac{d\mathbf{v}}{dt} = -\nabla P + \frac{1}{4\pi}[(\nabla \times \mathbf{B}) \times \mathbf{B}],$$
$$\frac{\partial \mathbf{B}}{\partial t} = \nabla \times [\mathbf{v} \times \mathbf{B}], \qquad (1)$$
$$\frac{\partial \rho}{\partial t} = -\nabla(\rho \mathbf{v}), \qquad \frac{d}{dt}\frac{P}{\rho^\gamma} = 0,$$

where $\mathbf{B}$ is the magnetic field, $\mathbf{v}$ is plasma velocity, ρ and P are the plasma density and pressure respectively, γ is the adiabatic index, and $d/dt \equiv \partial/\partial t + (\mathbf{v}\nabla)$ represents the Lagrangian derivative in moving plasma. In a steady state ($\partial/\partial t \equiv 0$), the set of equations (1) describes the distribution of the plasma equilibrium parameters $\mathbf{B}_0, \mathbf{v}_0, P_0$ and ρ_0.

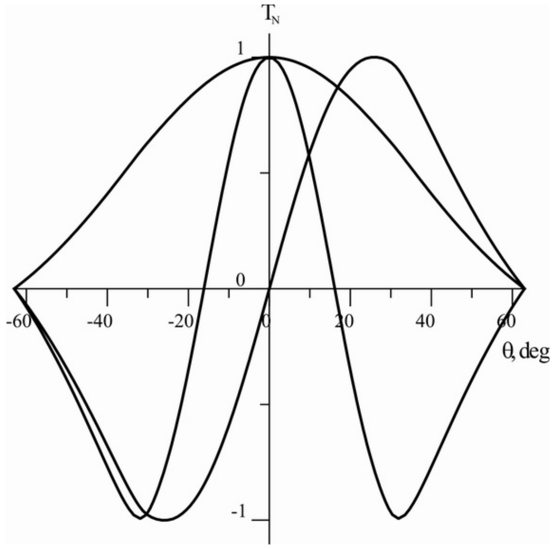

Figure 2. The structure of longitudinal harmonics $N = 1, 2, 3$ of resonant Alfvén oscillations at the shell $L = 6$ ($L = a / R_E$ — the McIlwain parameter of the magnetic shell with the equatorial radius a, R_E — the Earth radius).

The disturbed electric field of the oscillations can be represented as $\mathbf{E} = -\nabla\varphi + \nabla \times \boldsymbol{\psi}$, where the scalar potential φ corresponds to Alfvén waves and the vector potential $\boldsymbol{\psi} = (0, 0, \psi = \psi_F + \psi_S)$ — to magnetosonic oscillations, and ψ_F and ψ_S describes fast and slow magnetosonic oscillations respectively. Frequencies of Alfven waves are two orders of value higher than those of slow magnetosonic oscillations [6], therefore fast magnetosonic waves that are drivers for Alfvén oscillations cannot excite slow magnetosonic waves. Since Alfvén resonance is being considered, we will take in the further calculations $\psi_S = 0$.

After linearization of the set (1) relative to the smaller disturbances $\sim \exp(ik_2 x^2 - i\omega t)$ due to the MHD oscillations of plasma, we obtain:

$$\nabla_1 \hat{L}_T \left(\nabla_1 \varphi - (\nabla_1 \ln \overline{\omega})\varphi\right) - k_2^2 \hat{L}_P \varphi = \hat{L}_S \psi, \qquad (2)$$

where $\overline{\omega} = \omega - m\Omega$ is the oscillation frequency modified by Doppler effect, $\Omega = \Omega(x^1)$ is the angular speed of the plasma rotation, m is azimuthal wave number,

$$\hat{L}_T(\overline{\omega}) = \frac{\overline{\omega}^2}{A^2} p + \frac{1}{\sqrt{g_3}} \nabla_3 \frac{p}{\sqrt{g_3}} \nabla_3,$$

$$\hat{L}_p(\overline{\omega}) = \frac{\overline{\omega}^2}{A^2} p^{-1} + \frac{1}{\sqrt{g_3}} \nabla_3 \frac{p^{-1}}{\sqrt{g_3}} \nabla_3,$$

$$p = \sqrt{g_2 / g_1}.$$

The right-hand part of (2) is the driver of resonant Alfven oscillations — the field of monochromatic fast magnetosonic wave. The expression for the right-hand operator $\hat{L}_S$ and the equation describing fast magnetosonic waves are rather unwieldy and are not presented here. In the following calculations we will treat ψ as known.

Spatial structure of resonant Alfvén waves

We will consider the field-aligned structure of several first harmonics of standing Alfven waves. Typical field-aligned wave length of such oscillations is of the order of the field line length. The typical scale of resonant Alfven oscillations across magnetic shells is much smaller than their longitudinal wave length: $|\nabla_1 \varphi / \varphi| >> |\nabla_3 \varphi / \varphi|$ [8]. Therefore, a solution to (2) may be sought using the method of different scales, representing the potential in the form

$$\varphi = V(x^1) T(x^1, x^3) \exp(ik_2 x^2 - i\omega t), \qquad (3)$$

where the function $V(x^1)$ describes the small-scale transverse structure of oscillations along the x^1 coordinate in the main order, whereas the function $T(x^1, x^3)$ describes the oscillation structure along geomagnetic field lines.

Substituting (3) into (2), in the main order of perturbation theory we obtain longitudinal equation

$$\hat{L}_T(\Omega_N) T_N = 0. \qquad (4)$$

Boundary conditions on the ionosphere in the same approximation have a homogeneous form: $T_N(x_\pm^3) = 0$. The solutions of (4) are eigenfunctions T_N and corresponding eigenvalues $\overline{\omega} = \Omega_N$ ($N = 1, 2, 3...$ is the longitudinal wave number). Fig. 2 presents the structure of the first three longitudinal harmonics of resonant Alfvén waves. Fig. 3 shows the distribution of their eigenfrequencies $f_N = \Omega_N / 2\pi$ across the magnetic shells. The position x_{TN}^1 of magnetic shells where

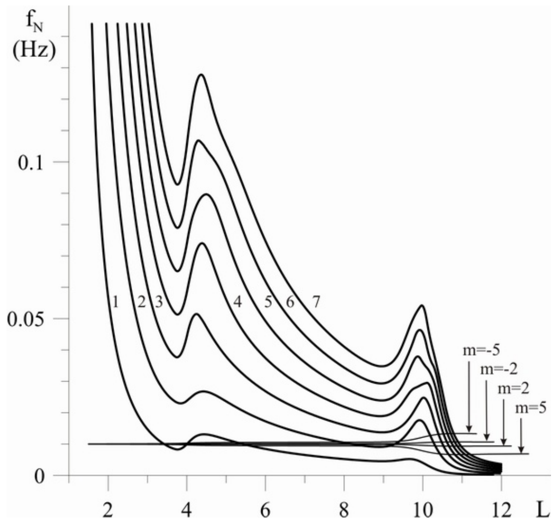

Figure 3. Distribution across the magnetic shells of the eigenfrequencies of resonant Alfvén waves for longitudinal harmonics $N = 1 - 7$ (thick lines) and of the Doppler-shifted frequency of a monochromatic fast magnetosonic wave with azimuthal wave numbers $m = \pm 2, \pm 5$ (thin lines).

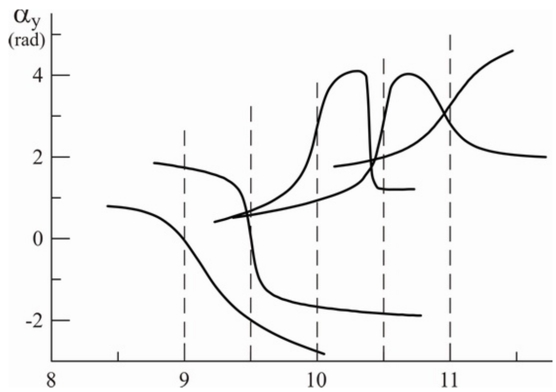

Figure 4. The behavior of the phase α_y of azimuthal magnetic component when the resonance condition is fulfilled at different shells.

fast magnetosonic waves with frequency ω can excite resonant Alfvén oscillations is determined by

$$\Omega_N(x^1_{TN}) = \omega - m\Omega(x^1_{TN}) .$$

In the next order of the disturbance theory (2) leads to an equation determining the structure of resonant Alfven oscillations across the magnetic shells:

$$\begin{aligned} &\nabla_1\left(\left(\bar{\omega} + i\gamma_N\right)^2 - \Omega_N^2\right)\nabla_1 V_N - \\ &-\left(\bar{\omega}^2 - \Omega_N^2\right)\left(\nabla_1 \ln \bar{\omega}\right)\nabla_1 V_N - \\ &-k_2^2\left[\alpha_N + \left(\left(\bar{\omega} + i\gamma_N\right)^2 - \Omega_N^2\right)\bar{\alpha}_N\right]V_N = i\mu_N \end{aligned} \qquad (5)$$

where γ_N is the decrement due to the dissipation of resonant waves in the ionosphere,

$$\alpha_N = \int_{\ell_-}^{\ell_+} T_N^2\left(\frac{\partial^2 p^{-1}}{\partial \ell^2}\right)d\ell , \quad \bar{\alpha}_N = \int_{\ell_-}^{\ell_+} \frac{T_N^2}{pA^2} d\ell ,$$

μ_N defines the magnitude of oscillations excited by a monochromatic fast magnetosonic wave. In the vicinity of the resonant shell the eigenfrequency can be presented in the form

$$\Omega_N \approx \bar{\omega}\left(1 - \frac{x^1 - x^1_{TN}}{L_N}\right),$$

where $L_N = \Omega_N / \nabla_1 \Omega_N$.

Introducing a dimensionless transverse coordinate $\xi = (x^1 - x^1_{TN})/L_N$ we can present (5) in the form

$$\begin{aligned} &\frac{\partial}{\partial \xi}(\xi + i\varepsilon)\frac{\partial}{\partial \xi}V_N + d_N(\xi + i\varepsilon)\frac{\partial}{\partial \xi}V_N - \\ &\qquad -\kappa_N^2\left(1 + \beta_N(\xi + i\varepsilon)\right)V_N = ib_N , \end{aligned} \qquad (6)$$

where

$$\varepsilon = \gamma_N / \Omega_N , \; d_N = -k_2\left(\nabla_1 \Omega\right) L_N / \Omega_N$$

$$\kappa_N = k_2 L_N \sqrt{\alpha_N} / \Omega_N , \; \beta_N = \bar{\alpha}_N \Omega_N^2 / \alpha_N .$$

Using Fourier transformation we find a solution to (6) in the form:

$$V_N(\xi) = -\int_0^\infty \frac{b_N \exp(ik(\xi + i\varepsilon))}{\sqrt{k^2 - id_N k + \kappa_N^2 \beta_N}} \times \left(\frac{\zeta(k) - i}{\zeta(k) + i}\right)^{a_N} dk , \qquad (7)$$

where

$$\zeta(k) = \frac{k\sqrt{\kappa_N^2 \beta_N + d_N^2/4}}{\kappa_N^2 \beta_N - ikd_N/2},$$

$$a_N = \frac{\kappa_N^2 + d_N/2}{\sqrt{\kappa_N^2 \beta_N + d_N^2/4}} .$$

The magnetic field in the vicinity of the resonance shell

When the spatial structure of the potential φ is found, the expressions for the physical components of the disturbed magnetic field can be obtained as follows:

$$B_x = \frac{B_1}{\sqrt{g_1}} = \frac{c}{\Omega_N}\frac{m}{\sqrt{g_2}}\frac{\partial T_N}{\partial \ell}V_N ,$$

$$B_y = \frac{B_2}{\sqrt{g_2}} = i\frac{c}{\Omega_N}\frac{1}{\sqrt{g_1}}\frac{\partial T_N}{\partial \ell} \times \left((\nabla_1 V_N) - \frac{m(\nabla_1 \Omega)}{\Omega_N}V_N\right)$$

In the neighborhood of the resonant shell ($\xi = 0$) function V_N has a logarithmic peculiarity while the derivative $\nabla_1 V_N$ has a stronger peculiarity $\sim \xi^{-1}$. Thus, the resonant Alfvén oscillations have toroidal polarization ($|B_2| >> |B_1|$).

Now we consider the behavior of the phase of the azimuthal component of the magnetic field $B_y = |B_y|\exp(i\alpha_y)$. It is known that the phase of azimuthal component of resonant Alfvén waves monotonically changes in the neighborhood of resonant magnetic shell and the phase shift is approximately π. Fig. 4 shows the transverse distribution of the phase α_y when the resonance condition is fulfilled at the shells $L = 9; 9{,}5; 10; 10{,}5; 11$. The phase α_y changes non-monotonically near the shell $L = 10$. It should be noted that the phase of the component B_x does not have that feature.

In the model magnetosphere we employed the shell $L = 10$ coincides with the magnetopause. In its vicinity the angular speed of plasma rotation changes rapidly and at the magnetopause the gradient of speed $\nabla_1 \Omega(x^1)$ is maximal. At the magnetic shell $L = 4$ (plasmopause) where the angular speed does not increase as steeply the phase of the component B_y changes monotonically. Therefore it can be supposed that the structure of the resonant Alfvén oscillations depends greatly on the value of the angular speed gradient at the resonant magnetic shell.

Fig. 5a represents the dependence of the phase α_y on azimuthal wave number m for the first longitudinal harmonic ($N=1$). The scale of non-monotonical changes decreases considerably while

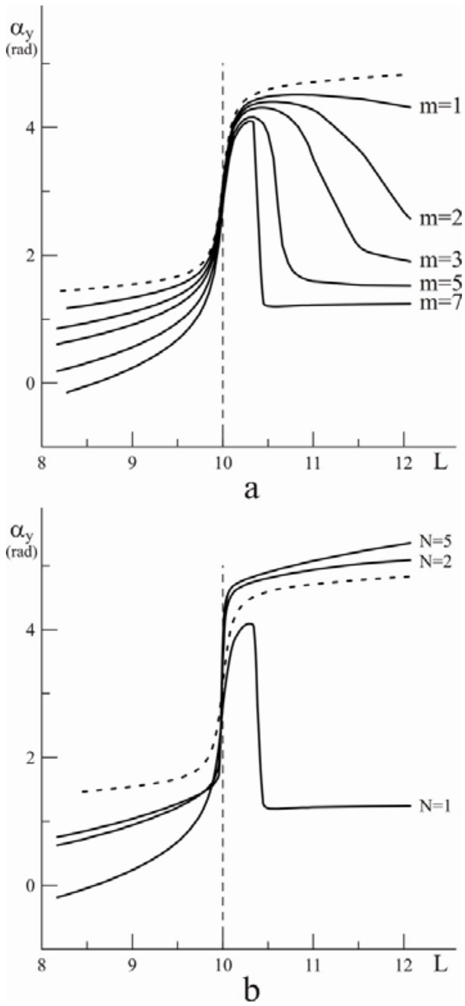

Figure 5. The dependence of the phase of the azimuthal magnetic component of resonant Alfvén waves (a) on the azimuthal wave number m for $N=1$ and (b) on the number of longitudinal harmonic N for $m=7$. The dashed line corresponds to a model with stationary plasma ($\Omega=0$).

m rises. Fig. 5b shows how the phase α_y changes when the number N increases. It is evident that the behavior of α_y becomes monotonical again for the harmonics $N \geq 2$.

Conclusion

The main results of this study may be summarized as follows.

1. The equation is derived that describes the structure of resonant Alfven waves in a model dipole magnetosphere with rotating plasma.

2. Solutions are found to the equations defining the structure of Alfven oscillations along magnetic field lines and across magnetic shells.

3. The magnetic field structure of Alfven waves is investigated in the neighborhood of resonant shell for different frequencies of an impinging fast magnetosonic wave. It is shown that the phase of the azimuthal magnetic component of the fundamental longitudinal harmonic changes non-monotonically if the resonance condition is fulfilled in a region where the angular velocity gradient is maximal.

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