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TRANSMISSION OF ALFVÉN WAVES THROUGH A STRONG SHOCK WAVE

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We investigate a transmission of Alfvén waves through the Earth's bow shock from the solar wind into the magnetosheath. For simplification, we assume that the Alfvén wave is a plane wave of small amplitude. We also consider the shock wave surface as a plane one. Besides we assume that the Mach and Mach – Alfvén numbers are very large (M_1, $M_{A1} \gg 1$) upstream from shock. The problem in similar statement was already investigated earlier.[1-4] However, our solution has three important distinctions. First, we use the more general geometry of a problem (McKenzie and Westphal[1] and Hassam[2] analyse the special cases when the wave vector of incident wave lies in a plane $(\vec{n},\vec{B})$ and is directed along $\vec{B}$, respectively). Second, all theoretically possible incident angles will be analysed (Hassam,[2] Pudovkin and Lubchich[3] analysed only the incident angles smaller than 61°, at which the wave vector of refracted fast magnetosonic wave is directed from a shock surface). Third, we take into account inertia of oscillating surface of a shock wave. Earlier (e.g., Ref. 2-4) this effect was not considered, which led to essential theoretical problems, namely the small oscillations of shock wave surface, considered in the problem as an individual linearly independent mode, actually were not linearly independent oscillations which lead to the conflict with the used linear approach. This problem has been investigated in detail by Lubchich and Pudovkin[5] where the necessity of the account of viscosity of medium inside a very thin front of a real shock wave was shown.

Let the YZ plane coincide with the surface of unperturbed shock, the plasma flow velocity $\vec{V}$ is directed along the X-axis (or along the normal $\hat{n}$ to the unperturbed shock surface), the wave vector $\vec{k}_A$ of incident Alfvén wave lies in the XY plane under the angle λ to X-axis, and the external (i.e., not wave) magnetic field $\vec{B}$ is directed arbitrarily in space (ψ is the angle between $\vec{B}$ and X-axis, α the angle between a projection of the magnetic field $\vec{B}_\tau$ on the shock plane and Y-axis).

Jump of the background (unperturbed) quantities on a strong shock wave is

$$B_{n2} = B_{n1}\,;\ \vec{B}_{\tau 2} = \frac{\gamma+1}{\gamma-1}\cdot\vec{B}_{\tau 1}\,;\ \vec{V}_{\tau 2} = \vec{V}_{\tau 1} = 0\,;\ \chi \equiv \frac{\rho_2}{\rho_1} = \frac{V_{n1}}{V_{n2}} = \frac{\gamma+1}{\gamma-1}\,;\ \frac{P_2}{P_1} = \frac{2\cdot\gamma}{\gamma+1}\cdot M_1^2 >> 1\,.$$

Here indexes 1 and 2 refer to the values in the solar wind and magnetosheath respectively, γ is the adiabatic index, i.e., we assume that the plasma state equation has the same form as perfect gas. Behind a strong shock wave we have

$$M_{2A} \sim M_{1A}, M_1, \text{ and } M_2 = \sqrt{\frac{\gamma-1}{2\cdot\gamma}} << M_1\,, \qquad (1)$$

as consequence, the plasma kinetic pressure is much greater than the magnetic pressure. In such medium the properties of magnetosonic wave are simplified. A fast magnetosonic wave converts into a sound wave:

$$V_{ph} = c_s\,;\ \delta P = c_s^2\cdot\delta\rho = \rho\cdot c_s\cdot\delta V_l\,;\ \delta\vec{B}\to 0\,.$$

By its properties a slow magnetosonic wave is close to the Alfvén wave, but has a different polarization:

$$V_{ph} = \pm V_{Al}\,;\ \delta V_j^{\pm} = \mp\frac{\delta B_j}{\sqrt{4\cdot\pi\cdot\rho}}\,;\ \delta P, \delta\rho \to 0\,.$$

As known, the polarization of the Alfvén wave is

$$V_{ph} = \pm V_{Ai}\,;\ \delta V_i^{\pm} = \mp\frac{\delta B_i}{\sqrt{4\cdot\pi\cdot\rho}}\,;\ \delta P, \delta\rho = 0\,. \qquad (2)$$

Here the index *l* refers to the component of vectors along a wave vector $\vec{k}$; the subscript *i* denotes the components along the vector $[\vec{k}\times\vec{B}]$, the index *j* refers to the components in the direction $[\vec{k}\times[\vec{k}\times\vec{B}]]$. Behind a shock wave it can also propagate an entropy wave for which

$$V_{ph} = 0\,;\ \delta P = 0\,;\ \delta\rho \neq 0\,.$$

The oscillations of shock wave surface propagate with the phase velocity $V_{ph} \equiv c_p = \frac{\omega}{k_y}$,

where ω and k_y are the frequency and tangential component of wave vector of incident wave in the reference frame connected with the unperturbed shock surface. The perturbation of medium parameters connecting with the shock surface oscillations is

$$\delta V_{x1} = \delta V_{x2} = -\delta V_{sh}\,;\ \frac{\delta V_{y1}}{V_{x1}} = \frac{\delta V_{y2}}{V_{x2}} = -\frac{\delta V_{sh}}{c_p}\,;\ \frac{\delta B_{x1}}{B_{y1}} = \frac{\delta B_{x2}}{B_{y2}} = \frac{\delta V_{sh}}{c_p}\,;\ \frac{\delta B_{y1}}{B_x} = \frac{\delta B_{y2}}{B_x} = -\frac{\delta V_{sh}}{c_p}\,, \text{ and} \qquad (3)$$

$$\frac{\delta P_{1(2)}}{\rho_1 \cdot V_{x1}} = 2 \cdot \left(\frac{1}{V_{x1(2)}} \cdot \frac{B_x \cdot B_{y1(2)}}{4 \cdot \pi \cdot \rho_{1(2)}} + c_p \right) \cdot \frac{\delta V_{sh}}{c_p} . \quad (4)$$

Here δV_{sh} is the velocity of oscillating surface of a shock wave along the X-axis. All authors (e.g., Ref. 2-4) take into account the perturbations (3). The conditions (3) describe the kinematic effects arising under a transition from a local reference frame connected with the perturbed shock surface to an initial frame related to the unperturbed plane shock. The pressure variations (4) are a consequence of dynamic effect associated with non-inertiality of the local reference frame. Usually authors disregard this effect and do that, in our opinion, unfounded. The condition (4) is written for a shock wave of arbitrary intensity. In case of a strong shock wave this equation is simplified:

$$\delta P_1 = \delta P_2 = 2 \cdot \rho_1 \cdot V_{x1} \cdot \delta V_{sh}$$

and coincides with the condition obtained earlier (Ref. 5) for a case of shock wave in usual (non-magnetic) hydrodynamics.

Generally, behind the front of the fast shock wave there arise six emanating waves. They are fast magnetosonic wave, forward and backward slow magnetosonic wave, forward and backward Alfvén wave, and entropy wave. Moreover the amplitude of shock surface oscillations is the seventh unknown quantity. The propagation direction for emanating waves is obtained from the conditions of continuity on a shock wave of the wave frequencies ω (in the reference system connected to the unperturbed discontinuity) and tangential components of wave vectors k_y. Behind the front of a strong shock wave five of six waves have a small phase velocity

$$\left| V_{ph} / V_2 \right| \leq V_{A2} / V_2 << 1$$

and consequently are propagating in the coincident direction

$$tg\lambda_{A\pm} = tg\lambda_{S\pm} = tg\lambda_E = \chi^{-1} \cdot tg\lambda . \quad (5)$$

The direction of wave vector of the fast magnetosonic wave is

$$\cos\lambda_F = \left(\cos\lambda_E \cdot \sqrt{M_2^2 - \sin^2\lambda_E} - \sin^2\lambda_E \right) \Big/ M_2 . \quad (6)$$

The Mach number M_2 is given by Eq. (1). At large incident angles λ and, hence, large refraction angle of entropy wave λ_e, the fast magnetosonic wave will be transformed to a surface wave damping when moving away from the discontinuity. However, Eq. (6) is also correct in this case, only the angle of λ_f will be a complex quantity.

After linearization of boundary conditions on a shock front we obtain a set of linear equations with respect to the small variations on two sides of the strong shock wave

$$\frac{\gamma}{\gamma-1} \cdot \frac{\delta P_2}{\rho_2 \cdot V_{x2}} + \frac{1}{2} \cdot V_{x2} \cdot \frac{\delta\rho_2}{\rho_2} + \left(\frac{3}{2} + \frac{2\cdot\gamma}{(\gamma-1)^2} \right) \cdot \delta V_{x2} = \chi \cdot \left\{ \frac{\gamma}{\gamma-1} \cdot \frac{\delta P_1}{\rho_1 \cdot V_{x1}} + \frac{1}{2} \cdot V_{x1} \cdot \frac{\delta\rho_1}{\rho_1} + \left(\frac{3}{2} + \frac{2\cdot\gamma}{(\gamma-1)^2} \right) \cdot \delta V_{x1} \right\},$$

$$\frac{\delta P_2}{\rho_2 \cdot V_{x2}} + V_{x2} \cdot \frac{\delta\rho_2}{\rho_2} + 2 \cdot \delta V_{x2} = \frac{\delta P_1}{\rho_1 \cdot V_{x1}} + V_{x1} \cdot \frac{\delta\rho_1}{\rho_1} + 2 \cdot \delta V_{x1} ,$$

$$V_{x2} \cdot \frac{\delta\rho_2}{\rho_2} + \delta V_{x2} = \frac{1}{\chi} \cdot \left(V_{x1} \cdot \frac{\delta\rho_1}{\rho_1} + \delta V_{x1} \right),$$

$$\delta V_{y2} = \delta V_{y1} , \quad (7)$$

$$\delta V_{z2} = \delta V_{z1} , \ \delta B_{x2} = \delta B_{x1} , \text{ and } \delta\vec{B}_{\tau2} = \chi \cdot \delta\vec{B}_{\tau1} . \quad (8)$$

As obvious, the equations are divided into two separate sets. The set (7) contains only the equations for the small perturbations of hydrodynamic quantities and completely coincides with the corresponding set from usual (non-magnetic) hydrodynamics. Eqs. (8) contain the Z component of velocity perturbations and magnetic field perturbations. The incident Alfvén wave is a transverse wave, and in our case it propagates with a small phase velocity $\left| V_{ph} / V_1 \right| \sim 1/M_{A1} << 1$. In the using approach it is possible to present the Alfvén wave as the sum of two independent "fictitious" waves propagating with zero phase velocity. One of them is the usual hydrodynamic vorticity wave carried the transverse perturbations of velocity and second wave is a “magnetic” wave carried the transverse perturbations of the magnetic field. We can consider the transmission of these two waves through the strong shock separately. We can also decompose the vorticity wave into two components linearly polarized in the XY plane and along the Z-axis, respectively. Obviously, their amplitudes are equal to $\delta V_{xy} = \delta V_{Al} \cdot \sin\varepsilon$ and $\delta V_z = \delta V_{Al} \cdot \cos\varepsilon$. Here ε is the angle between the velocity perturbations and the Z-axis. If the wave vector of incident wave is directed along the external magnetic field, the angle ε can be arbitrary. At any other orientation of $\vec{k}$ the angle ε is equal to

$$\cos\varepsilon = (\cos\lambda \cdot \sin\psi \cdot \cos\alpha - \sin\lambda \cdot \cos\psi) / \sin\theta ,$$

where θ is the angle between the wave vector of incident Alfvén wave and the external magnetic field $\vec{B}_1$. It can be obtained from the condition

$$\cos\theta = \cos\lambda\cdot\cos\psi + \sin\lambda\cdot\sin\psi\cdot\cos\alpha .$$

As follows from the solution of the system (7), the incident vorticity perturbations of δV_{xy} cause the generation of diverging fast magnetosonic wave with the amplitude equal to

$$\frac{\delta V_F}{\delta V_{Al}} = \sin\varepsilon\cdot\sin\lambda\cdot\frac{4\cdot M_2}{\gamma+1}\cdot\frac{\left(2\cdot\gamma^2+3\cdot\gamma+1\right)-2\cdot\gamma^2\cdot\chi\cdot\sin^2\lambda_E}{\left(2\cdot\gamma^2+3\cdot\gamma+1\right)+2\cdot(\gamma+1)^2\cdot M_2\cdot\cos\lambda_F-2\cdot\chi\cdot\sin^2\lambda_E},$$

the diverging entropy wave

$$\frac{\delta\rho_E}{\delta V_{Al}} = \frac{\rho_2}{V_{x2}}\cdot\left\{M_2\cdot\frac{\gamma^2+1}{\gamma^2-1}\cdot\frac{\delta V_F}{\delta V_{Al}}-\sin\varepsilon\cdot\sin\lambda\cdot\frac{4\cdot\gamma}{(\gamma+1)^2}\right\},$$

the shock surface oscillations

$$\frac{\delta V_{sh}}{\delta V_{Al}} = \frac{1}{4\cdot M_2}\cdot\frac{\delta V_F}{\delta V_{Al}}-\frac{\sin\varepsilon\cdot\sin\lambda}{\gamma+1},$$

and also the transmitted perturbations of velocity equal to

$$\frac{\delta V_{vor}}{\delta V_{Al}} = \frac{\sin\lambda_E}{\gamma+1}\cdot\left\{\left(\frac{\gamma-1}{\sin^2\lambda_E}+\frac{4\cdot\gamma^2}{\gamma^2-1}\right)\cdot\sin\varepsilon\cdot\sin\lambda-\frac{1}{M_2}\cdot\frac{1}{\gamma-1}\cdot\frac{\delta V_F}{\delta V_{Al}}\right\}.$$

Solving Eqs. (8), we obtain the vectors of "transverse" perturbations of velocity and magnetic field into the compressed medium

$$\frac{\delta\vec{V}_{tr}}{\delta V_{Al}} = \left(\frac{\delta V_{vor}}{\delta V_{Al}}\cdot\sin\lambda_E, -\frac{\delta V_{vor}}{\delta V_{Al}}\cdot\cos\lambda_E, \cos\varepsilon\right),$$

$$\frac{\delta\vec{B}_{tr}}{\delta B_{Al}} = \left(\sin\varepsilon\cdot\sin\lambda, -\chi\cdot\sin\varepsilon\cdot\cos\lambda, \chi\cdot\cos\varepsilon\right).$$

Here δV_{Al} and δB_{Al} are the perturbations of velocity and magnetic field carried by the incident Alfvén wave (see Eq. (2)). Since it is obvious that the vectors of "transverse" perturbations of velocity and magnetic field transmitting into the compressed medium are perpendicular to wave vectors of five emanating "slow" waves that propagated along the consistent directions (see Eq. (5)). The perturbations are carried by four waves, namely, the forward and backward Alfvén wave and forward and backward slow magnetosonic wave. The amplitude of waves is equal to

$\delta V^{tr}_{Al+} = 0.5\cdot\left(\delta\vec{V}_{tr}\mp\delta\vec{B}_{tr}\big/\sqrt{4\cdot\pi\cdot\rho_2}\right)\cdot\vec{i}_2$ for the forward Alfvén wave,

$\delta V^{tr}_{Al-} = 0.5\cdot\left(\delta\vec{V}_{tr}\pm\delta\vec{B}_{tr}\big/\sqrt{4\cdot\pi\cdot\rho_2}\right)\cdot\vec{i}_2$ for the backward Alfvén wave,

$\delta V^{tr}_{Sl+} = 0.5\cdot\left(\delta\vec{V}_{tr}\mp\delta\vec{B}_{tr}\big/\sqrt{4\cdot\pi\cdot\rho_2}\right)\cdot\vec{j}_2$ for the forward slow magnetosonic wave, and

$\delta V^{tr}_{Sl+} = 0.5\cdot\left(\delta\vec{V}_{tr}\pm\delta\vec{B}_{tr}\big/\sqrt{4\cdot\pi\cdot\rho_2}\right)\cdot\vec{j}_2$ for the backward slow magnetosonic wave.

Here $\vec{i}_2$ and $\vec{j}_2$ are the unit vectors along $\left[\vec{k}_E\times\vec{B}_2\right]$ and $\left[\vec{k}_E\times\left[\vec{k}_E\times\vec{B}_2\right]\right]$, respectively. The upper (lower) sign corresponds to incident of the forward (backward) Alfvén wave.

We obtain the direction of propagations and relative amplitudes of all emanating waves, i.e., completely solve the problem.

Figures 1 and 2 show the transmission coefficients versus the incident angle (Fig.1) and angle α (Fig. 2) for the case of incident forward Alfvén wave.

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Reference

[1]McKenzie J.F. and Westphal K.O., Transmission of Alfvén waves through the Earth's bow shock. Planetary and Space Science, 1969, V.17, P.1029-1037.

[2]Hassam A.B., Transmission of Alfvén waves through the Earth's bow shock: Theory and observation. JGR, 1978, V.83, P.643-653.

[3]Pudovkin M.I. and Lubchich A.A., Passage of an Alfvén wave through a strong shock-wave front. Geomagnetism and Aeronomy, 1989, V.29, P.197-203. [1989, V.29, P.176-180].

[4]Lubchich A.A., Interaction of magnetohydrodynamic waves with magnetohydrodynamic discontinuity, Ph.D. thesis, St. Petersburg State University, 1999.

[5]Lubchich A.A. and Pudovkin M.I., Interaction of small perturbations with shock waves. Physics of Fluids, 2004, V.16, P.4489-4505.

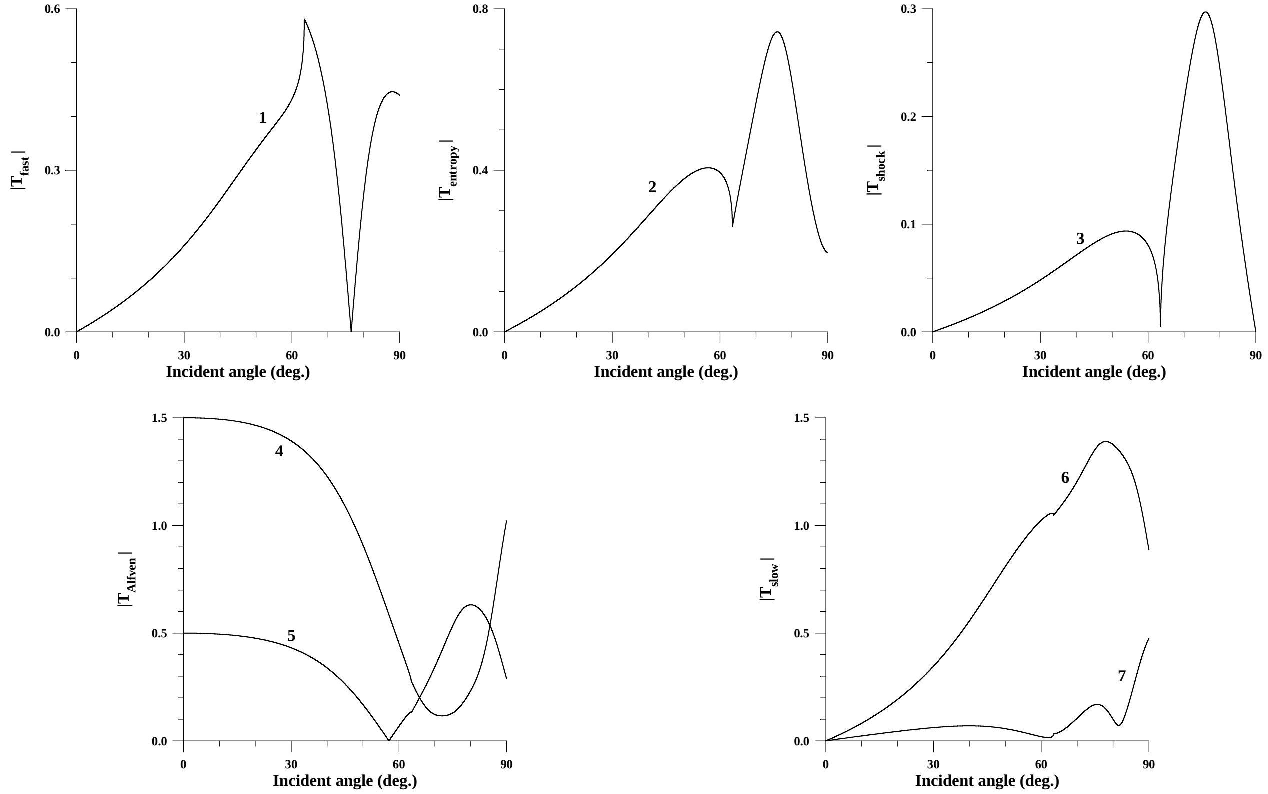

Figure 1. The relative amplitude of fast magnetosonic wave (1, $\left|\delta V_F / \delta V_{Al}\right|$), entropy wave (2, $\left|V_{x2} \cdot \delta\rho_E / \rho_2 \cdot \delta V_{al}\right|$), shock wave oscillations (3, $\left|\delta V_{sh} / \delta V_{Al}\right|$, forward and backward Alfvén wave (4 and 5, $\left|\delta V_{Al}^{tr} / \delta V_{Al}\right|$), and forward and backward slow magnetosonic wave (6 and 7, $\left|\delta V_{Sl}^{tr} / \delta V_{Al}\right|$) versus the incident angle of forward Alfvén wave under γ=5/3, ψ=60°, and α=30°.

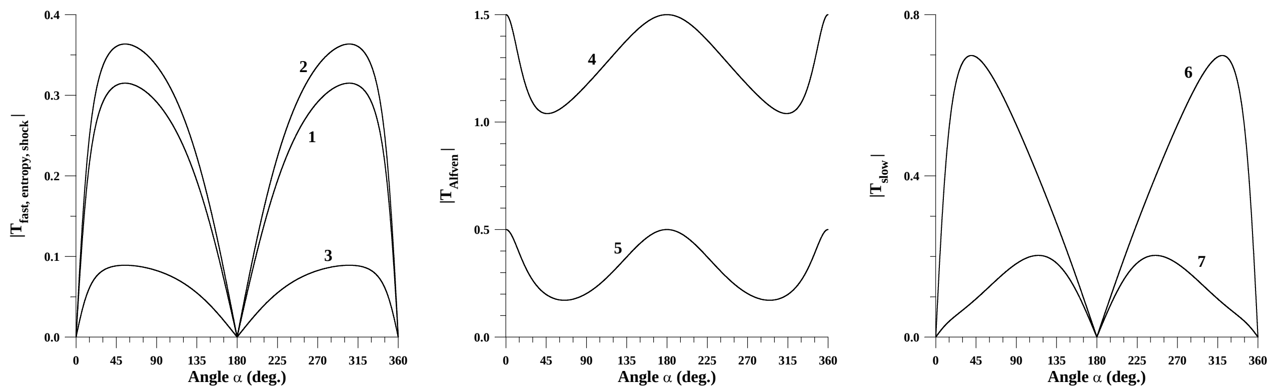

Figure 2. The relative amplitude of fast magnetosonic wave (1, $\left|\delta V_F / \delta V_{Al}\right|$), entropy wave (2, $\left|V_{x2} \cdot \delta\rho_E / \rho_2 \cdot \delta V_{al}\right|$), shock wave oscillations (3, $\left|\delta V_{sh} / \delta V_{Al}\right|$, forward and backward Alfvén wave (4 and 5, $\left|\delta V_{Al}^{tr} / \delta V_{Al}\right|$), and forward and backward slow magnetosonic wave (6 and 7, $\left|\delta V_{Sl}^{tr} / \delta V_{Al}\right|$) versus the angle α between the projections of the magnetic field $\vec{B}_\tau$ and wave vector $\vec{k}_A$ of the incident forward Alfvén wave on the shock plane under γ=5/3, ψ=60°, and λ=45°.