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MAGNETOSONIC SOLITONS IN HIGH-β NON-MAXWELLIAN SPACE PLASMAS

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Abstract. A nonlinear theory of large-amplitude magnetosonic (MS) waves in high-β space plasmas is revisited. It is shown that depending on the shape of the equilibrium ion distribution function these waves can exist in the form of "bright" or "dark" solitons in which the magnetic field is increased or decreased relative to the background magnetic field. The basic parameter that controls the shape of the nonlinear structure is the wave dispersion which can be either positive or negative. A general dispersion relation for MS waves propagating perpendicular to the external magnetic field in a plasma with an arbitrary distribution function is derived. The new dispersion relation allows the treatment of general plasma equilibria such as the Dory-Guest-Harris (DHG) or Kennel-Ashour-Abdalla (KA) loss cone equilibria, as well as distributions with power law velocity dependence that are modelled by the family of κ-distributions. It is shown that in bi-Maxwellian plasma the dispersion is negative, i.e. the phase velocity decreases with the increase in the wave number and thus the solitary solution in this case has the form of the "bright" solitons with the magnetic field increased. On the contrary in non-Maxwellian plasmas such as the ring-type distribution or DGH plasmas this solution may have the form of magnetic hole. The results of similar investigations based on nonlinear Hall-MHD equations are reviewed.

1. Introduction. The magnetosonic (MS) waves are frequently observed in the upstream and downstream regime of the terrestrial bow shock [Balikhin et al., 1997]. They are inherent to the magnetosheath, which is a turbulent layer formed downstream of the bow shock in the front of the magnetopause. The magnetosheath contains a large number of MS solitary structures of various shapes. Some of them belong to the class of magnetically rarefactive ("dark") solitons propagating at large angles to the ambient magnetic field whereas others are the waves with the magnetic field increased, the so-called "bright" solitons or magnetic holes [e.g., Lucek et al., 2005]. The analytical and numerical study of these nonlinear structures has been the subject of a great deal of research in recent years [e.g., Baumgartel, 1999; Baumgartel et al., 2005]. Using a nonlinear Hall-MHD model of the MS solitons in the high-β space plasmas it was assumed that the ion inertia can serve as the dominant dispersion effect that can prevent the MS structures from the wave breaking. Recently Pokhotelov et al. [2005a] noted that the Hall-MHD soliton model has a limited application and cannot be directly applied to high-β plasmas unless the effects due to magnetic viscosity or finite ion Larmor radius effects (FLR) are taken into account. For quasi-perpendicular propagation the FLR effects always prevail. The inconsistency between the exact kinetic treatment and the results of Hall-MHD theory in high-β plasmas has been also noticed by Krauss-Varban et al. [1994]. The dispersion of the MS waves propagating at large angle to the ambient magnetic field has been extensively discussed since the work of Kennel and Sagdeev [1967] where it was found that MS solitons in high-β Maxwellian plasma are rarefactive, i.e. represent the "dark solitons". This analysis has been further modified by Macmahon [1968] who found that MS solitons belong to the variety of "bright" solitons. A further discussion of this problem was given by Mikhailovskii and Smolyakov [1985]. Different types of MS wave forms were found in the foreshock regions as well as various ion populations: beam, intermediate and diffuse ions [e.g., Gosling et al., 1978]. The latter result from either the Fermi or the shock-drift acceleration mechanisms at the bow shock. In the presence of such distributions waves can be driven by the ion/ion beam [Gary, 1991] or halo [Pokhotelov et al., 2005b] instabilities.

The present paper provides the fully kinetic analyses of the MS waves in high-β plasma, taking into consideration both non-Maxwellian distributions and finite amplitude effects. Such a theory can give an answer to the question under which conditions the MS waves can propagate in space plasmas in the form of "bright" or "dark" solitons which can be registered by satellite magnetometers.

2. Linear MS dispersion relation in non-Maxwellian plasmas

We consider the case of utmost importance when MS waves propagate perpendicular to the external magnetic field. Then the MS dispersion relation for the arbitrary particle distribution functions is reduced to

$$N_\perp^2 = \varepsilon_{22} + (\varepsilon_{12})^2 / \varepsilon_{11} \tag{1}$$

where

$$\varepsilon_{22} = (c^2/v_A^2)[1+(\omega^2/\omega_{ci}^2)-(11/4)k_\perp^2\rho_i^2]-(k_\perp^2c^2/\omega^2)[\beta_\perp-(3/2)\lambda\beta_{\perp i}k_\perp^2\rho_i^2] \quad (2)$$

$$\varepsilon_{12} = -\varepsilon_{21} = i(c^2/v_A^2)(\omega/\omega_{ci})[1-(3/2)(\omega_{ci}^2/\omega^2)k_\perp^2\rho_i^2], \qquad \varepsilon_{11} = c^2/\omega^2 \quad (3)$$

Here v_A is Alfven velocity, ρ is the plasma mass density, ρ_i is the ion Larmor radius, ω_{ci} is the ion cyclotron frequency and parameter λ is given by

$$\lambda = n < W^2 F_i > /2(p_{\perp i})^2 \quad (4)$$

where W is the ion energy. Parameter λ deserves a special attention since it plays a central role in the theory of MS solitons. Substitution (2)-(3) with (1) gives the MS dispersion relation

$$\omega^2 = k_\perp^2 V_A^2(1-k_\perp^2 d^2) \quad (5)$$

where $V_A = v_A(1+\beta_\perp)^{1/2}$ stands for the generalized Alfven velocity which incorporates the high-β effect and d is the dispersion length given by

$$d^2 = (\rho_i^2/4)[1+\beta_\perp(6\lambda-7/2)]/(1+\beta_\perp) \quad (6)$$

From Eq. (6) it follows that the square of the dispersion length can be either positive or negative depending on the shape of the ion distribution function which is defined by the sign of the last term in the nominator of Eq. (6). The MS dispersion is negative if the parameter λ is relatively small, $\lambda < 7/12$. This corresponds to the necessary (but not sufficient) condition of existence of negative dispersion. In the opposite case the dispersion is positive. In this connection a few comments are in order. First, for canonical Maxwellian distribution function $\lambda = 1$ and $d^2 > 0$. Thus in such plasma the MS dispersion is negative as it was demonstrated by Macmahon [1968] who used simplyfied fluid considerations. As it will be shown below this may result in the appearence of "bright" solitons in high-β plasmas. The same is true for bi-Maxwellian distribution for which the parameter λ also equals unity and d^2 is given by

$$d^2 = (\rho_i^2/4)(1+5/2\beta_\perp)/(1+\beta_\perp) \quad (7)$$

Thus, even in non-equilibrium plasmas such as bi-Maxwellian plasmas the dispersion of MS waves is still negative, i.e. the phase velocity decreases with the increase in the wave number.

A positive dispersion may appear in a plasma with the ion ring-type distribution $F_i \propto \delta(v_\perp - v_{\perp 0})$, where $v_{\perp 0}$ is the ion "ring" velocity. For such distribution $\lambda = 1/2$ and the necessary condition for the reversal of the sign of d^2 is satisfied. Such distribution functions were systematically observed when MS waves were detected in the Earth's magnetosphere on board the GEOS 1 and 2 satellites. The ring type ion distributions can also be formed during the pick up processes when neutral atoms are injected from comets or the interstellar medium into solar wind and get trapped by solar wind plasma.

It should be noted that in majority of cases the measured particle distributions in the near-Earth environment considerably deviate from the bi-Maxwellian or ring type shape. They are better fitted by generalized loss-cone Dory-Guest-Harris distribution [e.g., Leubner and Schupfer, 2000]

$$F_{\kappa l}(v_\parallel, v_\perp) \propto (v_\perp/v_{T\perp})^{2l}[1+(v_\parallel^2/\kappa v_{T\parallel}^2)+(v_\perp^2/\kappa v_{T\perp}^2)]^{-\kappa-1} \quad (8)$$

where $v_{T\parallel}$ and $v_{T\perp}$ are the thermal ion speeds parallel and perpendicular to the magnetic field, the spectral index κ and loss-cone index l the condition $\kappa > l + 5/2$. The mixed loss-cone high-energy-tail distribution (8) reduces for $l = 0$ to the κ-distribution, for $\kappa \to \infty$ to the Dory-Guest-Harris type distribution and for $\kappa \to \infty$ and $l = 1$ to the bi-Maxwellian distribution. Using Eqs. (4) and (8) one finds easily the parameter λ which is $\lambda = (1+l/2)(\kappa - l - 3/2)/(1+l)(\kappa - l - 5/2)$. A close inspection of this expression shows that presence of the high-energy tails does not lead to the reversal of the sign of the square of the MS wave dispersion length. On the other hand the loss-cone effects do it if the loss cone index is sufficiently large. For example, in the limiting case $\kappa \to \infty$ the loss cone index should be greater than 5. The general case that demonstrates the necessary conditions for the existence of the reversal of the sign of the wave dispersion for the arbitrary l and κ is depicted in Figure 1(Left). One sees that larger *l* are in favour of positive dispersion whereas the smaller κ plays an opposite role.

Finally let us discuss one more ion distribution function that might be promising for the reversal of the sign of the MS wave dispersion. This is a partially filled loss cone distribution of Ashour-Abdalla and Kennel [1978], the so-called KA distribution discussed by Pokhotelov et al. [2002] in connection with the mirror instability

$$F_i \propto \exp(-v_\parallel^2/v_{T\parallel}^2)\{\eta\exp(-v_\perp^2/v_{T\perp}^2)+(1-\eta)/(1-\varsigma)[\exp(-v_\perp^2/v_{T\perp}^2)-\exp(-v_\perp^2/\varsigma v_{T\perp}^2)]\} \quad (9)$$

where parameter η is restricted, $0 \le \eta \le 1$, and ς takes positive values. It takes care of the formation of the loss cone and the degree of loss cone filling. In particular, the bi-Maxwellian is reproduced for either $\eta = 1$ or $\varsigma = 0$. Using Eq. (9) one easily finds $\lambda = [\eta + (1-\eta)(1+\varsigma+\varsigma^2)][\eta+(1-\eta)(1+\varsigma)]^{-2}$

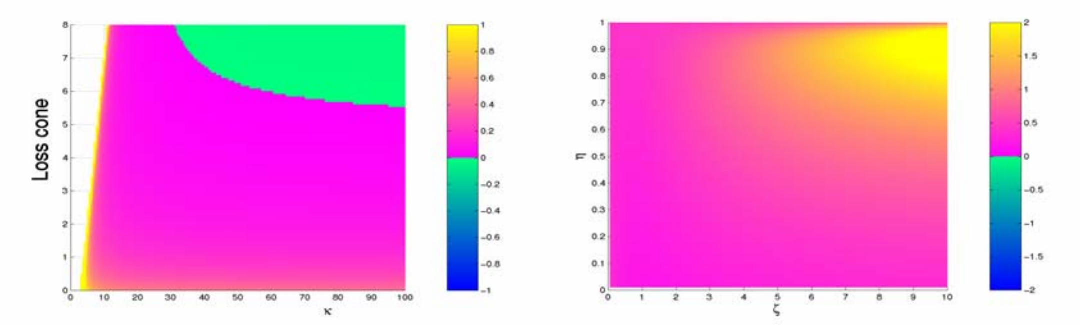

Fig. 1 Left –Plot of $\lambda - 7/12$ as a function of the loss cone index l and parameter for generalized loss-cone Dory-Guest-Harris distribution. On the right - The same as in Fig. 1 but for KA distribution function.

One sees that in case of KA distribution function the necessary condition for the reversal of the sign of the MS dispersion is not attained.

3. MS solitons in non-Maxwellian plasmas

The basic idea in deriving the model soliton-like equations is the reductive perturbation scheme of Korteweg and de Vries (KdV), who simplified general wave equations by expanding them in a perturbation theory expansion with nonlinear parameters and dispersion parameters being treated as small quantities of the same order of magnitudes [cf. Petviashvili and Pokhotelov, 1992]. Following this idea one can deduce the nonlinear terms independently neglecting the dispersion. The propagation of MS waves at large angle to the external magnetic field gives rise to a quadratic nonlinearity of the advective type (KdV type) associated with the $\mathbf{v}\cdot\nabla\mathbf{v}$ terms related to the ion inertia in the momentum equation. We now deduce a simplified form of this nonlinearity by disregarding the corrections due dispersion and diffraction. We start from the ion momentum equation $\rho(\partial_{\mathbf{t}}\mathbf{v} + \mathbf{v}\cdot\nabla\mathbf{v}) = \mathbf{J}\times\mathbf{B} - \nabla\cdot\hat{\mathbf{P}}$. The pressure tensor is defined as $\hat{\mathbf{P}} = p_\perp \hat{\mathbf{I}} + (\mathrm{p}_\parallel - p_\perp)\hat{\mathbf{b}}_i\hat{\mathbf{b}}_j$ where $\hat{\mathbf{I}}$ is the unit dyadic, $p_{\parallel(\perp)}$ the parallel (perpendicular) plasma pressure and $\hat{\mathbf{b}}_i$ the components of the unit vector $\mathbf{B}/B$. Since we are interested here solely in calculation of nonlinear corrections the terms related to FLR effects that corresponds to the collisionless magnetic viscosity in Eq. (12) are neglected. In this case the MS wave exhibits solely the y-component of the electric field and the z-component of the magnetic field. Thus, in the absence of dispersion only the x-component of the ion velocity survives and $\mathbf{v} \equiv \mathrm{v}\mathbf{x}$. Similarly, Eq. (12) shows that the only nonzero component of the electric current is the y-component given by $J = \rho B^{-1}(\partial_t v + v\partial_x v) + B^{-1}\partial_x p_\perp$. We note that the terms due to the pressure anisotropy do not contribute to the electric current in our approximation. Eq. (14) can be further simplified. First, we note that our plasma is frozen-in the magnetic field and thus $\rho/B = \rho_0/B_0$. Moreover, in the course of calculation of the second term on the right-hand side of Eq. (14) one can assume that plasma pressure varies adiabatically, i.e. $p_\perp/B^2 = \mathrm{const}$. This condition can be easily obtained from the perpendicular plasma pressure balance condition when the effects due to the thermal heat flux are neglected. The latter are of the same order as the dispersion corrections and thus vanish. Thus, one obtains $\mu_0 \mathrm{J} = \mathrm{B}_0 \mathrm{v}_\mathrm{A}^{-2}(\partial_\mathrm{t}\mathrm{v} + \mathrm{v}\partial_\mathrm{x}\mathrm{v}) + \beta_\perp \mathrm{B}_0 \partial_\mathrm{x} \mathrm{b}$. Here $\mathrm{b} = \delta\mathrm{B}_z/\mathrm{B}_0$ is a dimensionless wave amplitude. Substitution of this expression into Ampere's law gives $V_A^2\partial_x b + \partial_t v + v\partial_x v = 0$. We assume that all perturbed quantities vary as $\propto \mathrm{f}(\mathrm{x}-\mathrm{ut})$, where u is the wave velocity. Then, with the help of Faraday's law, we arrive to the condition $\mathrm{E_y} = \mathrm{u}\delta\mathrm{B_z}$ and thus $\mathrm{v} = \mathrm{u}(\mathrm{b}-\mathrm{b}^2)$. The substitution of these expressions with Ampere's law results in $(1-\mathrm{V_A^2}/\mathrm{u}^2)\partial_\xi \mathrm{b} - 3\mathrm{b}\partial_\xi \mathrm{b} = 0$. By supplementing this equation by dispersion correction one finds that $\mathrm{d}^2\partial_{\xi^2}^2 \mathrm{b} = (1-\mathrm{V_A^2}/\mathrm{u}^2)\mathrm{b} - 3\mathrm{b}^2/2$. This equation admits a nonlinear solution in the form of one-dimensional soliton $\mathrm{b} = \mathrm{b}_0/\cosh^2(\kappa\xi/2)$ where cosh (x) is the hyperbolic cosine and the parameters b_0 and κ are given by $b_0 = 1 - V_A^2/u^2$ and

$\kappa = (1 - V_A^2 / u^2)^{1/2} / d$. When dispersion is negative the MS structures are super-Alfvenic with the magnetic field increased ("bright" solitons). In the opposite case of positive dispersion they are sub-Alfvenic with the magnetic field decreased ("dark" solitons or magnetic holes).

4. Discussion and conclusions

The analysis presented above revealed the key role of FLR effects in the formation of solitary MS structures. It was shown that the shape of these structures is very sensitive to the details of the velocity space distribution. Contrary to previous statement that the ion inertia plays a major role in the formation of dispersive properties of MS waves in high-β space plasmas, our analysis shows that this role ultimately belongs to the FLR effect. Depending on the details of the background distribution function the dispersion corrections produce either positive or negative frequency shift. For instance, in Maxwellian plasma the MS dispersion is negative, i.e. the wave phase velocity in this case decreases with the increase in the wave number whereas the non-Maxwellian effects may lead to the reversal of the dispersion frequency shift. All these features give quite a complex picture of nonlinear phenomena in space plasmas. The clarification of the conditions under which the "dark" or "bright" solitons arise in space plasma environments is clearly of utmost importance. Our analysis makes these conditions more accurate. In most space plasmas where collisions occur very rarely, the measured velocity distributions of charged particles frequently deviate substantially from the canonical Maxwellian distributions. The simplest nonequilibrium distribution is generally assumed to be the bi-Maxwellian or kappa distributions (or generalized Lorenzian) where the kappa index determines the slope of their suprathermal energy distribution. The larger the value of kappa the lesser the excess of suprathermal particles. It was shown that for all these distributions the MS dispersion is negative and thus in such plasmas the solitary structures should represent the "bright" solitons with the magnetic field increased inside the structure. However, in some specific plasmas where the loss cone effects may play an essential role one should expect the appearance of the reversal of the sign of the dispersion frequency shift. This may result in the appearance of "dark" solitons with the magnetic field increased inside the structure. Similar phenomena can be observed in the pick up processes similar to those observed in the vicinity of comets. The model developed in this paper still remains oversimplified. For example, it is so far restricted by finite but small amplitudes of the solitons when the KdV expansion provides a useful guide for construction of nonlinear equations. We note that a detailed comparison of theoretical results with satellite observations is, however, outside the scope of the present study the intention of which is to provide a deeper insight into physics of nonlinear dynamics of MS waves in high-β plasmas.

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