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PROPERTIES OF NONLINEAR LOW-FREQUENCY WAVES IN HIGH-β SPACE PLASMAS: KINETIC AND FLUID DESCRIPTION

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Abstract. The theory of ion-cyclotron type modes accounting for the collionless magnetic viscosity in high-β Maxwellian space plasmas is developed. A comparison of the kinetic results with those obtained in the framework of Hall-MHD is carried out. It is shown that in order to coincide with the fully kinetic treatment the Hall-MHD equations must be supplemented with terms responsible for the effect due to the collisionless magnetic viscosity. This effect was not included in the previous analyses. This in turn leads to the modification of corresponding nonlinear equation that describes nonlinear dynamics of ion-cyclotron waves.

1. Introduction

The ion-cyclotron waves are the most important modes in high-β space plasmas. The mathematical description and identification of such waves in satellite data has been the subject of a great deal of research in the last four decades [e.g., *Barnes*, 1966; *Formisano and Kennel*, 1969; *Foote and Kulsrud*, 1979; *Gary*, 1986, 1999; *Leamon et al.*, 1998a,b, 1999, 2000; *Stawicki et al., 2001*; *Krauss-Varban et al.*, 1994, 1996; *Gary and Nishimura*, 2004; *Gary and Borovsky*, 2004]. All these studies were mainly based on numerical analysis of the coupled Vlasov-Maxwell system of equations. More then a decade ago *Krauss-Varban et al.* [1994] raised an interesting question whether the fully kinetic treatment of ion-cyclotron waves in high-β plasmas coincided with the results of a simpler description based on Hall magnetohydrodynamics (Hall-MHD). The fluid description is usually simpler than their kinetic counterparts. All terms in the fluid equations have a definite physical sense and the mode properties are well calculated, except for the case when waves are highly damped due to the wave-particle resonant interaction. In this respect it is of interest to investigate whether there is a relevant fluid approach that can be used for calculation of hydromagnetic wave dispersion in high-β plasmas.

2. Kinetic description

We investigate the ion-cyclotron waves in an electron proton plasma considering that all fluctuating quantities vary in time and space as $\exp(-i\omega t + i\mathbf{k}\cdot\mathbf{r})$, where ω and $\mathbf{k}$ are the wave frequency and the wave vector, respectively. We assume that $\mathbf{k} = k_\perp \hat{\mathbf{x}} + k_\parallel \hat{\mathbf{z}}$, $\hat{\mathbf{x}}$ and $\hat{\mathbf{z}}$ are the unit vectors perpendicular and parallel to the external magnetic field $\mathbf{B}_0$, $k_\perp$ and $k_\parallel$ are the perpendicular and parallel components of the wave vector. For the case of the quasi-parallel propagation, $k_\perp^2 / k_\parallel^2 << \omega^2 / \omega_{ci}^2$, the dispersion relation of the ion-cyclotron waves takes the form

$$k_\parallel^2 v_A^2 / \omega^2 = (\omega_{ci}^2 / \omega k_\parallel v_{Ti}) Z(\varsigma_\pm) \pm \omega_{ci} / \omega. \tag{1}$$

Here $\omega_{ci} = eB_0 / m_i$ is the ion cyclotron frequency, e and m_i the ion (proton) charge and mass, respectively, $v_A = B_0 / (\mu_0 n_0 m_i)^{1/2}$ the Alfvén velocity, μ_0 the permeability of free space, n_0 the equilibrium particle number density, $v_{Te} = (2T_e / m_e)^{1/2}$ and $v_{Ti} = (2T_i / m_i)^{1/2}$ the electron and ion thermal velocities, T_e and T_i the electron and ion temperature, respectively, m_e the electron mass, $\varsigma_\pm = (\omega \pm \omega_{ci}) / k_\parallel v_{Ti}$ the ion cyclotron resonance factor, and $Z(\varsigma_\pm)$ the plasma dispersion function (the Kramp's function). Eq. (1) yields the dispersion relation for the ion-cyclotron waves, $\pm$ sign corresponds to the right-hand polarized magnetosonic-whistler and left-hand circularly polarized Alfvén-cyclotron modes. Using the asymptotics of the Z function at large argument, one reduces Eq. (1) to

$$k_{\|}v_A/\omega=(1\pm\omega/\omega_{ci})\left[(1\pm\omega/\omega_{ci})^3\pm(\beta_i/2)\omega/\omega_{ci}\right]^{-1/2}, \qquad (2)$$

where $\beta_i=2\mu_0 n_0 T_i/B_0^2$ is the ion plasma beta. In the low-frequency limit, $\omega<<\omega_{ci}$, Eq. (2) reduces to

$$\omega/k_{\|}v_A\approx\left[1\pm(1+\beta_i/2)\omega/\omega_{ci}\right]^{1/2}. \qquad (3)$$

In the limiting case of very large ion betas, $\beta_i>>1$, this dispersion relation has been deduced by *Foote and Kulsrud* [1979]. The damping rate $\gamma_\pm$ of the ion-cyclotron modes can be expressed in terms of the cyclotron resonance factor $\varsigma_\pm$, i.e.

$$\gamma_\pm/\omega_{ci}\approx-(\pi^{1/2}/2)|\varsigma_\pm|\exp(-\varsigma_\pm^2), \qquad (4)$$

where $|\varsigma_\pm|\approx\beta_i^{-1/2}(\omega_{ci}/\omega)\left[(1\pm\omega/\omega_{ci})^3\pm(\beta_i/2)\omega/\omega_{ci}\right]^{1/2}$.

Figs. 1-3 show the plots of normalized ion-cyclotron wave number and damping rate as a function of normalized frequency calculated with the help of Eqs. (2) – (4). To compare our analytical results with previous analyses we have plotted similar curves obtained in numerical simulations of the coupled Vlasov-Maxwell system of equations [*Barnes*, 1966; *Gary*, 1999; *Gary and Borovsky*, 2004; *Krauss-Varban et al*., 1994; *Stawicki et al*., 2001]. One sees that both results are in rather a good agreement.

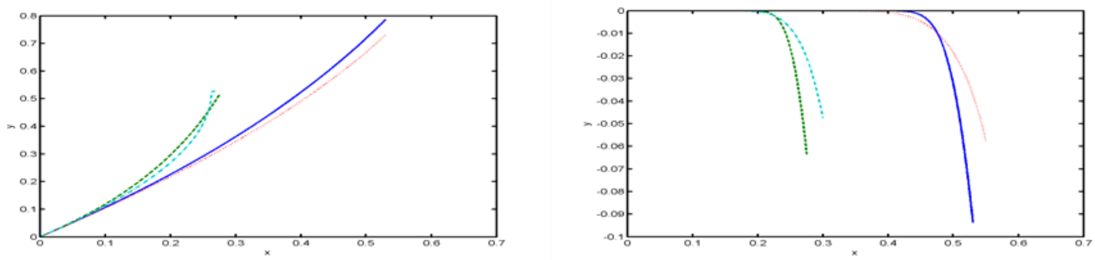

Fig. 1a (on the left). The Alfvén-cyclotron wave number normalized to the ion inertial length $y=k_{\|}c/\omega_{pi}$ as function of the dimensionless wave frequency $x=\omega/\omega_{ci}$. Solid and dashed curves correspond to $\beta_i=0.1$ and $\beta_i=1$, respectively. Dotted and dash-dotted curves are the results of numerical simulations *of Stawicki et al.* (2001) carried out for the same values of β_i. Fig. 1b (on the right). The same as in Fig. 1a but for a normalized Alfvén-cyclotron wave damping rate $y=\gamma_-/\omega_{ci}$.

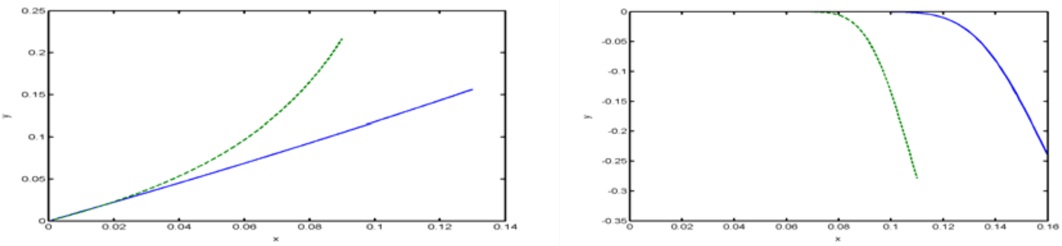

Fig. 2a (on the left). The Alfvén-cyclotron wave number normalized to the ion inertial length $y=k_{\|}c/\omega_{pi}$ as a function of $x=\omega/\omega_{ci}$. Fig. 2b (on the right). The same as in Fig. 2a but for a normalized Alfvén-cyclotron damping rate $y=\gamma_-/\omega_{ci}$. Solid and dashed curves correspond to $\beta_i=5$ or $\beta_i=10$, respectively.

Figs. 1b and 2b show that the frequency range of the weak damping of Alfvén-cyclotron waves is relatively narrow and it decreases with the increase in β_i. The transition from weak to strong damping starts abruptly with the increase of ω or $k_{\|}$.

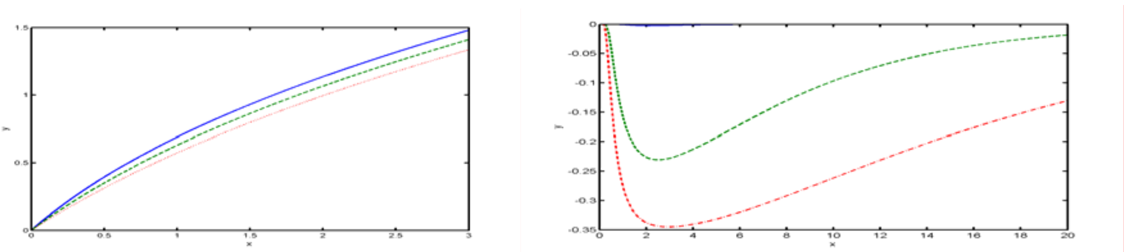

Fig. 3a (on the left). The magnetosonic-whistler wave number as a function of dimensionless frequency. Fig. 3b (on the right) The normalized growth rate $y = \gamma_+ / \omega_{ci}$ of the magnetosonic-whistler waves as a function of dimensionless frequency. Solid, dotted and dashed curves correspond to $\beta_i = 1$, $\beta_i = 5$ and $\beta_i = 10$, respectively.

One sees that magnetosonic-whistler waves possess a weak damping and propagate in a relatively wide range of the wave frequencies. The maximum damping in plasma with $\beta_i = 5$ or $\beta_i = 10$ is attained at $\omega \approx (2-3)\omega_{ci}$.

3. Hydrodynamic description

Now we study the ion-cyclotron waves in high-β collisionless plasmas in the framework of two-fluid MHD. To describe the wave dispersion in such plasma we make use of Braginskii's type hydrodynamic equations. The first of them is the particle continuity equation, $\partial n_\alpha / \partial t + \nabla \cdot (n_\alpha \mathbf{v}_\alpha) = 0$, another is the particle momentum equation $m_\alpha n_\alpha d\mathbf{v}_\alpha / dt + \nabla p_\alpha + \nabla \cdot \hat{\pi} = q_\alpha n_\alpha [\mathbf{E} + (\mathbf{v}_\alpha \times \mathbf{B})]$ and finally, the equation of the state is $p_\alpha = K n_\alpha^\gamma$. Here n_α, $\mathbf{v}_\alpha$ and p_α are the particle number density, velocity and pressure of the α th species. The subscript α takes the value i or e for the ions and electrons, m_α and q_α the particle mass and charge, respectively, $\mathbf{E}$ and $\mathbf{B}$ the electric and magnetic fields, $d/dt = \partial/\partial t + \mathbf{v} \cdot \nabla$ the convective time derivative, $\hat{\pi}$ is the collisionless magnetic viscosity tensor, K the constant value and γ the ratio of specific heats. We neglect the electron inertia and magnetic viscosity in the electron momentum equation. The components of the ion magnetic viscosity tensor $\hat{\pi}$ are $\hat{\pi}_{zx} = -(n_i T_i / \omega_{ci}) \partial_z v_y$, $\hat{\pi}_{zy} = (n_i T_i / \omega_{ci}) \partial_z v_x$, $\partial_z \equiv \partial / \partial z$. We investigate the waves propagating along the ambient magnetic field $\mathbf{B}_0 \parallel \hat{\mathbf{z}}$ in the low-frequency, $d/dt << \omega_{ci}$, and long-wavelength approximation, considering that $\varepsilon = \omega_{ci}^{-1} d/dt \approx \rho_i \partial / \partial z << 1$, where $\rho_i = (2T_i / m_i)^{1/2} \omega_{ci}^{-1}$ is the ion Larmor radius. A power series expansion of Eq. (11) on the parameter ε yields the perpendicular ion velocity [cf. Onishchenko et al., 2001] $\mathbf{v}_\perp = \mathbf{v}_E + \mathbf{v}_\pi + \mathbf{v}^I + \mathbf{v}^{II} + v_z \mathbf{B}_\perp / B_0$. Here $\mathbf{v}_E = (\mathbf{E} \times \hat{\mathbf{z}}) / B_0$ is the electric drift velocity, $\mathbf{B} = \mathbf{B}_\perp + B_0 \hat{\mathbf{z}}$, $\mathbf{B}_\perp$ is the wave magnetic field, $\hat{\mathbf{z}}$ is the unit vector along the ambient magnetic field $\mathbf{B}_0$, $\mathbf{v}_\pi = (m_i n_i \omega_{ci})^{-1} \hat{\mathbf{z}} \times \nabla \cdot \hat{\pi}$ is the ion diamagnetic drift velocity due to the magnetic viscosity, $\mathbf{v}^I$ is the ion polarization drift velocity $v^I = \omega_{ci}^{-1} \hat{z} \times \partial_t v_E$ and the correction to the ion polarization drift velocity is $\mathbf{v}^{II} = \omega_{ci}{}^{-1} \hat{\mathbf{z}} \times \partial_t \mathbf{v}^I = -\omega_{ci}{}^{-2} \partial_{tt}{}^2 \mathbf{v}_E$, where $\partial_t \equiv \partial / \partial t$. Using these Eqs. one can obtain that $\mathbf{v}_\pi = -(\rho_i^2 / 2) \partial_{zz}^2 \mathbf{v}_E$. One sees from this that $\mathbf{v}^{II}$ and $\mathbf{v}_\pi$ are small values of an order of ε relative to the ion polarization velocity $\mathbf{v}^I$. Considering the circularly polarized waves we introduce a complex function $A_\pm = A_x \pm i A_y$ for the arbitrary two-dimensional vector $\mathbf{A} = (\mathbf{B}_\perp, \mathbf{E}_\perp, \mathbf{v}_\perp, \mathbf{j}_\perp)$, where $\mathbf{j}_\perp$ is the perpendicular electric current. Using these notations we write Faraday's and Ampere's laws as $\partial_z E_\pm = i \mu_0 \partial_t B_\pm$ and $c^{-2} \partial_t E_\pm - i \mu_0 \partial_z B_\pm + \mu_0 j_\pm = 0$. Then we decompose the electric current as $\mathbf{j} = \mathbf{j}^I + \mathbf{j}^D + \mathbf{j}^{NL}$ where $\mathbf{j}^I = e n_0 \mathbf{v}^I$, $\mathbf{j}^D = e n_0 (\mathbf{v}^{II} + \mathbf{v}_\pi)$ and $\mathbf{j}^{NL} = e n_0 (v_z \mathbf{B} + \tilde{n} \mathbf{v}^I / n_0)$ are the polarization, dispersion and nonlinear parts of the ion electric current, v_z and $\tilde{n}$ are the parallel ion velocity and wave perturbation of the ion density. Neglecting nonlinear effects one obtains from here a dispersion relation (3). The term proportional to the ion beta β_i in Eq. (3) is related to $\mathbf{v}_\pi$ and therefore cannot be obtained in the framework of the Hall-MHD. To investigate nonlinear waves we introduce the spatial variable $\varsigma = z - Vt$ and the slow time τ and obtain $\tilde{n}/n_0 = v_z/V = (1/2)V^2(V^2 - c_s^2)|B|^2/B_0^2$, where $c_s^2 = \gamma p_i/n_i$. From here the derivative nonlinear Schrödinger (DNLS) equation [cf. *Spangler and Sheerin,* 1982] follows

$$i\partial_\tau B_\pm + ib\partial_\varsigma(B_\pm |B|^2) \pm a\partial_{\varsigma\varsigma}^2 B_\pm = 0, \qquad (5)$$

where $a = (V^2 + v_{Ti}^2/2)/2\omega_{ci}$ and $b = V^3/4(V^2 - c_s^2)$. It should be noted that Eq. (5) may be used solely when the phase velocity $V \approx v_A >> c_s$ that corresponds to the applicability of hydrodynamic description of collisionless plasmas.

4. Discussion and conclusions

The linear theory of uniform, magnetized, collisionless high-β isotropic Maxwellian plasma has been used to the study dispersion and damping of the ion-cyclotron waves in an electron-proton plasma. A compact expression for the wave dispersion relation in the quasi-parallel approximation has been obtained. There has been expressed in terms of the tabulated plasma dispersion Z-function and thus it can be analysed numerically for different arguments. The dispersion relations and damping rates for the left- and right-hand polarized waves have been obtained for large arguments of the Z-function when damping is weak. In the low-frequency approximation and the limiting case of very large β_i the dispersion relation (3) reduces to that previously obtained by *Foote and Kulsrud* [1979]. For the finite value of β_i the term that describes the wave dispersion coincides with that in the nonlinear Schrödinger equation (DNLS) discussed by *Mjølhus and Wyller* [1988]. We showed that dispersion relation for the ion-cyclotron waves in the high-β plasma can be obtained with the help of Braginskii type MHD where the term containing parameter β_i arises from the collisionless magnetic viscosity and thus cannot be obtained in the framework of standard Hall-MHD.

Figs. 1a, b illustrate the dispersion and damping of the left-hand circularly polarized Alfvén-cyclotron waves due to the proton cyclotron resonance in a plasma with $\beta_i = 0.1$ and $\beta_i = 1$. From Figs. 1a and 1b one sees that dispersion relation (2) and the growth rate (4) for the Alfvén-cyclotron modes in high-β plasmas are in a good agreement with the numerical simulations [Gary, 1993; *Gary and Borovsky*, 2004; *Krauss-Varban et al*., 1994; *Stawicki et al*., 2001] who proposed the fit functions for the dispersion $\omega/\omega_{ci} = k_\parallel c/\omega_{pi} - (0.34 + 0.61 \times \beta_i^{0.61})k_\parallel^2 c^2/\omega_{pi}^2$ and the growth rates $\gamma/\omega_{ci} = -0.6\beta_i^{0.36}(k_\parallel c/\omega_{pi})^{2m}\exp(-m_3\omega_{pi}^2/k_\parallel^2 c^2)$ with $m = 0.77\beta_i^{0.03}$ and $m_3 = 0.32\beta_i^{-0.65}$ in the domain $0.01 < \beta_i < 2.5$. Strong proton cyclotron damping starts abruptly when $\omega/\omega_{ci} \cong 0.5$ (when $k_\parallel c/\omega_{pi} \cong 0.7$) or $\omega/\omega_{ci} \cong 0.25$ (when $k_\parallel c/\omega_{pi} \cong 0.45$) in a plasma with $\beta_i = 0.1$ or $\beta_i = 1$. Figs. 2a, b show the plots of the dispersion and damping rates of the Alfven-cyclotron waves in a plasma with $\beta_i = 5$ and 1. The waves with $k_\parallel c/\omega_{pi} \geq 0.15$ and frequencies $\omega/\omega_{ci} \geq 0.12$ or $\omega/\omega_{ci} \geq 0.085$ are highly damped (with $|\gamma|/\omega \approx 1$) in a plasma with $\beta_i = 5$ and 10, respectively. As it follows from Figs. 1b and 2b the transition from the weak to strong damping starts abruptly with the increase in $k_\parallel$. The hydrodynamic description for such highly damped waves due to the proton cyclotron resonance cannot be used. Contrary to the Alfven-cyclotron waves the magnetosonic-whistler waves in high-β plasmas at quasi-parallel wave propagation are weakly damping in a wide range of the wave frequencies and wave numbers. Figs. 3a, b show the wave dispersion and proton cyclotron damping of the magnetosonic-whistler waves in plasmas with $\beta_i = 1,\ 5$ and 10.

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