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PROPAGATION OF THE BALLOONING WAVES IN THE EARTH'S MAGNETOTAIL

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Abstract. The current interest in the theory of the ballooning waves is inspired by the fact that these waves are a natural framework for the interpretation of the flapping motions in the magnetotail frequently observed by Cluster. Flappings manifest as low-frequency oscillations of the plasma sheet generated by some impulsive sources in the center of the magnetospheric tail (e.g., substorm activations or bursty bulk flows), and propagating predominantly toward the flanks. The group velocity is found to range from a few tens to a few hundreds km/s. Previously, we showed that the ballooning mode unlike a number of other modes matches these observed characteristics [*Golovchanskaya and Maltsev*, 2005]. Here, more rigorous formulas for the velocities of the ballooning wave propagation in the inhomogeneous high-β plasma confined in the curvilinear magnetic field are derived. The consideration is a generalization of the treatment of *Liu* [1997] for the case of non-zero component of the perturbation wave vector along the magnetic tension force. The Cauchy problem is formulated for the partial derivative equation of the 4^{th} order describing the linear ballooning wave propagation in the plane perpendicular to the background magnetic field, and the properties of the solution are illustrated.

1. Introduction

The first evidence of the excitation of the ballooning waves in the Earth's magnetotail were reported long ago [*Speiser and Ness*, 1967; *Lui et al.*, 1978]. However the directions and velocities of the wave propagation were difficult to reliably determine from single-spacecraft measurements. The situation changed radically with the start of Cluster spacecraft system operation. It was established that the up-down oscillations of the magnetotail current sheet with the periods of a few minutes observed under both quiet and disturbed geomagnetic conditions is a common feature of magnetotail dynamics [*Zhang et al.*, 2002; *Runov et al.*, 2003; *Sergeev et al.*, 2003, 2004; *Runov et al.*, 2005]. These oscillations referred to as the flapping motions were found to propagate predominantly in the $\pm y_{GSM}$ directions, that is, toward dawn in the dawn sector and toward dusk in the dusk sector, the velocity of propagation increasing with geomagnetic activity. *Golovchanskaya and Maltsev* [2005] demonstrated that the ballooning mode unlike a number of other modes matches these observed characteristics. The dispersion relation derived in [*Golovchanskaya and Maltsev*, 2005] indicates propagation in the positive/negative azimuthal direction with a group velocity dependent on the half-thickness of the current sheet, thermal velocity of ions in the neutral sheet and wave number across the magnetic field. The group velocity was estimated to range from 40 km/s for quiet magnetotail configurations to 400 km/s for more stretched active plasma sheet, which is consistent with the observations. However, aiming to emphasize a close analogy between the ballooning waves in the magnetosphere and internal gravitational waves in the upper atmosphere [e.g. *Hargreaves*, 1979], those authors made some simplifying assumptions concerning the form of the ballooning perturbation in deriving the expression for the group velocity. Namely, the term $\sim 4\cdot k_{\|}^2 k_y^2/(k_n^2 + k_y^2)$ in the dispersion relation, $k_{\|}$, k_n, and k_y being the components of the wave vector along the magnetic field **B**, in the direction **n** of the magnetic tension force and in the y_{GSM} direction, respectively, was neglected. In the present study this restriction is relaxed, the dispersion relation is considered in a more general form (section 2) and more rigorous formulas for the ballooning propagation velocities are obtained (section 3). The Cauchy problem is formulated for the partial derivative equation of the 4^{th} order describing linear ballooning wave propagation in the plane perpendicular to the background magnetic field. The problem is solved for the initial perturbation of a special form allowing the analytical solution (section 4). The results are summarized in section 5.

2. Dispersion relation for the ballooning waves

Similarly as it was done by Liu [1997] and keeping his denotations, we start from the one-fluid ideal MHD equations. They are

$$\frac{\partial \rho}{\partial t} = -\mathbf{u} \bullet \nabla \rho - \rho \nabla \bullet \mathbf{u} \tag{1}$$

$$\rho\left(\frac{\partial \mathbf{u}}{\partial t} + (\mathbf{u} \bullet \nabla)\mathbf{u}\right) = -\nabla\left(p + \frac{B^2}{2\mu_0}\right) + \frac{(\mathbf{B} \bullet \nabla)\mathbf{B}}{\mu_0} \tag{2}$$

$$\frac{\partial p}{\partial t} = -\mathbf{u} \bullet \nabla p - \gamma p \nabla \bullet \mathbf{u} \tag{3}$$

$$\frac{\partial \mathbf{B}}{\partial t} = -\mathbf{B}\nabla \bullet \mathbf{u} - (\mathbf{u} \bullet \nabla)\mathbf{B} + (\mathbf{B} \bullet \nabla)\mathbf{u} \tag{4}$$

where p is the plasma pressure, ρ the mass density and γ the adiabatic compression index.
From the force balance condition for the equilibrium we have

$$\frac{\beta}{2} k_p + k_b - k_c = 0 \tag{5}$$

As in the typical plasma sheet configurations around the equatorial plane the gradients of the magnetic field and plasma pressure as well as the magnetic curvature vector are directed toward the Earth, k_p, k_b, and k_c in (5) are positive.

In the following derivation we will use the expressions for the Alfven velocity $c_A^2 = \frac{B^2}{\mu_0 \rho}$, the sound velocity $c_s^2 = \frac{p}{\rho}$, the β-parameter $\beta = \frac{2\mu_0 p}{B^2}$ and the identities $\beta = \frac{2c_s^2}{c_A^2}$, $k_p = \frac{c_A^2}{c_s^2}(k_c - k_b)$.

Meaning the analogy between the internal gravitational waves (IGW) in the upper atmosphere and ballooning waves in the plasma sheet, the role of gravity in the latter case taken by the magnetic tension force (the last term in the RHS of the momentum equation (2)), and taking into account that the IGW propagation velocity is non-zero only if there is a wave vector component along the gravity, we set small perturbations of the quantities entering (1) – (4) in the form ~$\exp(i\, k_n\, n + i\, k_y\, y + i\, k_{||}\, s - i\, \omega\, t)$. The wavenumber k_n along the magnetic curvature vector as well as the azimuthal wavenumber k_y are supposed to be large, so that $k_\perp = (k_n^2 + k_y^2)^{1/2} \to \infty$. Further **u** denotes the velocity perturbation (the background velocity is supposed to be zero) and prefix δ is used to mark the perturbations of other quantities.

The linearization of the n-and y-components of the momentum equation (2) within the local approximation, that is, suggesting $\nabla_{||}\delta X = i\cdot k_{||}\, \delta X$ (see also [*Ohtani and Tamao*, 1993]) yields

$$[c_A^2 2k_c(k_b + k_c - ik_n) - (\omega^2 - c_A^2 k_{||}^2)]u_n + [c_A^2(2k_c - ik_n)(1 - \frac{\gamma c_s^2 k_{||}^2}{\omega^2}) - i\gamma c_s^2 k_n]\nabla \bullet \mathbf{u} = 0 \tag{6}$$

$$[(\omega^2 - c_A^2 k_{||}^2)(k_b + k_c - ik_n) - 2c_A^2 k_c k_y^2]u_n + \{[(\omega^2 - c_A^2 k_{||}^2) - c_A^2 k_y^2](1 - \frac{\gamma c_s^2 k_{||}^2}{\omega^2}) - \gamma c_s^2 k_y^2\}\nabla \bullet \mathbf{u} = 0 \tag{7}$$

In deriving formulas (6) and (7) the following relations were used (see also [*Liu*, 1997])

$$\delta p = -\frac{ip}{\omega}(k_p u_n + \gamma \nabla \bullet \mathbf{u}) \tag{8}$$

$$\mathbf{n} \bullet \big((\delta \mathbf{B} \bullet \nabla)\mathbf{B} + (\mathbf{B} \bullet \nabla)\delta \mathbf{B}\big) = 2B\delta B_{||} k_c + ik_{||} B \delta B_n \tag{9}$$

$$\delta B_{||} = \frac{iB}{\omega}(-\nabla \bullet \mathbf{u} + ik_{||}u_{||} - (k_b + k_c)u_n) \tag{10}$$

$$\delta B_n = -\frac{B}{\omega} k_{||} u_n \tag{11}$$

$$\delta B_y = -\frac{1}{k_y}(k_{||}\delta B_{||} + k_n \delta B_n) \tag{12}$$

$$u_{||} = -\frac{ik_{||}\gamma p}{\omega^2 \rho}\nabla \bullet \mathbf{u} \tag{13}$$

$$u_y = -\frac{\omega \delta B_y}{k_{||} B} \tag{14}$$

From (6) and (7) the following dispersion relation can be obtained for the slow mode with dominant perpendicular wavenumbers

$$(\omega^2 - c_A^2 k_\parallel^2)(k_n^2 + k_y^2)[c_A^2(\omega^2 - \gamma c_s^2 k_\parallel^2) + \gamma c_s^2 \omega^2]$$

$$= 2k_c c_A^2 k_y^2 [(k_b - k_c) c_A^2 (\omega^2 - \gamma c_s^2 k_\parallel^2) + (k_b + k_c)\gamma c_s^2 \omega^2] \quad (15)$$

Since it is known that the flapping motions, which we identify with the ballooning waves, develop in the background configurations with high β (e.g. [*Sergeev et al.*, 2004]), we will use at this step the condition β>>1 or identically $\frac{c_A^2}{c_s^2} << 1$. Then (15) is reduced to

$$\omega^4 - \omega^2 2c_A^2 [k_\parallel^2 + k_c(k_b + k_c)\frac{k_y^2}{k_n^2 + k_y^2}] + c_A^4 [k_\parallel^4 + 2k_c(k_b - k_c)\frac{k_y^2 k_\parallel^2}{k_n^2 + k_y^2}] = 0 \quad (16)$$

which solutions are

$$\omega^2 = c_A^2 [k_\parallel^2 + k_c(k_b + k_c)\frac{k_y^2}{k_n^2 + k_y^2} \pm \sqrt{k_c^2 (k_b + k_c)^2 \frac{k_y^4}{(k_n^2 + k_y^2)^2} + 4k_\parallel^2 k_c^2 \frac{k_y^2}{k_n^2 + k_y^2}}] \quad (17)$$

One can see that for k_n=0 solution (17) coincides with that of *Liu* [1997] (his formula (26)).
Further we choose the positive sign before the square root in the RHS of (17) implying the propagation, i.e. real ω case. The minus sign refers to destabilization of the ballooning mode, which was previously considered by *Liu* [1997].

3. Propagation velocities of the ballooning waves predicted from the linear analysis

The formulas for the group velocities of the ballooning waves in the plane perpendicular to the magnetic field were derived by *Golovchanskaya and Maltsev* [2005] in neglecting the term ~ $O(k_\parallel)$ in (17). In such a case (17) takes the form of the well-known dispersion relation for the IGWs in the upper atmosphere

$$\omega^2 = \omega_g^2 \frac{k_y^2}{k_n^2 + k_y^2} \,, \quad (18)$$

In the atmosphere $\omega_g^2 = g/H$, where g is the gravity acceleration, H the characteristic scale of density variation with height. For the ballooning waves in this approximation $\omega_g^2 = 2c_A^2 k_c(k_b+k_c)$, or supposing $k_b << k_c$ for the high-β case, $\omega_g \approx (\sqrt{2})\, c_A k_c$.

Here we derive a more rigorous formula for the group velocities of the ballooning waves, while keeping preserved all the terms in (17).

The group velocity in the y direction is given by

$$v_y^{group} = \frac{\partial\omega}{\partial k_n} = \frac{1}{\omega}\frac{c_A^2 k_c k_y k_n^2}{(k_y^2 + k_n^2)^2}\{k_b + k_c + [(k_b + k_c)^2 + 4k_\parallel^2 \frac{(k_n^2 + k_y^2)}{k_y^2}]^{1/2} - \frac{2k_\parallel^2 (k_y^2 + k_n^2)}{k_y^2}[(k_b + k_c)^2 + 4k_\parallel^2 \frac{(k_n^2 + k_y^2)}{k_y^2}]^{-1/2}\} \quad (19)$$

$$v_n^{group} = \frac{\partial\omega}{\partial k_n} = -\frac{1}{\omega}\frac{c_A^2 k_c k_n k_y^2}{(k_y^2 + k_n^2)^2}\{k_b + k_c + [(k_b + k_c)^2 + 4k_\parallel^2 \frac{(k_n^2 + k_y^2)}{k_y^2}]^{1/2} - \frac{2k_\parallel^2 (k_y^2 + k_n^2)}{k_y^2}[(k_b + k_c)^2 + 4k_\parallel^2 \frac{(k_n^2 + k_y^2)}{k_y^2}]^{-1/2}\} \quad (20)$$

$$v_\parallel^{group} = \frac{\partial\omega}{\partial k_\parallel} = \frac{c_A^2}{\omega}\{k_\parallel + 2k_c k_\parallel [(k_b + k_c)^2 + 4k_\parallel^2 \frac{(k_n^2 + k_y^2)}{k_y^2}]^{-1/2}\} \quad (21)$$

From (19) and (20) it is seen that both v_y^{group} and v_n^{group} vanish for k_n=0. We further simplify formulas (19), (20), (21) by taking $k_n = k_y \equiv \kappa$ and adopting $k_b << k_c$. The formulas are reduced to

$$v_y^{group} = -v_n^{group} = \frac{c_A k_c}{4\kappa} \frac{\{k_c(8k_\|^2 + k_c^2)^{1/2} + 6k_\|^2 + k_c^2\}}{(8k_\|^2 + k_c^2)^{1/2}\{k_\|^2 + \frac{1}{2}k_c^2 + \frac{1}{2}k_c(8k_\|^2 + k_c^2)^{1/2}\}^{1/2}} \tag{22}$$

$$v_\|^{group} = \frac{c_A k_\| \{1 + 2k_c(8k_\|^2 + k_c^2)^{-1/2}\}}{\{k_\|^2 + \frac{k_c^2}{2} + \frac{k_c}{2}(8k_\|^2 + k_c^2)^{1/2}\}^{1/2}} \tag{23}$$

We consider (22), (23) for $k_\| \sim O(k_c)$, a case more consistent with the WKB approximation than the condition $k_\|^2 << k_c^2$ previously suggested by *Golovchanskaya and Maltsev* [2005] and resulting in

$$v_y^{group} \approx \frac{c_A k_c}{2k_y} \tag{24}$$

Then the following estimates keep true

$$v_y^{group} = -v_n^{group} = \frac{5c_A k_c}{6\sqrt{3}\kappa} \tag{25}$$

which is only decreased by ~ 4 per cent compared to v_y^{group} given by (24).

For the parallel group velocity we have from (23) for $k_\| \sim O(k_c)$

$$v_\|^{group} = \frac{5}{3\sqrt{3}} c_A \tag{26}$$

which is close to c_A.

Relations (24), (25) permit to reasonably explain the main observed features of the flapping motions, provided we identify them with the ballooning waves [*Golovchanskaya and Maltsev,* 2005]. Indeed, (24), (25) predict azimuthal propagation velocities of a few tens km/s for quiet conditions, an increase in the propagation velocities up to 160 - 400 km/s for disturbed conditions (because of k_c increasing), a dispersion over the wavelengths, namely, larger propagation velocities for longer wavelengths.

4. Ballooning wave propagation as a solution of the Cauchy problem

In neglecting the terms containing the field-aligned wave numbers, the dispersion relation (17) corresponds to the following linear partial derivative equation describing the transverse propagation of the ballooning waves

$$\frac{\partial^2}{\partial t^2}\left(\frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2}\right) + \omega_g^2 \frac{\partial^2 f}{\partial y^2} = 0 \tag{27}$$

where $\omega_g^2 = 2 \cdot c_A^2 k_c^2$. As was earlier mentioned, the sources of the ballooning waves in the magnetosphere are most likely substorm and/or bursty bulk flow activities. Since both are maximum at the center of the magnetotail, we refer the *y*-coordinate of the source (*y* = 0) to the midnight meridian. According to spacecraft observations, the source position along the tail (further *x* = 0) typically varies from $x_{GSM} \sim -20$ R_E, R_E being the Earth's radius, down the tail.

The general solution of eq. (27) can be found by performing a direct Fourier transform over *f* and then a reverse Fourier transform. As a result, the solution is expressed via the initial conditions for the function f (*x, y, t*=0) and for its time derivative f_t (*x, y, t*=0) as

$$f(x,y,t) = \iint f(x', y', t=0) \frac{\partial S_1(x-x', y-y', t)}{\partial t} dx'dy' + \iint f_t(x', y', t=0) \cdot S_1(x-x', y-y', t) dx'dy' \tag{28}$$

in which

$$S_1(x,y,t) = \frac{1}{2\pi} \iint \frac{\sin\left(\frac{\omega_g k_y}{\sqrt{k_x^2 + k_y^2}} t\right)\sqrt{k_x^2 + k_y^2}}{\omega_g k_y} e^{i\cdot(k_x x + k_y y)} dk_x dk_y \tag{29}$$

Generally, (28), (29) describe three types of the solutions with regard their spatial behavior: (i) oscillations in the whole space with finite amplitude everywhere; (ii) oscillations with finite amplitude at the source point and growing at infinity; (iii) oscillations singular at the source point and fading at infinity. Accounting for the nature of the source launching the ballooning waves, we will consider only the solutions of the third type as physically meaningful for our problem.

For the initial conditions chosen in a special form (see Figure 1 for the function $f_t(x, y, t=0)$)

$$f(x, y, t=0) = 0 \qquad (30)$$

$$f_t(x, y, t=0) = \frac{\omega_g x y^2}{(x^2 + y^2)^2} \qquad (31)$$

eq. (27) allows an analytical solution. The solution can be written as

$$f(x, y, t) = \frac{y^2}{(x^2 + y^2)^{3/2}} \sin\left(\frac{\omega_g x \cdot t}{(x^2 + y^2)^{1/2}} \right) \qquad (32)$$

At the source point (*x=0, y=0*), the function *f*(*x, y, t*) given by (32) has a singularity of the order ~ 1/r, where r is the radial distance from the origin of coordinates. Figure 2 shows the temporal behavior of the solution, the coordinates are in R_E, time t is normalized by $t_g = 2 \cdot \pi / \omega_g$. Within the approximation that we use in this section, the frequency ω_g can be taken in the form $\omega_g = \sqrt{2} \frac{v_T}{a}$. Here v_T is the thermal velocity of ions in the neutral sheet and *a* is the characteristic width of the magnetotail current sheet. The typical value of ω_g is ~ 10^{-1} s^{-1} for quiet conditions and several times higher for more stretched tail configurations under disturbed conditions. Correspondingly, the characteristic time t_g varies from 1 minute to a few tens of seconds.

From Figure 2 it is seen that the perturbation given at t=0 by (30), (31) propagates predominantly in the ±*y* directions, exhibiting dispersion over the wavelengths and forming fan-like wave structures with time.

5. Summary and discussion

Till recently the theory of ballooning perturbations in the magnetosphere has mostly been developing meaning the application to the substorm problem. For this purpose it is not very important whether a variation of the perturbation along the magnetic curvature vector is included. Even if taken into account, this effect can not be crucial for the occurrence of the ballooning instability, which is now considered as one of the candidates for substorm triggers.

The situation changed with the appearance of Cluster observations. It was revealed that so-called flapping waves routinely observed in the magnetosphere are inconsistent with any conventional magnetospheric wave mode. In sections 2 and 3 above we extended the theory of *Liu* [1997], developed for the ballooning instability in a high-β plasma, by including a non-zero k_n component of the wave vector. The obtained dispersion relation, here considered in a more rigorous form than previously in [*Golovchanskaya and Maltsev*, 2005], indicates the ballooning waves, which match the observed flapping characteristics.

In section 4 we showed that even in the linear (small amplitude) case the solution of the equation describing ballooning wave propagation is rather complicated and has little in common with the solution of the traditional wave equation. The ballooning wave formulation, however, should be further complicated to account for the observational evidence of large amplitudes of the waves. Thus, a development of non-linear ballooning theory is needed to make a comparison with the observations more complete.

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References

Golovchanskaya, I. V., and Y. P. Maltsev, On the identification of plasma sheet flapping waves observed by Cluster, *Geophys. Res. Lett., 32*, L02102, doi:10.1029/2004GL021552, 2005.

Hargreaves, J. K., *The Upper Atmosphere and Solar-Terrestrial Relations*, Van Nostrand Reinhold Company, New York – Cincinnati – Toronto – London – Melbourne, 1979.

Lui, A., C.-I. Meng, and S.-I. Akasofu, Wavy nature of the magnetotail neutral sheet, *Geophys. Res. Lett.*, 5, 279 – 282, 1978.

Ohtani, S., and T. Tamao, Does the ballooning instability trigger substorms in the near-Earth magnetotail? *J. Geophys. Res., 98*, No A11, 19,369-19,379, 1993.

Runov A., R. Nakamura, W. Baumjohann, T. L. Zhang, M. Volwerk, and H.-U. Eichelberger, Cluster observation of a bifurcated current sheet, *Geophys. Res. Lett., 30*, No 2, 1036, doi:10.1029/2002GL016136, 2003.

Runov, A., V. A. Sergeev, W. Baumjohann, R. Nakamura, S. Apatenkov, Y. Asano, M. Volwerk, Z. Vörös, T. L. Zhang, A. Petrukovich, A. Balogh, J.-A. Savaud, B. Klecker, and H. Rème, Electric current and magnetic field geometry in flapping magnetotail current sheets, *Ann. Geophys.*, 23, 1391 – 1403, 2005.

Sergeev, V., A. Runov, W. Baumjohann, R. Nakamura, T. L. Zhang, M. Volwerk, A. Balogh, H. Rème, J. A. Sauvaud, M. André, and B. Klecker, Current sheet flapping motion and structure observed by Cluster, *Geophys. Res. Lett.*, 30(6), 1327, doi:10.1029/2002GL016500, 2003.

Sergeev, V., A. Runov, W. Baumjohann, R. Nakamura, T. L. Zhang, A. Balogh, P. Louarnd, J.-A. Sauvaud, and H. Rème, Orientation and propagation of current sheet oscillations, *Geophys. Res. Lett., 31*, No 5, L05807, doi:10.1029/2003GL019346, 2004.

Speiser, T., and N. Ness, The neutral sheet in the geomagnetic tail: its motion, equivalent currents, and field line reconnection through it, *J. Geophys. Res.*, 72, 131 – 141, 1967.

Zhang, T., W. Baumjohann, R. Nakamura, A. Balogh, and K.-H. Glassmeier, A wavy twisted neutral sheet observed by Cluster, *Geophys. Res. Lett*., 29, 1899, doi:10.1029/2002GL015544, 2002.

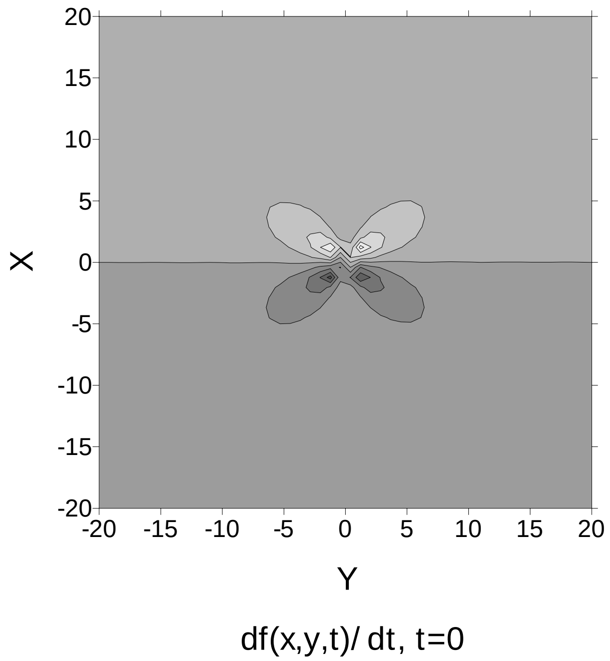

Figure 1. Initial condition for the time derivative of the ballooning perturbation. The function $f_t(x, y, t = 0)$ is shown in relative units, with negative (positive) values marked dark (light) grey. The coordinates are in R_E. The origin of coordinates is at the source point.

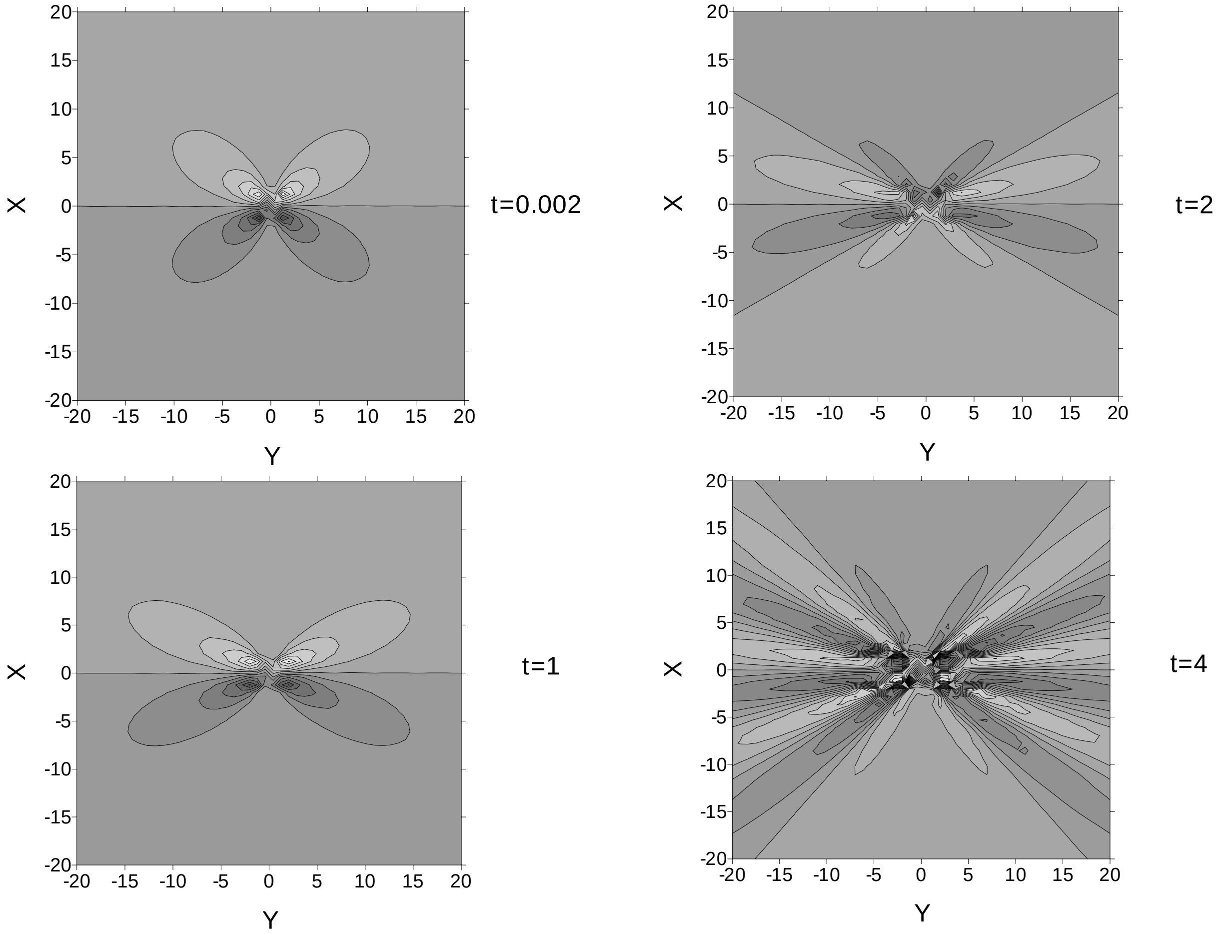

Figure 2. Solution of the equation for ballooning wave propagation at different times. The function $f(x,y,t)$ is in relative units, time t is normalized by $t_g = 2\cdot\pi/\omega_g$ (see the text).