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ON THE ALTITUDE DEPENDENCE OF SCHUMANN RESONANCE FREQUENCIES

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Abstract. Based on a realistic conductivity profile, the numerical modelling of the electromagnetic wave propagation in the cavity bounded by the Earth' conducting surface and the ionosphere has suggested that Schumann frequencies depend on height. At altitudes above 70 km, where the conductivity becomes high, a gradual decrease in the frequency has been found. This can be attributed to the fact that although the peaks for the total electromagnetic energy stored inside the cavity define a unique value for each Schumann resonance, the experimental determination of Schumann frequencies related with the maxima in the amplitude of the electromagnetic field Fourier transform spectrum is sensitive to the conductivity profile and will thus change with altitude. We show that a similar frequency shift can be obtained by using a simple analytical model for the Earth-ionosphere cavity.

1. Introduction

The realistic modelling of the propagation of electromagnetic waves through the ionosphere of the Earth is a complex challenge and requires the application of numerical methods. In particular, the Schumann frequencies in the lossy cavity bounded by the Earth' surface and the ionosphere have been successfully determined by using the so-called Transmission Line Matrix (TLM) method (Morente et al. 2003). In this numerical method, the medium is substituted by an equivalent three-dimensional transmission line mesh formed by repeating elementary transmission line structures (nodes), while the electromagnetic field is modelled by analogous electrical signals, voltage and current pulses which propagate through the transmission line mesh. Pulses are scattered at the center of each node according to a scattering matrix associated with each node and upon arriving of these pulses at the adjacent nodes, the next time step in the simulation is initiated. Moreover, a broadband signal located at an arbitrarily chosen mesh point is introduced to model the excitation of the atmosphere due to lightning. By using the conductivity profile of Schlegel and Füllekrug (1999), Morente et al. (2004) have numerically modelled the Schumann frequencies in the Earth-ionospheric cavity and found a height-dependent shift of the resonance frequencies to smaller values. Fig. 1 displays the altitude profile for the first six peak frequencies obtained by means of the TLM method. While the frequency decrease is rather small at lower altitudes, it becomes more pronounced above ~ 70 km, where the conductivity, as shown by the dashed line in Fig. 1, starts to increase significantly. At the upper boundary, the resonant frequencies are reduced by about 10% of its corresponding values at the ground. This behaviour can be understood by noting that the experimental determination of the Schumann frequencies is carried out by local rather than global measurements of the magnetic or electric field components. However, these quantities depend on the conductivity, which varies with height, and therefore in a dissi-pative system, the peak frequencies may also vary with height. In the following section we briefly summarize the reasoning of Morente et al. (2004), for more details the reader is referred to this paper.

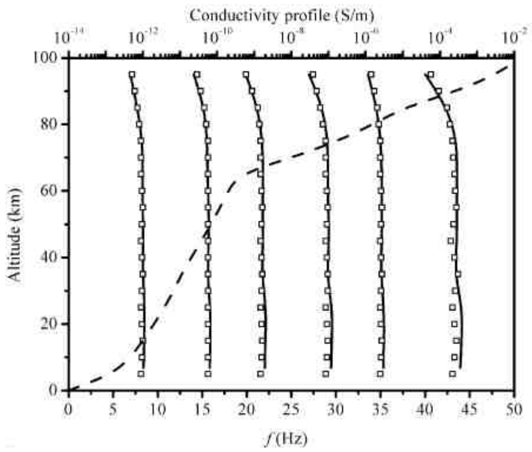

Figure 1: First six Schumann resonance frequencies of the azimuthal component of the magnetic field as a function of altitude. The solid lines correspond to a discrete Fourier transform of the entire time series of ~ 106 time steps, while the squares correspond to a reduced window from 103 -106 time steps. The dashed line illustrates the conductivity profile used in the simulation (from Morente et al. 2004).

2. Resonant electrical circuit

Let us consider a series of parallel connected capacitors, conductivities and resistivities as shown in Fig. 2. Each sub-circuit consists of an inductance L, a capacitance C, and a resistance R_i which correspond to the magnetic permeability, the electric permittivity, and to the atmospheric conductivity, respectively. *L* and *C* are assumed to be constant, whereas R_i is considered to be different in each stage, thereby representing an altitude depending conductivity. The whole circuit is excited by an external current source I = $I_0 e^{i\omega t}$ and shows some characteristics similar to those of the medium in the Earth ionospheric cavity.

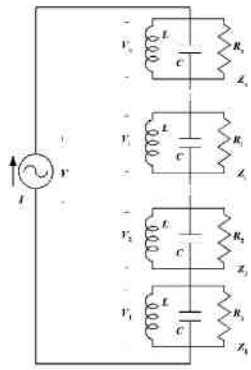

Figure 2: Resonant circuits corresponding to different altitudes. While *L* and *C* are constant, the resistivity R_i changes with height, making the different circuits to resonate at different frequencies (from Morente et al. 2004).

The impedance Z_i of the *i*-th circuit is given by

$$Z_i = \frac{i\omega L}{1-\omega^2 LC + i\omega L / R_i} \tag{1}$$

and the maximum amplitude $|Z_i|$ occurs at a frequency

$$\omega \equiv \omega_0 = \frac{1}{\sqrt{LC}} \tag{2}$$

It should be noted that the maximum energy transfer from the source into the system takes place at that frequency. However, the maximum amplitude of the current $(I_L)_i$ through the *i*-th inductor occurs at a different frequency

$$(\omega_L)_i = \omega_0 \sqrt{1-2\delta_i^2}\ , \tag{3}$$

where the damping constant is given by

$$\delta_i = \frac{1}{2R_i}\sqrt{\frac{L}{C}}\ . \qquad 4)$$

Finally the current through the capacitor at the *i*-th stage is maximum at the frequency

$$(\omega_c)_i = \frac{\omega_0}{\sqrt{1-2\delta_i^2}}\ , \tag{5}$$

showing that the capacitors, inductors and the global circuit all resonate at different frequencies; only for vanishing damping these frequencies coincide. Thus, in a lossy system, the resonance frequency of the magnetic field associated with the currents through the capacitors and inductances of the different stages will be different from the peak frequency corresponding to the maximum energy transfer due to the position depend resistivity. A similar behavior appears in the terrestrial ionospheric cavity in regions of enhanced energy dissipation as illustrated in Fig. 1.

3. A simple analytical model

We consider a simple spherical symmetric two-layer waveguide model analogue to the one treated by Roldugin et al. (2003) (Fig. 3). We extend this model by introducing a line current ***j*** at $z = b$ which is assumed to have only a θ-component j_θ. The inner boundary at $z = 0$ is represented by the surface of the Earth, where the electric conductivity σ is assumed to be infinite $(\sigma = \infty)$. Up to an altitude $z = a$ the relative electric permittivity is given by $\varepsilon_r = \varepsilon_i = 1$, above this altitude $(z >$ a), in the ionosphere, ε_r is assumed to have a different, however spatial independent value $\varepsilon_r = \varepsilon_i$. Since the main contribution to ε_r is due to the free electrons, the permittivity in the ionosphere can be written as

$$\varepsilon_i = 1 - \frac{\omega_p^2}{\omega(\omega - i\nu)}\ , \tag{6}$$

where $\omega_P = (Ne^2/\varepsilon_0 m)^{1/2}$ is the plasma frequency of the electrons (with *e*, *m* and *N* being the electron charge, mass and number density, respectively) and ν is the collision frequency between the electrons and the other constituents. Because we are only interested in the very low frequency solutions, (6) can be approximated for ω $\omega / i\nu <<$ 1 by

$$\varepsilon_i \cong \frac{\omega_p^2}{i\nu\omega} = -\frac{i\sigma}{\omega} \tag{7}$$

with $\sigma = \omega_p^2 / \nu$.

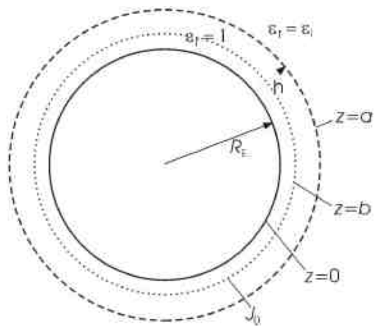

Figure 3: Two-layer model of the ionosphere with an external line current.

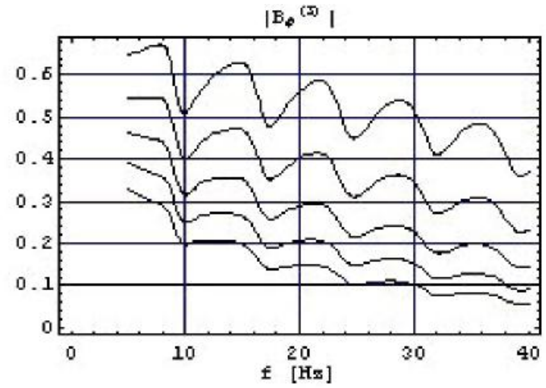

Figure 4: Magnitude of B_ϕ for different altitudes in the ionosphere. For the five lines from top to bottom the altitude increases in steps of 10 km from z = 80 to z = 120 km.

For a harmonic time dependence ($\mathbf{B} = \mathbf{B}_0 e^{i\omega t}$) and a spatially constant dielectric permittivity, the Maxwell equations read

$$\nabla \times \nabla \times \mathbf{B} = \frac{\omega^2}{c^2} \varepsilon_r \mathbf{B} + \mu_0 \nabla \times \mathbf{j}. \quad (8)$$

If we consider only TM^r modes (i.e. B_r = 0), assume that the field is independent from the azimuthal angle and that $h \ll R_E$ we obtain the homogeneous equation ($\boldsymbol{j}$ = 0) (Roldugin et al., 2003)

$$\frac{\partial^2 B_\phi}{\partial r^2} + (\varepsilon_r k^2 - k_s^2) B_\phi = 0\,, \quad (9)$$

with the abbreviations

$$k = \frac{\omega}{c}, \quad k_s = \frac{\sqrt{n(n+1)}}{R_E}\,, \; n=1,2,\ldots \quad (10)$$

Including the current, we search for solutions of the form

$$B_\phi(0<z<b) = C_1 \cos(k_z z) \quad (11\text{-a})$$

$$B_\phi(0<z<b) = C_2 [\, e^{ik_z(z-a)} + R_i e^{-ik_z(z-a)} \,] \quad (11\text{-b})$$

$$B_\phi(z>a) = C_i e^{ik_{zi}(z-a)}\,, \quad (11\text{-c})$$

where

$$k_z = \sqrt{k^2 - k_s^2}\,, \qquad k_{zi} = \sqrt{\varepsilon_i k^2 - k_s^2} \quad (12)$$

and R_i is the reflection coefficient; C_1, C_2 and C_i are unknown amplitudes yet to be determined. The boundary conditions read

$$B_\phi\big|_{z=a-0} = B_\phi\big|_{z=a+0} \quad (13\text{-a})$$

$$\frac{\partial B_\phi}{\partial z}\bigg|_{z=a-0} = \frac{1}{\varepsilon_i} \frac{\partial B_\phi}{\partial z}\bigg|_{z=a+0} \quad (13\text{-b})$$

$$B_\phi\big|_{z=b+0} - B_\phi\big|_{z=b-0} = \mu_0 J_\theta \quad (13\text{-c})$$

$$\frac{\partial B_\phi}{\partial z}\bigg|_{z=b-0} = \frac{\partial B_\phi}{\partial z}\bigg|_{z=b+0} \quad (13\text{-d})$$

ffrom which the reflection coefficient and the amplitudes of (11-a-c) can be obtained. Using (13-a-b) yields

$$R_i = \frac{\varepsilon_i k_z - k_{zi}}{\varepsilon_i k_z - k_{zi}} \quad (14)$$

while (13-d) involves

$$C_1 = -i \frac{C_2}{\sin(k_z b)} [e^{ik_z(b-a)} - R_i e^{-ik_z(b-a)}]. \quad (15)$$

Finally from (13-a,c) and (15) it follows

$$C_i = -\frac{i(1+R_i)\mu_0 J_\theta \sin(k_z b)}{e^{-ik_z a} - R_i e^{ik_z a}} \quad (16)$$

Fig. 4 illustrates the magnitude of the B_ϕ-component in the ionosphere as a function of frequency according to (11-c). The different lines show the B_ϕ-spectrum for different altitudes, where the top line corresponds to the lowest altitude. A slight shift of the peak frequencies to lower values with increasing height can be observed. In order to find an analytical expression for this resonant frequency change we first note that $|\varepsilon_i| \gg 1$ and the ionospheric wave number can hence be expressed as

$$k_{zi} = \frac{\omega_p}{c} \sqrt{\frac{\omega}{2\nu}} (1+i)\,. \quad (17)$$

Thus we arrive at

$$\left| \frac{B_\phi(z>a)}{\mu_0 J_\theta} \right| \equiv A(z>a) = A_0(z,\omega) e^{-(\omega_p/c)\sqrt{\omega/2\nu}(z-a)}\,, \quad (18)$$

where

$$A_0(z,\omega) = \left| \frac{C_i}{\mu_0 J_\theta} \right| \left| e^{i(\omega_p/c)\sqrt{\omega/2\nu}(z-a)} \right|. \quad (19)$$

Now, if we approximate the amplitudes A_0 in the vicinity of the maxima by a Gaussian function, we obtain together with (17)

$$A(z>a) = e^{-[(\omega-\omega_r)^2/\Delta\omega^2 + (\omega_p/c)\sqrt{\omega/2\nu}(z-a)]}\,, \quad (20)$$

with ω_r being the corresponding resonance frequency at $z = a$ and $\Delta\omega$ its half-width.

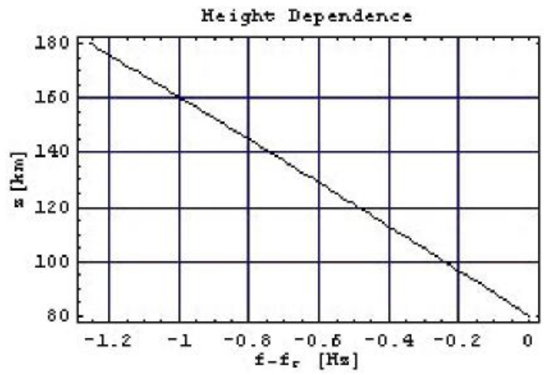

Figure 5: Shift of the first peak frequency of the azimuthal magnetic field versus height. The following model parameters have been used: ω_P = 1.8 x 10^7 Hz, ν = 2 x 10^8 s^{-1}, a = 80 km, ω_r = 94.2 Hz, $\Delta\omega$ = 31.4 Hz.

The maximum of this function is obtained by the condition ∂A(z > a)/∂ω = 0 and yields the resonance frequency at altitude $z > a$

$$\omega = \omega_r - \frac{z-a}{4}\frac{\omega_p}{c}\frac{\Delta\omega^2}{\sqrt{2\nu\omega}}, \qquad (21)$$

In Fig. 5, the altitude dependence of the first Schumann resonance is shown, leading to a reduction of ~ 1.2 Hz within 100 km or about 10% with respect to the peak frequency at z = 80 km.

Conclusion

The decrease of the forced Schumann resonance frequencies first found by Morente et al. (2004) via numerical simulations is obtained by means of a simple analytical model of the terrestrial ionospheric cavity. In addition to the results of Morente et al. (2004), it is shown that the resonance frequency shift can also occur in a system with constant (i.e. altitude independent) damping as long as the latter depends on frequency.

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References

[1] Morente, J.A., G.J. Molina-Cuberos, J.A. Port, B.P. Besser, A. Salinas, K. Schwingenschuch, and H. Lichtenegger, A numerical simulation of Earth's cavity with the Transmission Line Matrix method: Schumann resonances, J. Geophys. Res., 108(A5), 1195-1205 (2003).

[2] Morente, J.A., J.A. Port, A. Salinas, G.J. Molina-Cuberos, H. Lichtenegger, B.P. Besser, K. Schwingenschuh, Do Schumann resonance frequencies depend on altitude?, J. Geophys. Res., 109, A05306, doi:10.1029/2003JA010305 (2004).

[3] Roldugin, V. C., Y. P. Maltsev, A. N. Vasiljev, A.V. Shvets, and A. P. Niko-laenko, Changes of Schumann resonance parameters during the solar proton event of 14 July 2000, J. Geophys. Res., 108(A3), 1103, doi:10.1029/2002JA009495 (2003).

[4] Schlegel, K., and M. Füllekrug, Schumann resonance parameter changes during high-energy particle precipitation, J. Geophys. Res., 104(A5), 10111-10118 (1999).