"Physics of Auroral Phenomena", Proc. XXVIII Annual Seminar, Apatity, pp. 100-103 , 2005

Polar Geophysical Institute

INTERACTION OF THE FIELD-ALIGNED CURRENT FRONT WITH THE ANOMALOUS RESISTANCE LAYER

N. Mazur[1], E. Fedorov[1], V. Pilipenko[1], and T. Uozumi[2]

[1]*Institute of the Physics of the Earth, Moscow, Russia*
[2]*Space Environment Research Center, Kyushu University, Fukuoka, Japan*

Introduction: Onset of Anomalous Resistance in Space Plasma

Anomalous resistance due to the high-frequency turbulence is an ubiquitous element of a collisionless space plasma. Though various aspects of the anomalous resistance in plasma have been extensively examined by plasma theorists, the initial moment of the anomalous resistance establishment and subsequent changes of MHD properties of a medium have not been considered. From a general physical point of view it seems natural, that a sudden occurrence of a finite conductivity in a part of a system might exert a backward influence on an external current producing this conductivity. We attempt to consider self-consistently the interaction of a current front with a region where anomalous resistance could occur and accompanying transient MHD disturbances.

The anomalous resistance on auroral field lines is one of the key elements of the magnetosphere-ionosphere interaction. The emergence of an anomalous resistivity layer (ARL) with finite field-aligned conductivity $\sigma_{\|}$ results in the occurrence of anomalous electric field $E_{\|} \cong j_0/\sigma_{\|}$. This field-aligned $E_{\|}$ accelerates down-going electrons, which, in their turn, cause additional ionization of the ionosphere and activation of the auroral activity. Ionospheric ionization and relevant modification of the conductance provide a feedback in the coupled ionosphere-magnetosphere system. After turn-on of the positive feedback, a global instability of the ionospheremagnetosphere system goes on to an explosive phase with a much higher, may be non-linear, growth rate.

Another aspect of the ARL occurrence on auroral field lines is the possibility of an additional mechanism of Pi2 generation [*Arykov and Maltsev*, 1983]. Later on, *Pilipenko et al.,* [2005] developed a mathematical formalism for the description of the Alfven impulse (AI) generation by this mechanism and provided some observational evidence in favor of this hypothesis. However, in realistic situation, the induced AI may influence the total field-aligned current and establishment of anomalous resistance regime. In this paper we consider the self-consistent problem on excitation of both the anomalous resistance and AI at the front of field-aligned current entering the region with favorable conditions for the excitation of plasma turbulence.

Model and Basic Equations

We use a mathematical formalism for the description of the AI generation during the switch-on of anomalous resistivity described by *Pilipenko et al.* [2005]. In the model used, the homogeneous geomagnetic field is directed vertically up, $\mathbf{B}_0 = B_0\hat{z}$. The homogeneous magnetospheric plasma has a vanishing transverse static conductivity and infinite field-aligned conductivity $\sigma_{\|}. = \infty$. Similar to realistic magnetospheric situation, it is assumed that the threshold for the excitation of the anomalous resistance is lower within a certain interval of altitudes, further named as ARL. So, at the altitude $z = 0$ the finite $\sigma_{\|}.$ occurs inside the layer with the thickness b.

The mathematical approach is based on Maxwell's equations augmented by the ideal MHD equations. In this system the possible MHD disturbances are described by the following decoupled equations for Alfven waves, carrying the field-aligned current jz, and compressional waves, carrying the field-aligned magnetic field disturbance B_z

$$\partial_{tt} j_z - V_A^2 \partial_{zz} j_z = \frac{c^2}{4\pi} \nabla_\perp^2 \partial_t (\sigma_{\|}^{-1} j_z),$$
$$\partial_{tt} B_z - V_A^2 \nabla^2 B_z = 0. \qquad (1)$$

We assume that the ARL is a thin layer as compared to the Alfven wave length. Therefore, the thin layer approximation can be used, that is the ARL thickness $b \to 0$, whereas its resistivity $Q = b/\sigma_{\|}$ remains finite. In this approximation the Alfven wave equation is valid both in the upper ($z > 0$) and lower ($z < 0$) hemi-spaces:

$$\partial_{tt} j_z - V_A^2 \partial_{zz} j_z = 0. \qquad (2)$$

This equation must be supplemented with two boundary conditions at the interface $z = 0$ between two hemispaces, separated by a thin layer ($b \to 0$) with the resistivity Q. The first condition is the requirement of the continuity of j_z across the ARL. The second boundary condition is obtained by integration of the first equation from the system (1) across the layer and subsequent transition to the limit $b \to 0$, as follows:

$$\{\partial_z j_z\}_{z=0} + \nabla_\perp^2 \partial_t [R\, j_z^{(0)}] = 0. \qquad (3)$$

Here $R(x, y, t) = \dfrac{c^2}{4\pi V_A^2} Q = \Sigma_A Q V_A^{-1}$ is the normalized resistance of the ARL, $\Sigma_A = c^2/(4\pi V_A)$ is the Alfven wave conductance, $\{\partial_z j_z\}_{z=0} = \partial_z j_z(x, y, +0, t) - \partial_z j_z(x, y, -0, t)$ is the jump of the current density derivative across the ARL, and $j_z^{(0)}$ is the current at the upper ARL boundary ($z = 0$).

Self-consistent model of the current front interaction with ARL

Let us assume that from the magnetosphere a jz front propagating with Alfven velocity impinges the topside ionosphere

$$j_0(x, y, z, t) = J\,(z - z_0(x, y) + V_A t)\,. \qquad (4)$$

Function $J(z)$ is assumed to be monotonically growing; thus the field-aligned current density in the layer at $z = 0$ gradually increases. As soon as the current density through the ARL $j_z^{(0)}$ exceeds a threshold value j_*, an anomalous field-aligned resistance ignites. This process will be described by the simplified two-step model

$$R(j_z^{(0)}) = \begin{cases} 0 & \text{if } j_z^{(0)} < j_*, \\ R_0 & \text{if } j_z^{(0)} > j_*\,. \end{cases} \qquad (5)$$

Thus we take into account that the ARL resistance depends on magnitude of current through the layer, that is, we consider a self-consistent problem.

We suppose that the monotonic function $J(z)$ is such that $J(0) = j_*$. Then the wave "critical" front, i.e. the surface where $j_0(x, y, z, t) = j_*$ is $z = z_0 - V_A t$, and at $t = 0$ it intersects with the plane $z = 0$ at a point $x = 0$, $y = 0$. Below the critical front ($z < z_0 - V_A t$) the solution of (2, 3), where $R(x, y, t)$ is given by (5), is an undisturbed current front (4). The region $D_{AR}(t)$ in the plane $z = 0$, where anomalous resistivity has been switched-on, $R \neq 0$, is bounded by the curve $z_0 = V_A t$.

The disturbance caused by the turn-on of anomalous resistance propagates from the plane $z = 0$. The source of this disturbance is the growing with time region $D_{AR}(t)$. The solution of (2) for $t \geq 0$ evidently has the form

$$j_z(x, y, z, t) = J(z - z_0(x, y) + V_A t) + j_z^{(A)}\,(x, y, t \mp z/V_A) \text{ for } z \geq 0 \text{ and } z \leq 0. \qquad (6)$$

This function $j_z^{(A)}(x, y, t)$ characterizes the disturbance of field-aligned current, transported by induced AI propagating away from both sides of the ARL.

The second boundary condition (3) enables us to obtain an equation to determine the induced current $j_z^{(A)}(x, y, t)$ and total current through the layer at $z = 0$. Substituting $\{\partial_z j_z\}_{z=0}$ from (6) into the condition (3), one obtains

$$V_A \nabla_\perp^2 (R j_z^{(0)}) - 2 j_z^{(0)} = -2J_0\,, \qquad (7)$$

where $J_0(x, y, t) = J\,[V_A t - z_0(x, y)]$ is a given external current in the plane $z = 0$.

It is more convenient to consider (7) as an equation in respect to the variable $R\, j_z^{(0)}$ which is proportional to the potential drop across the layer. For an easy comparison with relevant relationships from [*Arykov and Maltsev,* 1983] and [*Pilipenko et al.,* 2005] we use the potential at the upper boundary of the ARL, i.e. $\varphi = \varphi(x, y, +0, t)$, related to $R\, j_z^{(0)}$ by a relationship $R\, j_z^{(0)} = -2\Sigma_A V_A^{-1} \varphi$. Then (7) is reduced to the following form

$$\Sigma_A \nabla_\perp^2 \varphi + j_z^{(0)} = J_0\,. \qquad (8)$$

Here the current $j_z^{(0)}(\varphi)$ in accordance with (5) has the form

$$j_z^{(0)}(\varphi) = \begin{cases} j_*(\varphi/\varphi_*) & \text{if } \varphi \leq \varphi_*, \\ j_* & \text{if } \varphi_* \leq \varphi < 0, \end{cases} \qquad (9)$$

where $\varphi_* = -0.5\,\Sigma_A^{-1} V_A R_0 j_* = -0.5 Q_0 j_*$. The occurrence of permanent value of $j_z^{(0)}(\varphi) = j_*$ when a potential drop across the ARL is less than some critical value (9) corresponds physically to the plasma state near the instability threshold. In this state the resistance R has some intermediate value between 0 and R_0.

The potential $\varphi(x, y)$ is to be continuous upon the transition across the boundary $J_0 = j_*$. Thus, the boundary condition for (8) is $\varphi(x, y) = 0$ at the boundary of D_{AR}. The current $j_z^{(0)}(x, y)$ is also a continuous function in the entire plane $z = 0$.

1D step-wise external current front

Let us consider the case when the external current (4) does not depend on y coordinate. In this case (8) has the form

$$\varphi'' = \Sigma_A^{-1}[J_0(x,t) - j_z^{(0)}(\varphi)], \qquad (10)$$

where $J_0(x, t) = J(V_A t - z_0(x))$, and the region D_{AR} is reduced to the interval $x_- < x < x_+$. If the integral curve of (10), connecting $(x_-, 0)$ and $(x_+, 0)$ does not go below the level $\varphi = \varphi_*$, then with regard for (9) the initial stage of the process can be described by a simple equation

$$\varphi'' = \Sigma_A^{-1}\Delta_0(x,t), \qquad (11)$$

where $\Delta_0(x, t) = J_0(x, t) - j_*$ is the surpass of the external current above the threshold.

After the minimum reaches the level φ_*, the integral curve $\varphi = \varphi(x)$ in its middle part goes to the region $\varphi > \varphi_*$, where owing to (9), it is determined by the following linear equation

$$\varphi'' = \lambda_A^{-2}\varphi + \Sigma_A^{-1}J_0(x,t), \qquad (12)$$

Here the scale $\lambda_A = \sqrt{V_A R_0 / 2} = \sqrt{\Sigma_A Q_0 / 2}$ related to the field-aligned resistivity is the Alfven resistive scale, introduced by *Vogt* [2000] and *Fedorov et al.* [2000]. The solution of (12) can be expressed analytically via quadratures. The complete solution is to be obtained by the smooth joining the solutions (11) and (12).

As an example, we consider a step-wise spatial distribution of the external current

$J_0(x, t) = J_0(t)\,\eta(\Lambda_\perp - |x|)$,

Let variation of $J_0(t)$ has a typical time scale T_0. The external current front (4) with $z_0(x) = 0$ is assumed to be flat and non-vanishing inside a limited region $|x| < \Lambda_\perp$.

First we consider a more simple case when the solution $\varphi(x)$ does not go below φ_*. The solution of (11) with the conditions $\varphi(-\Lambda_\perp) = \varphi(\Lambda_\perp) = 0$ is

$$\varphi(x) = \frac{1}{2}\varphi_*\lambda_A^{-2}[j_*^{-1}J_0(t) - 1](\Lambda_\perp^2 - x^2). \qquad (13)$$

When the external current increases in time linearly, that is $J_0(t) = j_*\,(1 + t/T_0)$, the value $|\min \varphi(x)| = 0.5\varphi_*(\Lambda_\perp/\lambda_A)2(t/T_0)$ also grows in time linearly. The minimum of $\varphi(x)$ reaches the level φ_* when $(\Lambda_\perp/\lambda_A)^2(t/T_0) = 2$. Further, the current $j_z^{(0)}$ remains at the threshold level j_* during the retardation time

$$T_d = 2T_0\,(\lambda_A/\Lambda_\perp)^2.$$

If the current $J_0(t)$ never exceeds the value of $j_*[1 + 2(\lambda_A/\Lambda_\perp)^2]$, then the value of $\varphi(x)$ does not go below φ_*, whereas $j_z^{(0)}$ remains at the threshold level j_*, and begins to decrease only since $J_0(t)$ will become less than j_*.

Fig. 1 shows the time evolution of potential φ, current surplus $j_z^{(0)}$ -j_* (solid line), and $\Delta_0 = J_0 - j_*$ (dotted line) under the parameter $\Lambda_\perp/\lambda_A = 4.0$. The difference between dashed and solid curves in Fig. 1 is caused by the AI generation.

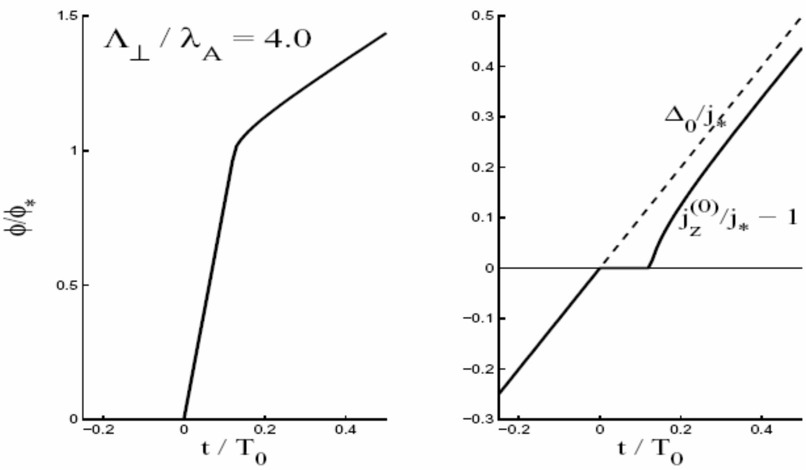

Figure 1. Time evolution of potential (left) and current (right) at $x = 0$.

Discussion: Consequences of the model

The intensity of field-aligned current front may become sufficient for the excitation of an anomalous resistance on auroral field lines. The sudden switch-on of $\sigma_\|$ results in the excitation of AI. Therefore, the occurrence of AI signifies the "switch-on" of an anomalous resistivity and thus may be considered as an indicator of the transition of a global magnetospheric instability into the ionosphere-coupled phase. The occurrence of resonant features of the ionosphere-magnetosphere system can produce oscillatory transient response. This mechanism of AI generation may contribute additionally to the Pi2 wave forms observed at auroral latitudes [*Arykov and Maltsev*, 1983].

The analytical solution of the self-consistent problem has shown that upon the entrance of field-aligned current front into the ARL the AI is generated. The impulse duration T_d depends on the ratio between the Alfven damping scale λ_A and external current width $\Lambda_\perp$. The interaction of current with ARL results in the delay of the current growth in the ARL by the time about T_d.

We suppose that at auroral latitudes Pi2 transient disturbance in fact comprises at least several possible driver mechanisms. Because of multiple nearly simultaneous contributions to Pi2 pulsations, there is no confirmative physical interpretation of their generation mechanism yet. A ”classical” Pi2 waveform — isolated damping quasi-sinusoidal train, is commonly observed at middle latitudes only. The fine temporal structure of auroral Pi2 may be used as a clue to the understanding and monitoring of a substorm process.

Acknowledgements. The discussions with A. Leonovich and D. Klimushkin are appreciated. This research is supported by the RFBR grant 04-05-64321 (NM) and INTAS grant 03-05-5359 (EF,VP).

References

Arykov A.A. and Maltsev Yu.P. Generation of Alfven waves in an anomalous resistivity region. Planet. Space Sci., 31, 267-273, 1983.

Fedorov, E., V. Pilipenko, and M.J. Engebretson, ULF wave damping in the auroral acceleration region, J. Geophys. Res., 106, 6203- 6212, 2001.

Pilipenko, V., N. Mazur, E. Fedorov, T. Uozumi, and K. Yumoto, Excitation of Alfven impulse by the anomalous resistance onset on the auroral field lines, Ann. Geophys., 23, 1455-1465, 2005.

Vogt, J., Alfven wave coupling in the auroral current circuit, Surveys of Geophysics, 23, 335-377, 2002.