"Physics of Auroral Phenomena", Proc. XXVII Annual Seminar, Apatity, pp. 83 - 86 , 2004

Polar Geophysical Institute

MEAN VARIATIONS OF THE SOLAR ACTIVITY CYCLES: ANALYTICAL REPRESENTATIONS

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Abstract. Attempts to approximate by different analytical functions the 11-year cyclic variations of the annual mean values of the solar activity Wolf numbers *W* are made. Three functions used to this purpose are 1) gamma distribution depending on a square of the argument, 2) lognormal function, and 3) exponential distribution depending on cubic polynomial. Parameters of these three distributions for the solar activity cycles 1 – 22 are calculated. All three functions make it possible to obtain a good approximation of the annual mean Wolf numbers of each 11-year solar cycle. However only the last function corresponds to the linear parabolic diffusion equation describing the process of the diffusion and heat and mass transfer of the disturbance travelling from the basis of the solar convection zone into the photosphere.

Introduction

Investigation of the solar activity variations during the 11-year solar cycles have revealed a definite law of the mean pattern of the activity indexes, such as the sunspot Wolf numbers W or the radio emission flux at 10.7 cm wavelength. Soon after the activity minimum there is a rather fast increase following by a slower decrease, i.e. in a whole a simple aperiodic process is observed. Attempts to obtain an analytical approximation of the observed time dependence, both for a separate cycle, and for a sequence of cycles, have been done many times. However these researches have given no satisfactory results possible to describe the 11-years cycle properties.

Statement of the problem

Among various functions, capable to describe the aperiodic character of variations of indexes of solar activity, first of all two functions have to be considered. First, a distribution depending upon a square of argument *t* (more generally known as Nakagami distribution [Vadzinsky, 2001]). For the first time it was used by Kostitzin in 1932, (see Waldmeier [1955, S.155]), however, after that time apparently nobody have drawn any attention to this distribution. The other distribution is the so-called lognormal one. They look like

$$G(t) = B \cdot t^{2k_G} \exp\left(-\frac{t^2}{T_G^2}\right) \text{ and } L(t) = \frac{A}{t} \cdot \exp\left[-\left(k_L \cdot \ln\frac{t}{T_L}\right)^2\right].$$

Here the time *t* (in years) for *G(t)* is measured from the moment of a cycle minimum, the argument *t* for *L(t)* is measured from the moment of the previous minimum of activity.

In these cases the integrals S_G and S_L over these distributions applied to the Wolf numbers for all cycles, values of argument corresponding to the maxima of these distributions t_{MG} and t_{ML} (i. e. their modes) and the maximal function values G_M and L_M are

$$S_G = \frac{G_M}{2} \cdot T_G \cdot e^{k_G} \cdot k^{-k_G} \cdot \Gamma(k_G + \tfrac{1}{2}), \quad \text{mode} \quad t_{MG} = \sqrt{k_G} \cdot T_G \text{, and } G_M = B \cdot k_G^k \cdot T_G^{2k_G} \cdot e^{-k_G}$$

$$S_L = \frac{\sqrt{\pi} \cdot L_M \cdot T_L}{k_L} \cdot \exp\left(-0.25 k_L^{-2}\right), \quad \text{mode} \quad t_{ML} = T_L \cdot \exp\left(-0.5 k_L^{-2}\right),$$

$$L_M = \frac{A}{T_L} \cdot \exp\left(0.25 k_L^{-2}\right).$$

Here the value Γ is the gamma function.

Exponential function

Consideration of physical mechanisms in a convection zone of the Sun results in the analysis of processes of diffusion. Kononovich [2003] has considered the joint action of the variations caused by the 11-years cycle and the variations of smaller time scale. The obtained decision contains exponential function from the cubic polynomial, representing an 11-year cycle, namely,

$$E(t) = H \cdot \exp\left[-k_E r_E (t - t_0) + \frac{D_T}{3} \cdot k_E^2 (t - t_0)^3\right] = H \cdot \exp\left[-m(t - t_0) + n(t - t_0)^3\right], \text{ where}$$

$$k_E = \sqrt{\frac{3n}{D_T}}, r_E = m\sqrt{\frac{D_T}{3n}}.$$

The characteristics of this distribution are

the mode $t_{ME} = t_0 - \sqrt{\frac{r_E}{D_T k_E}}$, maximum of the distribution $E_M = H \cdot \exp\left[\frac{2 \cdot r_E}{3}\sqrt{\frac{k_E \cdot r_E}{D_T}}\right]$.

The arguments for $E_M / 2$ are equal

$$t_{E1} = t_0 - 2\sqrt{\frac{r_E}{D_T \cdot k_E}} \cdot \cos\left[60 - \frac{1}{3}\arccos(1 - \frac{3\ln 2}{2 \cdot r_E}\sqrt{\frac{D_T}{k_E \cdot r_E}})\right], t_{E2} = t_0 - 2\sqrt{\frac{r_E}{D_T k_E}} \cdot \cos\left[60 + \frac{1}{3}\arccos(1 - \frac{3\ln 2}{2 \cdot r_E}\sqrt{\frac{D_T}{k_E \cdot r_E}})\right].$$

The half width of distribution W_E and the asymmetry are

$$W_E = 2\sqrt{\frac{3r_E}{D_T k_E}} \cdot \sin\left[\frac{1}{3}\arccos(1 - \frac{3\ln 2}{2 \cdot r_E}\sqrt{\frac{D_T}{k_E \cdot r_E}})\right]; P_E = \frac{t_{E2} - t_{ME}}{W_E} = \frac{1 - 2\cos\left[60 + \frac{1}{3}\arccos(1 - \frac{3\ln 2}{2 \cdot r_E}\sqrt{\frac{D_T}{k_E \cdot r_E}})\right]}{2\sqrt{3}\sin\left[\frac{1}{3}\arccos(1 - \frac{3\ln 2}{2 \cdot r_E}\sqrt{\frac{D_T}{k_E \cdot r_E}})\right]}.$$

The integral is

$$S_E = E_M \cdot e^{-\frac{2 \cdot r_E}{3}\sqrt{\frac{k_E \cdot r_E}{D_T}}} \cdot \int_0^{15} \exp\left[-k_E r_E (t - t_0) + \frac{D_T}{3} k_E^2 (t - t_0)^3\right] dt .$$

Full duration of a cycle T_E essentially depends upon the choice the moment of the W minimal values which usually are about 5 – 7 % of the E_M value, i.e. 1/15 - 1/20. In this case

$$T_E = 2\sqrt{\frac{3r_E}{D_T k_E}} \cdot \sin\left[\frac{1}{3}\arccos(1 - \frac{3\ln 20}{2 \cdot r_E}\sqrt{\frac{D_T}{k_E \cdot r_E}})\right].$$

Time of activity rise is $T_{E1} = t_{EM}$, and the branch of decrease is $T_{E2} = T_E - T_{E1}$.

On the basis of the considered functions the long-term data of the mean-annual Wolf numbers W [Vitinsky, 1973] for all available cycles 1-22 has been analyzed. Mean-annual data have been used for 11-years cycles since the natural others short-time variations have been smoothed. Using the above mentioned formulae the sets of parameters have been obtained, allowing to calculate the average "background" distribution of Wolf numbers for the all 11-year cycles. Examples of the approximations are shown on Figure 1.

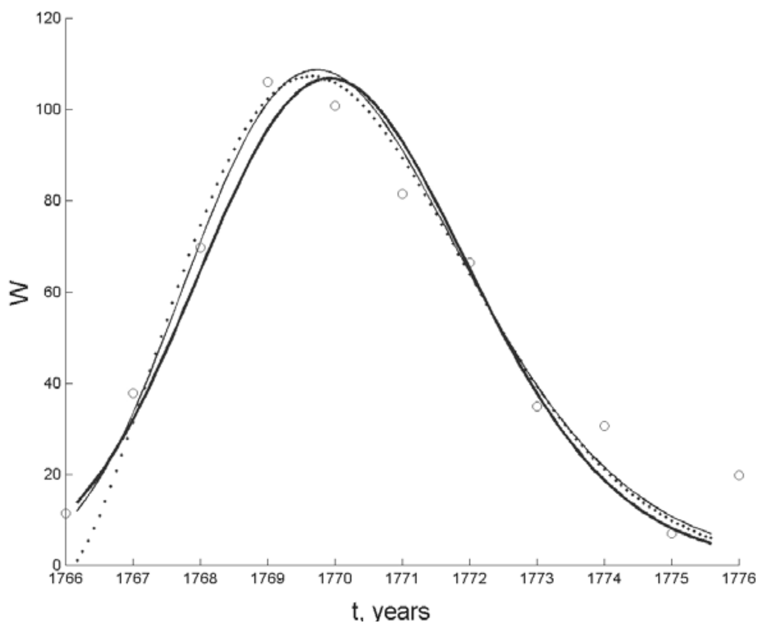

Figure 1. Examples of approximation of the mean-annual Wolf number (circles) for the cycle 2 (1766 – 1775) by the exponent of cubic polynomial *E(t)* – thick solid line, *G(t)* – thin line, and *L(t)* – dotted line.

The analysis of the data

The results of the approximations described above were correlated between each other for the all 22-cycle time interval covering the years 1755 –1996. The correlation between the key parameters of these distributions is high enough. The regression lines are also within the limits of errors and pass through coordinate origin. Here below are the equations of all these lines.

$L_M = (1.09 \pm 0.05)\, G_M,\ r = 0.983 \pm 0.007,$ $\sigma_L = (0.98 \pm 0.08)\, \sigma_G,\ r = 0.835 \pm 0.015$

$S_L = (1.13 \pm 0.09)\, S_G,\ r = 0.940 \pm 0.025,$ $W_L = (0.98 \pm 0.12)\, W_G,\ r = 0.884 \pm 0.048$

$E_M = (1.07 \pm 0.05)\, L_M,\ r = 0.977 \pm 0.010,$ $\sigma_E = (0.89 \pm 0.06)\, \sigma_L,\ r = 0.602 \pm 0.14$

$S_E = (1.01 \pm 0.02)\, S_L,\ r = 0.996 \pm 0.002,$ $W_E = (0.81 \pm 0.12)\, W_L,\ r = 0.816 \pm 0.073$

$E_M = (1.11 \pm 0.06)\, G_M,\ r = 0.970 \pm 0.013,$ $\sigma_E = (0.79 \pm 0.13)\, \sigma_G,\ r = 0.801 \pm 0.078$

$S_E = (1.16 \pm 0.09)\, S_G,\ r = 0.951 \pm 0.021,$ $W_E = (0.92 \pm 0.10)\, W_G,\ r = 0.911 \pm 0.037$

$S_E = (1.09 \pm 0.02)\, E_M \cdot W_E,\ r = 0.996 \pm 0.002;$ $\overline{S}_E = 584 \pm 195;\ \overline{E_M \cdot W_E} = 537 \pm 178.$

Apparently, on the average, the data for all cycles practically have identical dispersions σ. However, the average maximal E_M values are higher, than those for L_M and G_M, and the half widths W_E it is less, than W_L and W_G, the integrals of distributions S_E are higher, than the S_L and S_G values approximately by 10 %.

Discussion

The cyclic variations of the annual mean values of Wolf numbers W actually present the behavior of the latitude variations of the solar surface strip where the sunspots occur (so called “King’s zone”). This evidently follows from the time-space structure of the spot group distribution presented by “Maunder butterflies”. This diagram displays the complex processes of vertical movements of the disturbances occurring in the subphotospheric solar convection zone.

The integral aperiodic cyclicity of the solar activity with the time scales about 11 and 22 years is the main puzzle of the Sun. The other one are the quasi-biennial oscillations (QBO) – also aperiodic variations of the main cycle fine structure with time scale about 2-3 years. The QBO analysis has shown, that the QBO minimums coincide with the main 11-year activity cycles minims. In this case the beginning of the QBO pulses occur 1-2 years after the moment of a cycle minimum. It means, that the occurrence of a new cycle of activity and QBO most likely occur practically simultaneously as a result of pulse process and owing to variation of velocity distribution of the plasma babbles in the convection zone. At surface layers of the Sun the QBO signify the beginning of a new cycle. It is especially fair, as breakage of the QBO occurs 22 years after the cycle beginning. At this time the subsequent cycle has already started approximately 5 years after the maximum of the subsequent cycle [Kononovich and Shefov, 2003].

From physical point of view it is necessary to emphasize, that among the above considered functions *G(t), L(t)* and *E(t)* only the last distribution presents a solution of the parabolic differential equation which describes the process of diffusion [Polyanin, 2001]. This equation is

$$\frac{\partial E}{\partial t} = D_T \frac{\partial^2 E}{\partial r_E^2} - k_E \cdot r_E \cdot E\ ,$$

D_T is the diffusion coefficient (cm^2 / c). Substituting $E = r_E K$ one can obtain the following equation

$$\frac{\partial K}{\partial t} = D_T \left(\frac{\partial^2 K}{\partial r_E^2} + \frac{2}{r_E} \frac{\partial K}{\partial r_E} \right) - k_E \cdot r_E \cdot K,$$

to describe the non-stationer heat and diffusion processes in the central symmetrical media.

It is important, that there is a significant negative correlation ($r = -0.481 \pm 0.168$) between amplitude of maximum E_M of the 11-year cycles and scale parameter r_E, and a positive correlation between the $1/\sqrt{E_M}$ and the time of maximum t_{EM}
($r = 0.748 \pm 0.094$). It determines the following relationships of regression

$$E_M = (140 \pm 15) - (3.74 \pm 1.52)\, r_E, \qquad \sqrt{E_M} \cdot t_{EM} = 48.6 \pm 8.5.$$

Hence, the cycle maximum value is connected with the depth of the initial disturbance and with the speed of its movement to the surface.

Beside these correlations it is interesting is to compare the parameters k_E and r_E with the cycle maximum t_{EM} i.e. with the duration of the solar activity phase of growth. It allows applying these correlation patterns not only to the previous solar cycles before the year 1755, but also to cycles of several sun like variable stars. There are definite correlations of the star variability parameters, resembling those for the Sun. This opens a new opportunities to better understanding the nature of the Solar and stellar activity.

In a number of works the available long-term measurements of the solar neutrino has been published. According to the data [Sakurai, 1979; Lanzerotti and Raghavan, 1981; Haubold and Gerth, 1983] existing data are sufficient to reveal various possible components including the quasi-biennial oscillations (QBO). But the authors do not specify their character. We have tried to smooth all these data using the sliding filter 1 : 3 : 5 : 3 : 1, and have correlated them with the QBO component ΔW [Kononovich and Shefov, 2003] of the Wolf number index of the solar activity. It was revealed that there is a distinct concurrence of both variations, but in an antiphase with the time shift about 1.4 years. The correlation coefficient of these data from 1974 till 1982 is -0.84 ± 0.10. Thus, the same phase of the QBO component of the W solar activity index occurs earlier than the corresponding phase of the neutrino flux variations. The revealed time difference make it possible to suggest a method to estimate the speed and the depth of the unknown disturbance provoking, on one side, the solar activity effects over the solar surface layers and, on the other side, the neutrino flux variations in the solar center.

The conclusion

The analysis of the long-term data of the mean values of the solar annual Wolf numbers has revealed the certain preference among different analytical presentations of the activity cycles for the sun and sun like stars as well. The most physically sounded function is revealed to be the exponential distribution depending upon the cubic polynomial $E(t)$.

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