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# RECONSTRUCTION OF THE RECONNECTION RATE FROM PERTURBATIONS IN THE AMBIENT MAGNETIC FIELD

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**Abstract.** The aim of this work is to describe the behaviour of flux transfer events based on a time-dependent Petschek-type model of reconnection. In the frame of this model, we are able to evaluate the magnetic field configuration and the plasma flow components, as well as the shape of the Petschek shocks. We consider two different kinds of hypothetical measurements, namely along a profile $B_z(x)$ and along a trajectory $B_z(t)$. By using a discrete Fourier transformation, we are able to calculate the magnetic field along a certain profile $B_z(x)$. Out of this profile, we reconstruct the reconnection electric field at the reconnection site. This is an ill-posed inverse problem, which we treat with the method of regularisation. But satellite measurements are always trajectories and therefore we use the Cagniard-deHoop method to calculate the magnetic field configuration along a trajectory $B_z(t)$. The solution is given as a convolution integral, which is a well-known problem in the theory of inverse problems. By using a regularisation operator, we can reconstruct the reconnection electric field for different initial electric field configurations.

## 1. Introduction

In a search for reconnection at the dayside magnetopause, Russell and Elphic (1978) noticed that there appear localized, transient reconnection events, which can be identified by an isolated bipolar variation of the magnetic field component normal to the magnetopause and a simultaneous deflection in the tangential components, which can be interpreted as disturbances caused by a moving flux tube passing by the satellite. They named these characteristic events "flux transfer events" (FTEs). The implications of time-varying, localized reconnection models for the interpretation of FTEs have been discussed by Semenov et al. (1992).

After the observation of these FTE signatures, some attempts were made to reconstruct different features of the reconnection process involved.

Walthour et al. (1994) developed a method for inferring the cross-sectional size, shape, and the speed of propagation of a thin, infinitely long obstacle corresponding to a flux tube. Since the analysis is confined to perturbations outside the obstacle, the method is referred to as a remote sensing method. Lawrence (1998) analyzed a series of FTE-like events generated by a time-dependent model of reconnection, where he studied the effects of three different reconnection fields on the perturbations. Hau and Sonnerup (1999) developed a method to reconstruct two-dimensional space plasma structures in magnetohydrostatic equilibrium, and applied this model to two magnetopause crossings by the spacecraft AMPTE/IRM (Hu and Sonnerup, 2003). But these models do not intend to reconstruct the reconnection electric field. Here we present two different models to investigate the reconnection rate, which is the most important feature to describe the reconnection process, from perturbations in the ambient magnetic field.

## 2. MHD description of asymmetric magnetic reconnection

The basic configuration used in our model is shown in Fig. 1. The magnetic fields are orientated antiparallel and have different field strength, namely $B_a$ in the upper half plane, and $B_b$ in the lower half plane.
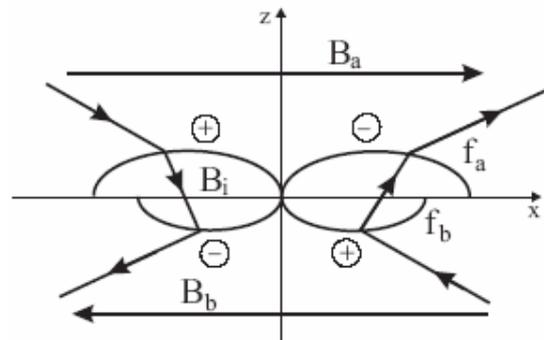


Fig. 1: Magnetic field configuration for asymetric reconnection.

The background magnetic fields and the total pressure $P$ are assumed to be constant. Additionally, we consider a fixed plasma, meaning that $v = 0$ in the

inflow region in zero order. The shocks $f_a$ and $f_b$ bound the outflow region, where the magnetic field is $B_i$. The current sheet separating the two different plasmas is located in the *x-y* plane.

We consider the incompressible case, and introduce normalized quantities (Semenov et al., 2003) giving the MHD equations for an ideal fluid as

$$\frac{\partial \mathbf{v}}{\partial t} + (\mathbf{v} \cdot \nabla)\mathbf{v} = -\nabla P + (\mathbf{B} \cdot \nabla)\mathbf{B}, \qquad (1)$$

$$\frac{\partial \mathbf{B}}{\partial t} = \nabla \times (\mathbf{v} \times \mathbf{B}), \qquad (2)$$

$$\nabla \cdot \mathbf{v} = \nabla \cdot \mathbf{B} = 0, \qquad (3)$$

where *B*, *v*, and *P* are the magnetic field, the plasma flow velocity, and the total pressure, respectively.

If we perform an order-of-magnitude estimation, we can use the assumption for weak reconnection that quantities perpendicular to the current sheet are small compared with the tangential components. Now the problem can be separated in two different steps. First we can evaluate the tangential components $B_{ix}$ and $v_{ix}$ from the non-linear system of MHD equations for the zero order by assuming that these quantities are constant. If they are constant, they can be found from the Rankine-Hugoniot relations directly. In a second step, we can determine the components $B_{iz}$ and $v_{iz}$ from the linearized system of MHD equations in the first order approximation.

After some algebra (Semenov et al., 2004) the boundary conditions for the magnetic field components at *z=0* in the two inflow and the outflow region as

$$B_{az0} = c_1 E(t - |x|/c)\,\mathrm{sgn}(x) - \\ - c_2 E(t - |x|/B_a)\,\mathrm{sgn}(x), \qquad (4)$$

$$B_{bz0} = c_3 E(t - |x|/c)\,\mathrm{sgn}(x) - \\ - c_4 E(t - |x|/B_b)\,\mathrm{sgn}(x), \qquad (5)$$

$$B_{ix} = c_5 E(t - |x|/c)\,\mathrm{sgn}(x), \qquad (6)$$

where *E* is the reconnection electric field, which is a function of its argument, and $c_1$-$c_5$ are constants depending on the magnetic field configuration. These three equations are the Dirichlet boundary conditions needed to solve the Laplace equation to get the magnetic field configuration in the whole space. The solution of the Dirichlet problem in both half planes is given by the Poisson integral, which is a special form of a convolution integral in space, for the x- and z-components of the perturbed magnetic field. For the computational treatment of the problem and for solving the inverse problem it is more convenient to calculate the Fourier transform of the magnetic field at the reconnection line and solve the problem in Fourier space (Semenov et al., 2004). The Fourier transform of the magnetic field is found to be

$$B_{az,bz}(k) = B_{az0,bz0}(k)e^{-\gamma z}, \qquad (7)$$

where $B_{az0,bz0}$ is the magnetic field at z=0 and for each half space, and $\gamma$ is a parameter from the Fourier transformation. From this equation, the magnetic field along a profile can be found for the whole half space.

## 3. The inverse problem applied to a magnetic field profile

An inverse problem is given if we consider a certain phenomenon, which cannot be observed directly. The indirect observed attributes of the phenomenon are correlated with the phenomenon itself via an certain operater. To achieve informations of the process it is necessary to find the inverse of the operator. If this is not possible for the whole space, and if the solution is not stable, such problems are called ill-posed inverse problems (Tikhonov and Arsenin, 1977). In our problem, the observed attributes are the magnetic field measurements $B_z$ at a certain distance, and the phenomenon we reconstruct is the magnetic field at the reconnection site. Therefore, this inverse problem is of the form

$$B_{az0,bz0}(k) = \frac{B_{az,bz}(k)}{e^{-\gamma z} + M}, \qquad (8)$$

where *M* is an regularisation operator (Tikhonov and Goncharsky, 1987), which is used to avoid that the solution becomes unstable or goes to infinity. In this case, it is useful to consider this operator as a small constant value. Based on the reconstruction of the magnetic field configuration at the reconnection site, we developed an iteration method to reconstruct the reconnection electric field, which works good for asymmetric magnetic field configurations in both half planes. But for nearly symmetric conditions, many iteration steps become necessary, and the results are not satisfying anymore (Penz, 2002). Furthermore, realistic satellite measurements are always a trajectory and not a profile.

## 4. The Cagniard-deHoop method for incompressible plasma

To calculate the magnetic field components along a trajectory, we use the so-called Cagniard-deHoop method, which is used in seismology to describe elastic waves. The method was applied to reconnection problems by Heyn and Semenov (1996). This method will give us the magnetic field components as a convolution integral in time, which can be treated convenient in the theory of inverse problems.

It is possible to find a solution for the displacement vector, from which the magnetic field and plasma flow parameters can be derived easily, in Fourier-Laplace space. The Cagniard-deHoop method is used to perform the inverse Laplace transform analytically, which gives the magnetic field in the upper half plane as

$$B_{az}(x,z,t) = c_6 \mathrm{Re} \int_0^t g(x,z,\tau) E(t-\tau) d\tau, \quad (10)$$

where $g(x,z,\tau)$ is the integration kernel, which depends mainly on the magnetic field configuration and the relative position of the spacecraft, $c_6$ is a constant, and $E(t-\tau)$ is the reconnection electric field, which we reconstruct in the following.

## 5. Reconstruction of the reconnection rate out of a trajectory

In nature, the shocks are moving with velocities of some hundred km/s, while the satellite's velocity is only some km/s. Therefore, we can consider the satellite as fixed, meaning that $x=const$ and $z=const$ in this case. Now the magnetic field is only a function of time $B_z(x,z,t) = B_z(t)$. In Laplace space, the convolution integral can be written as

$$B_z(p) = K(p)E(p), \quad (11)$$

which is similar to Equation (7) in the Fourier method. Here, $K(p)$ is the integration kernel from Equation (10). To reconstruct the reconnection electric field we introduce again a regularisation operator $M(p)$ giving

$$E(p) = \frac{B_z(p)}{K(p) + M(p)}, \quad (12)$$

This operator is choosen that it does not influence the electric field for small values of $p$, but when the functions $B_z(p)$ and $K(p)$ reach small values, the denominator is forced to go to infinity, so that the reconnection electric field is zero in Laplace space and large oscillations are suppressed.

At first, we use an initial electric field of the form

$$E(t) = \frac{b^2 e^2}{20} t^2 e^{-bt}, \quad (13)$$

with $b=4$, which corresponds to a reconnection rate of approximately 0.1. We can use the magnetic field trajectories obtained via the Cagniard-deHoop method to reconstruct the electric field. If we use a magnetic field configuration with $B_a=2$ and $B_b=-1$ and reconstruct the electric field from a satellite position at $x=5$ and $z=2$ the result is very satisfying. For a stronger asymmetric magnetic field configuration yields even better results. Also for sine-shaped reconnection electric field the method works compareable to the case of an exponential reconnection electric field, but the smooth behaviour of the exponential field reduces oscillations.

Additionally, we can model the case of two reconnection pulses. Here we use a initial reconnection electric field of the form of two sine-shaped pulses, which are separated by a certain time interval. If we apply our reconstruction method to such a electric field, we can reconstruct the electric field for $z=2$ very good and for $z=5$ qualitatively for a magnetic field $B_a=2$ and $B_b=-1$. If the time separation between two reconnection pulses is

smaller than 1 time unit, the method works only for small distances above the reconnection site.
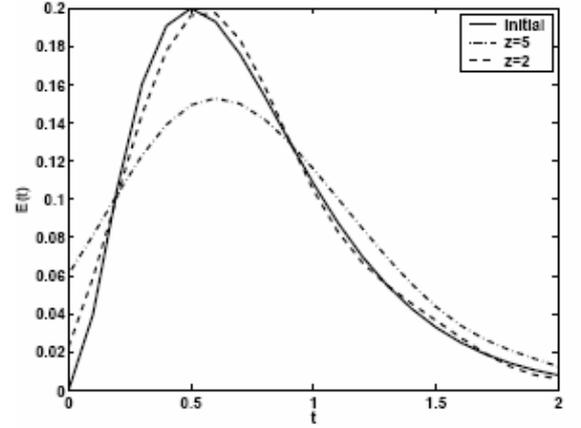


Fig. 2: Reconstructed and initial electric field for a position $x=5$ and $z=2$ (dotted line) and $z=5$ (dashed-dotted line) for a magnetic field configuration $B_a=2$ and $B_b=-1$.
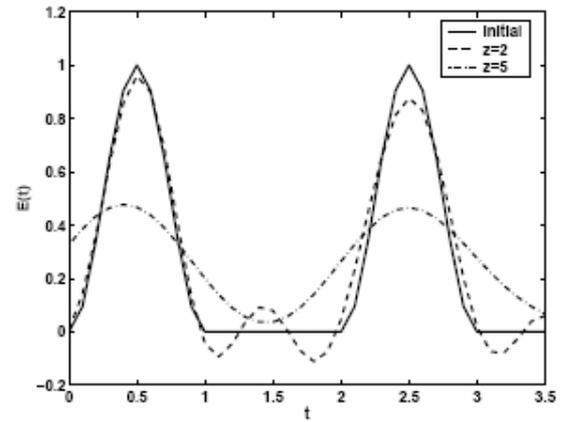


Fig. 3: Reconstructed and initial electric field two sine-shaped reconnection pulses for $B_a=2$ and $B_b=-1$ for a distance of $z=2$ (dotted line) and $z=5$ (dashed-dotted line).

## 6. Conclusions

We presented a new method for the reconstruction of the reconnection rate from disturbances in the ambient magnetic field, which can be considered as the measurements of a hypothetical satellite. First, the magnetic field components at the reconnection site are determined from a profile. The convolution integral for this case is given as a Poisson integral. A solution of the Poisson integral can be found in Fourier space rather than in Laplace space by using a regularisation method to solve the inverse problem. For this case we developed a iteration method to reconstruct the electric field for most magnetic field configurations. But in this case we reconstruct the data from profiles, which are not realistic for satellite measurements. In a second approach, we use the Cagniard-deHoop method to calculate the magnetic

field components along a trajectory. In this case, the magnetic field is given as a convolution integral, which can be solved by applying a regularisation operator. It was shown that this method gives the reconnection electric field quantitatively good for values of $z<5$, corresponding to 50 times the height of the outflow region. We used different initial electric fields, and found that the method works better if the initial field has a smooth behaviour. The magnetic field configuration also influence the results in a way that a larger ratio between the magnetic fields on both sides of the discontinuity leads to a better reconstruction of the initial reconnection electric field. Also for reconnection pulses with a time separation of $t>1$, the method works sufficiently.

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