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TRANSFORMATION OF IONOSPHERIC PLASMA TRANSPORT EQUATIONS TO A CONSERVATIVE FORM

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1. Introduction

Magnetic hydrodynamic equations written in the dipole coordinate system are typically used in numerical modeling of the ionosphere, plasmasphere and inner magnetosphere in calculating spatial and temporal variations of thermal plasma parameters [1,2,3]. This coordinate system is distinguished because the geomagnetic field in the near Earth space is to a good accuracy approximated by the field of a dipole located in the Earth's center. If the radial distance, co-latitude and azimuthal angle of some point in the solar-magnetic coordinate system are denoted by r, θ, and γ, respectively, then the dipole coordinates of the point are [4]

$$\alpha = \frac{r}{R\sin^2\theta}, \qquad \beta = -\left(\frac{R}{r}\right)^2 \cos\theta, \qquad \gamma = \lambda,$$

where R is the Earth radius. The coordinate β varies along geomagnetic field lines. The metric coefficients of the orthogonal coordinate system α, β, γ can be written as [1,2]

$$h_\alpha = R\frac{\sin^3\theta}{\sqrt{\delta}}, \quad h_\beta = \frac{r^3}{R^2\sqrt{\delta}}, \quad h_\gamma = r\sin\theta, \qquad \delta = \sqrt{1+3\cos^2\theta}\,.$$

In the height range h ≥ 150 km, where the transport processes are important, thermal plasma is magnetized and, hence, the hydrodynamic velocity of the *i*-th ion species is represented as

$$\vec{V}_i = V_i\vec{e}_\beta + W_\alpha\vec{e}_\alpha + W_\gamma\vec{e}_\gamma, \qquad \vec{W} = \frac{c}{B^2}(\vec{E}_m \times \vec{B}) + \vec{\omega},$$

where $\vec{E}_m$ is the magnetospheric convection electric field, and $\vec{\omega}$ is the corotation velocity.

2. Transformation of transport equations

In the reference frame of dipole plasma tube drifting across the field with the velocity $\vec{W}$, density of the *i*-th ion species is described by the system of continuity and momentum equations

$$\frac{dn_i}{dt} + \frac{1}{Ah_\beta}\frac{\partial}{\partial\beta}(An_iV_i) = -n_i\vec{\nabla}\cdot\vec{W} - \frac{n_i}{\tau_i} + q_i, \tag{1}$$

$$-B_in_iV_i = \frac{1}{h_\beta}\frac{\partial n_i}{\partial\beta} + K_in_i, \tag{2}$$

Here $A = \varphi(\alpha)h_\alpha h_\gamma$ is the cross-section of the tube determined according to (1) and accurate to an arbitrary function $\varphi(\alpha)$; and $d/dt = \partial/\partial t + \vec{W}\cdot\vec{\nabla}$ is the Lagrangian derivative.

Let the function $\varphi(\alpha)$ be defined in such a way that the cross-section of the tube is

$$A = \left(\frac{r}{R}\right)^3 \frac{1}{\sqrt{\delta}}.$$

Instead of the dipole coordinates α, β, γ, we introduce the new spatial variables L, x, y via the transformation

$$L = \alpha, \qquad x = X(\alpha,\beta), \qquad y = \gamma, \tag{3}$$

and transfer from the density n_i and flux n_iV_i to the functions:

$$F_i = A\ell n_i, \qquad \Phi_i = -A(n_i V_i + \frac{\ell}{\tau}\psi n_i). \tag{4}$$

The quantities ℓ, τ and ψ in (4) are determined by the relations

$$\frac{\psi}{\tau} = \frac{W_\alpha}{h_\alpha}\frac{\partial X}{\partial \alpha}, \qquad \frac{1}{\ell} = \frac{1}{h_\beta}\frac{\partial X}{\partial \beta}. \tag{5}$$

Having transformed to new variables and functions, instead of equations (1) and (2) we obtain equations for the quantities F_i and Φ_i in a conservative form

$$\frac{dF_i}{dt} = \frac{\partial \Phi_i}{\partial x} - \frac{F_i}{\tau_i} + A\ell q_i, \tag{6}$$

$$C_i\Phi_i = \frac{\partial F_i}{\partial x} + H_i F_i . \tag{7}$$

Here: $C_i = \ell^2 B_i$; $\quad H_i = \ell\left(K_i - \frac{\ell}{\tau}\psi B_i\right) - \chi$; $\quad \chi = \frac{\partial}{\partial x}\ln(A\ell)$.

In a similar manner, the conservative form can be obtained for the heat balance equation. Let $p_a = n_a k T_a$ be the pressure of plasma component *a*. The heat balance equation for this component may be written as (1) and by adding to it the transport equation of internal energy along geomagnetic field lines, we obtain the following system of equations:

$$\frac{\partial p_a}{\partial t} + \frac{1}{Ah_\beta}\frac{\partial}{\partial \beta}(A\hbar_a) + \vec{\nabla}\cdot(p_a\vec{W}) = Q_a - \frac{p_a}{\tau_a}, \tag{8}$$

$$-B_a\hbar_a = \frac{1}{h_\beta}\frac{\partial p_a}{\partial \beta} + K_a p_a, \tag{9}$$

where $B_a = \frac{3}{2}\frac{kn_e}{\kappa_a}$; $\quad K_a = -\frac{3}{2}\frac{kn_e}{\kappa_a}V_a - \frac{1}{h_\beta}\frac{\partial}{\partial \beta}\ln n_e$; $\quad Q_a$ - is the heating source; τ_a is the characteristic cooling time due to collisions; and κ_a is the thermal conductivity coefficient of electrons or ions along the geomagnetic field.

If now instead of pressure p_a and energy flux $\hbar_a$, we introduce the new functions

$$E_a = A\ell p_a, \qquad \Gamma_a = -A(\hbar_a + \frac{\ell}{\tau}\psi p_a),$$

then, by applying transform (3), we get for the quantities E_a and Γ_a a conservative system of equations in the form of (6), (7).

3. Application of the transformed equations in numerical models of the ionosphere

In order to use the above transport equations of particles and energy in the dipole geomagnetic field, it is necessary to specify the form of transform (3), i.e. to set the function $X(\alpha, \beta)$. Further two cases will be considered, which we refer to as ionosphere case and plasmasphere case. The former situation is typical for $h \leq 1000$ km, whereas the latter is related to greater heights, where field line curvature is the largest.

Ionosphere. In this height range, magnetic field lines at geomagnetic latitudes $\Lambda \geq 40°$ are close to straight lines, so that it is convenient to choose

$$X(\alpha,\beta) = \frac{r(\alpha,\beta)}{R}.$$

Then:
$$\frac{1}{\tau} = \frac{W_{\alpha 0}}{R}\left(\frac{R}{r_0}\right)^2 \sqrt{\delta_0}\sin\theta_0; \qquad \ell = \frac{R\sqrt{\delta}}{2\cos\theta};$$

$$\psi = \left(\frac{r}{R}\right)^2 \frac{1}{\delta}; \qquad \chi = \left(\frac{R}{r}\right)\frac{1+5\cos^2\theta}{2\cos^2\theta}.$$

Here $W_{\alpha 0}$ is the drift velocity component in the direction α on a sphere of radius $r = r_0$, i.e. at the bottom of the field line (as is the case for all quantities with the index "0"). Hence, it follows, in particular, that the parameter τ plays the role of a time scale of coordinate changes in the transverse transport of plasma between the tubes of a different volume. Of course, this scale does not change along field lines.

Plasmasphere. In the lower part of this height range the field lines are nearly vertical, whereas in the near-equator region they are virtually horizontal. Therefore, the coordinate along a field line can be presented by a dimensionless quantity

$$X(\alpha,\beta) = \frac{\beta}{\beta_*},$$

where: $\beta_* = \left(\frac{R}{r_*}\right)^2 \sqrt{1-\frac{r_*}{R\alpha}}$.

Then:
$$\frac{1}{\tau} = \frac{W_{\alpha 0}}{R}\left(\frac{R}{r_0}\right)^2 \sqrt{\delta_0}\sin\theta_0; \qquad \ell = R\left(\frac{R}{r_*}\right)^2 A\cos\theta_*;$$

$$\psi = \frac{x}{2}\left(\frac{r_*}{R}\right)^2 \frac{1}{\cos\theta_*}; \qquad \chi = 6\beta_*\left(\frac{r}{R}\right)^2 \cos\theta\,\frac{(3+5\cos^2\theta)}{(1+3\cos^2\theta)^2}.$$

Here the quantities marked with an asterisk refer to a sphere of radius r_*, which is the lower boundary of the plasmaspheric region. Note, that with such a choice of $X(\alpha, \beta)$, any dipole field line is mapped into a fixed segment $-1 \le x \le +1$.

4. Summary

We have transformed the ionospheric transport equations to the conservative form of (6), (7), which is a standard form for numerical flow run method. The use of this method in problems of modeling the mid-latitude ionosphere and plasmasphere is described in detail in [1]. When treating phenomena in the high-latitude ionosphere, it is necessary to solve numerically the transport equations for plasma tubes, whose length can increase or decrease greatly as they drift across the magnetic field. This encounters a number of serious difficulties associated with restructuring spatial networks and violation of the approximation, and ultimately gives rise to numerical instabilities. The use of the flow run method with a preliminary transformation of the transport equations to a conservative form showed that this method is always stable and, thus, the most effective when applied in modeling phenomena in plasma tubes of a very large size.

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