«*Physics of the Auroral Phenomena*», *Proc. XXV Annual Seminar, Apatity, pp.107-110, 2003*

Polar Geophysical Institute

‘ABSOLUTE’ AND ‘CONVECTIVE’ INSTABILITIES IN THE NUMERICAL MODEL OF VLF EMISSIONS GENERATION

B.V. Kozelov and E.E. Titova *(Polar Geophysical Institute, Apatity, Murmansk region, 184209 Russia)*

Abstract. A new numerical semi-empirical model is suggested to describe the joint generation of different types of VLF emissions; namely, hiss and choruses. The hiss generation is assumed to provide the convective instability at cyclotron interaction of energetic particles with VLF waves. The discrete chorus-like emissions are excited in the process of absolute instability development (by analogy with back-wave oscillator) under step-like velocity distribution of energetic particles. The transition from hiss to discrete type of generation happens when the efficiency of convective interaction decreases and the absolute instability has conditions to start. The model allows us to describe such features of natural VLF emissions, as: 1) Long-period self-modulation of hiss under weak input fluxes. 2) Increase of maximum amplitude and frequency in hiss near beginning of a discrete element. 3) Increase in succession frequency of the chorus elements with increase of hiss amplitude. 4) Power-law distributions of time intervals between chorus elements.

1. Introduction

Dissipation of plasma energy by the cyclotron interaction of energetic particles with low frequency waves is a widespread phenomenon and often such interaction results in generation of separate discrete elements. Examples of such a phenomenon are the ELF-VLF chorus emissions, which represent sequence of elements in frequency band 10^2 - 10^4 Hz and with duration 0.1-1 seconds, which are boosted on frequency. The mechanism of chorus generation is based on cyclotron interaction of (10-100 keV) electrons of radiation belts with low frequency waves near equatorial plane [Bespalov and Trakhtengerts, 1986]. However, the basic problem of chorus generation, i.e. how the sequence of discrete chorus elements is formed, is not solved yet. Trakhtengerts [1999] has offered the solution of this problem on the basis of a backward-wave oscillatior (BWO) mode of cyclotron magnetospheric maser. The BWO generation mode was discovered in electron devices long ago and, in details, was considered in paper [Ginzburg and Kuznetsov, 1981].

In [Kozelov et al., 2001] by the data of MAGION-5 satellite in the morning sector at 30-40^o from the equatorial plane it is revealed, that the time intervals between chorus elements have a power-law distribution with index ≈1.5. Thus, a physical mechanism of chorus generation should ensure certain dynamic properties, which are not included in the frameworks of laboratory research of BWO. In the previous paper [Kozelov et al., 2002] we offered an explanation of the dynamics of VLF chorus emissions as a regime of “on-off” inter-mittency in the BWO generator, controlled by a noise. Unlike other known types of intermittency, the “on-off” intermittency is the only one, which is really dynamical [Heagy etc., 1994].

In this paper we offer a semi-empirical numerical model, which enables to reproduce many features of joint dynamics of VLF hiss and chorus emissions.

2. Numerical model

Let's consider a discrete 1-D set which contains N cells. With each cell we will associate some velocity of electrons $v_{||}$ or (that is equivalent in this model) corresponding wave frequency f. Each cell at each time step will be characterized by particle flux $F(t)$ and wave amplitude $A(t)$. We suppose that in the interaction region the particle flux may amplify the wave amplitude in the same cell only, but there are two reasons for the amplification, which are analogues to convective and absolute instabilities.

For development of convective instability it is necessary, that the particles and waves returned to the interaction region. Let T_A be the periods of group oscillations of wave packet, T_F be the periods of bounce oscillations of energetic particles. An elementary time of the interaction we shall take equal to T_0 (~ 0.1s). Then the particle orbit consists of T_F/T_0 cells, and for the waves - of T_A/T_0 cells. The cells sequentially pass the interaction region in opposite directions (see, Fig.1). In the interaction region we set the following rules for changing wave amplitude and particle flux:

$$A(t) = k\,(F(t - T_F) + \Phi) + \delta A \qquad (1)$$

$$F(t) = (1 - k)\,(F(t - T_F) + \Phi) \qquad (2)$$

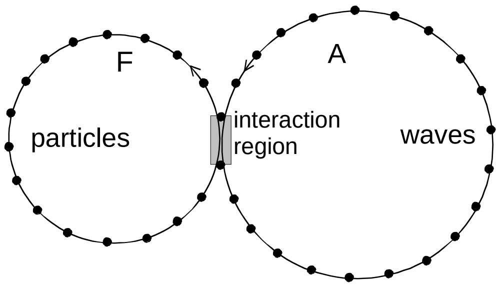


Fig.1. Sketch of feedbacks for ‘convective’ instability.

Here $A(t)$ is the wave amplitude, $F(t)$ is the particle flux at time step t. Let these values be expressed in identical energetic units. Then k means a transmission factor of energy from particle flux to waves. Obviously, that k is not greater than 1. The external sources are described by two addends: Φ is the external flux of new particles, δA=10^{-6} is the background wave included to avoid the trivial regime without generation.

Let's consider that

$$k = k_1 + k_2, \qquad (3)$$

where the first addend is determined by the convective instability, the second one - by the absolute instability.

For the convective instability:

$k_1 = A(t - T_A)\, q,$ (4)

where the factor q determines a fraction of the wave amplitude which has come back to the interaction region. On the other hand, from (1) and (4) one can see that

$A(t) = A(t - T_A)\, q\, F(t - T_F),$

where the product $q\, F\, (t - T_F)$ is the wave amplification at the current time step, therefore, the growth rate for the convective instability is equal to $q\, F(t - T_F)\, /T_0$. The q value for each $v_{||}$ cell is set as an external parameter. It can be expected, what this value depends on the anisotropy of the particle flux and reflection of the waves, so $q < 0.05$. For the cells, in which $q = 0$, the convective instability does not work.

For the absolute instability the feedback appears at exceeding of a certain threshold Δ_{thr} already by the nearest groups of particles and waves sequentially passing the interaction region. The efficiency of the interaction depends on ΔF, which is the height of the step on the distribution of the particle flux over $v_{||}$:

$$k_2 = \begin{cases} A(t - T_0)\left(\dfrac{\Delta F}{\Delta_{thr}}\right)^s & for\ \ \Delta F \geq \Delta_{thr} \\ 0 & for\ \ \Delta F < \Delta_{thr} \end{cases} \quad (5)$$

Here $s \geq 1$ is a nonlinearity parameter; $\Delta F = F_i(t-T_0) - F_{i+1}(t-T_0)$, where i is the index of $v_{||}$ cell. Thus, in this case the wave from the previous cell is amplified, and the amplification depends on particle flux in the adjacent cell of $v_{||}$. In this model it is the only connection between $v_{||}$ cells.

3. Examples of model dynamics

Let's consider examples of the model dynamics for the following parameters: $N = 40$, $T_F / T_0 = 7$, $T_A / T_0 = 11$, $\Delta_{thr} = 10$, $s = 2$. The dependence of the q parameter on the index of $v_{||}$ cell is shown in Fig.2.

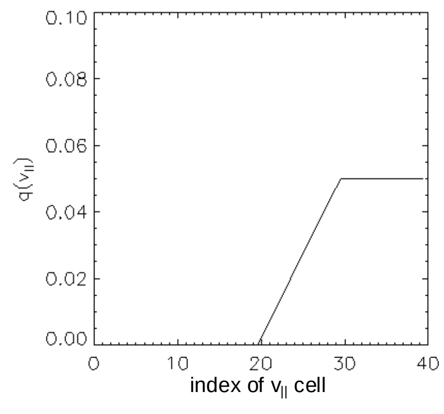


Fig.2. Dependence of the q parameter (see, Eq.4) on the index of $v_{||}$ cell.

Let's set a constant particle flux Φ in all cells of the system. As the parallel velocity of particles in the numerical model is uniquely connected to the frequency of the corresponding wave, it is convenient to present dynamics of the system by "a dynamical spectrum", the axis of frequencies for which being just the inverted axis of $v_{||}$. Examples of such spectra for wave amplitude A in a steady regime for several values of Φ are presented in Fig.3. The time scale is expressed in periods of T_0.

We note several features of the model dynamics having analogies in VLF observations:

1) Hiss-like generation in the bottom half of the frequency range.

2) Chorus-like discrete elements in the top half of the frequency range.

3) Long-period self-modulation of hiss under weak input fluxes, see Fig.3-a,b.

2) Increase of maximum amplitude and frequency in hiss near beginning of a discrete element, Fig.3 and Fig.5 (see discussion in section 4.1).

3) Increase in succession frequency of the chorus elements with increase of hiss amplitude, Fig.3.

4. Discussion

4.1. *Formation of step-like distribution*

For the case of $\Phi = 2$ the distribution of particle flux F is presented in Fig.4 the corresponding "dynamic spectrum" for the wave amplitude being shown in Fig.3-b. The vertical axes in the figures are the same. It is seen, that the convective instability at frequencies < 20 leads to distortion of the particle flux and forms a step-like distribution on the border between the resonant (for the convective instability) and non-resonant particles. However, this step-like deformation appears only in the average flows. It is interesting, that the slope of this step-like distribution is much greater than the slope of $q(v_{||})$ plot, shown in Fig.2.

For more precise illustration, we present in Fig.5 the distributions of amplitude and particle flux that are averaged over 30 time points starting from the moments $t = 70, 110, 140, 170$. One can see the change of the distribution shape near beginning of a discrete element.

4.2. *"On-off" intermittency and power-law distribution of the intervals*

From expressions (1) and (5) one can see, that for the absolute instability:

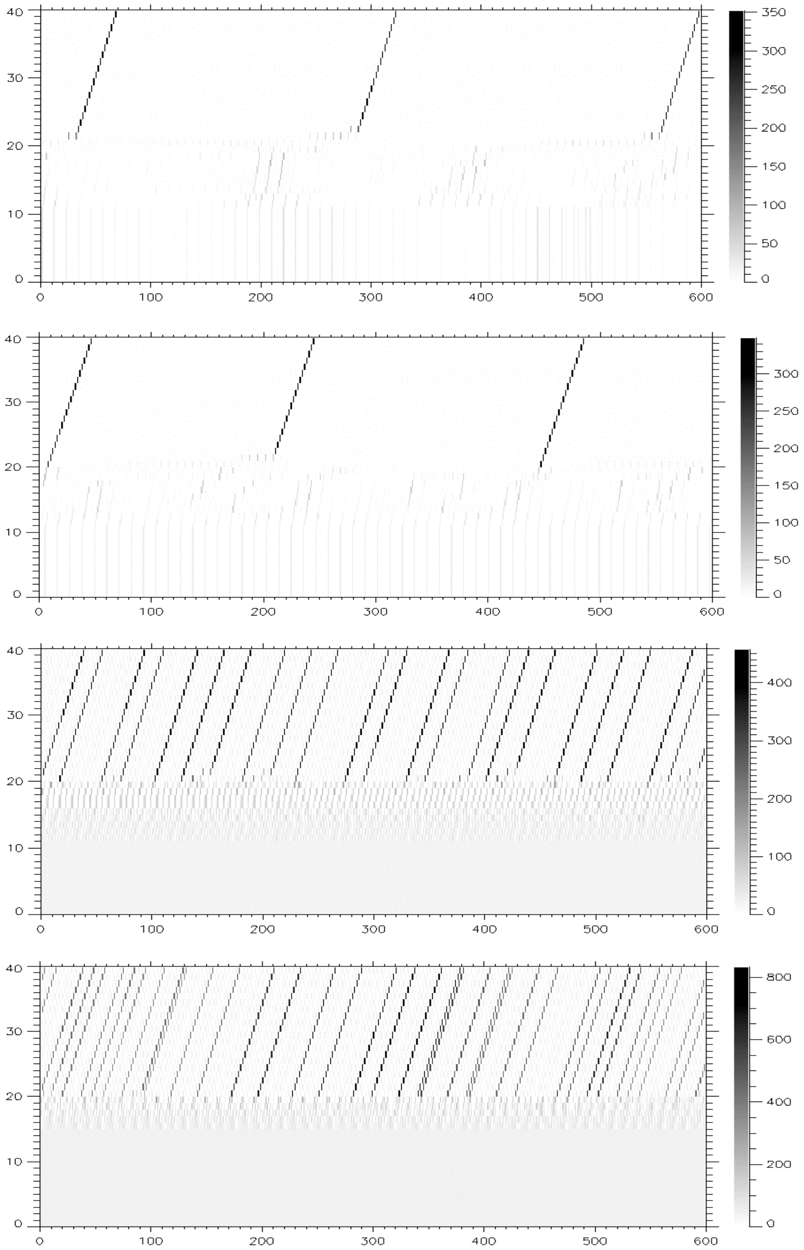
$A(t) = A(t-T_0)\, \{(F_i(t-T_0) - F_{i+1}(t-T_0))/\, \Delta_{thr}\}^s\, (F_i(t - T_F) + \Phi) + \delta A$

That is the amplification factor (denoted by λ) is:

$\lambda = \{(F_i(t-T_0) - F_{i+1}(t-T_0))/\, \Delta_{thr}\}^s\, (F_i(t - T_F) + \Phi)$ (6)

Having introduced a mean particle flux in a given cell and by expanding this expression into a series, it is possible to reduce (6) to the form $\lambda = \langle\lambda\rangle + d\lambda$, where $\langle\lambda\rangle$ is a constant averae value, $d\lambda$ is a fluctuation which dependends on fluctuations of the particle flux in adjacent cell of $v_{||}$.

For the cell, which is a boundary between resonant and nonresonance particles with regard to the convective instability, the amplification factor is determined only by the absolute instability, i.e. equation (6). However, dλ will depend on the dynamics of particle flux in an adjacent cell due to convective instability. As it has been noted in paper [Kozelov et al., 2002], in such a case it is possible to expect appearance of "on-off" intermittency mode that leads to power-law distributions of time intervals between chorus elements.



a. Φ =1.5

b. Φ =2.

c. Φ =20.

d. Φ =40.

Fig.3. "Dynamical spectrum" for the wave amplitude A in a steady regime for several values of Φ. The vertical axis of frequencies is the inverted axis of $v_{||}$. The abscissa axis is time expressed in periods of T_0.

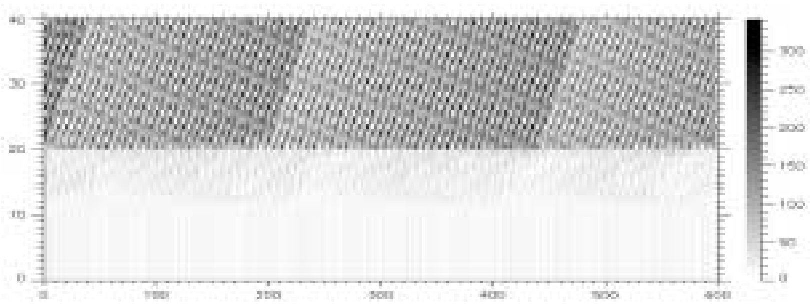


Fig.4. "Dynamical spectrum" of particle flux in steady regime for Φ=2, see Fig.3-b. The axes are the same as in Fig.3.

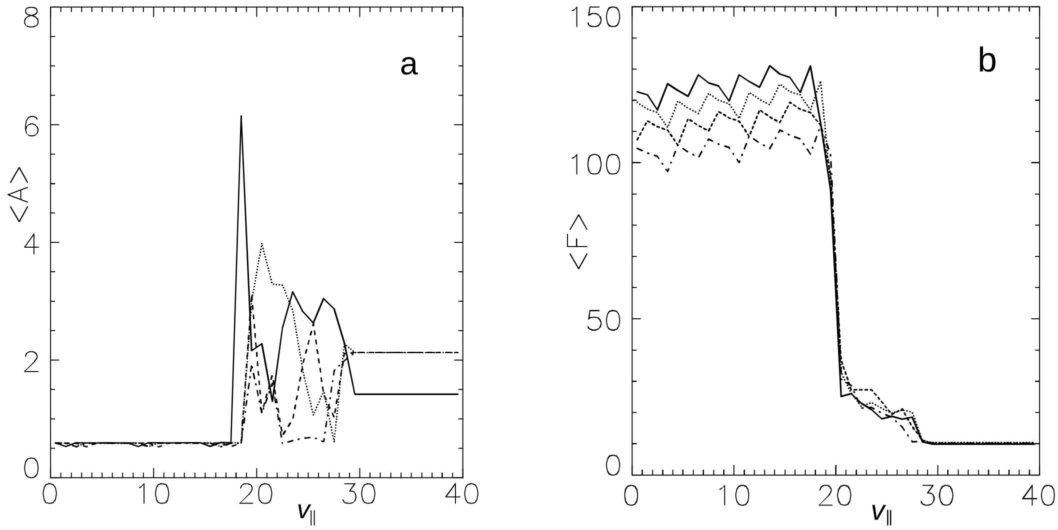


Fig.5. Averaged distributions of the wave amplitude (a) and particle flux (b) for the case *Φ* =2, time intervals: dash-dot line – *t*=70-100; dashed line - *t*=110-140; dotted line - *t*=140-170; solid line - *t*=170-200.

5. Conclusions

The discrete dynamical model of VLF emissions generation is presented. The model if based on principal concepts on generation of VLF hiss and discrete choruses:

1) the feedback due to bounce-oscillations of the waves is necessary for hiss generation;

2) at hiss generation a step-like deformation of particle distribution over parallel velocity is forming on the boundary between resonant and non-resonance particles;

3) having achieved a certain threshold, the generation transfers to absolute instability mode, which depends on parameters of step-like distribution of the particle flux.

The model allows us to describe such features of natural VLF emissions, as:

1) Long-period self-modulation of hiss under weak input fluxes.

2) Increase of maximum amplitude and frequency in hiss near beginning of a discrete element.

3) Increase of the succession frequency of chorus elements with increase of hiss amplitude.

4) Power-law distributions of the intervals between chorus elements.

Acknowledgements. Authors thank V.Yu. Trakhtengertz for very fruitful discussion. The work is supported by grants RFBR-01-05-64382-a and INTAS 99-0502.

References

Bespalov, P.A., and V.Y Trakhtengerts, The cyclotron instability in the Earth radiation belts, in: Reviews of Plasma Physics, Ed. M.A. Leontovich, vol.10, 155-192, Plenum, New York, 1986.

Ginzburg, N.S., and S.P. Kuznetsov, Periodic and stochastic regimes in electron generators with distributed interaction, in Relativistic HF Electronics, Institute of Applied Physics, Gorky, USSR, 101-104, 1981, (in Russian).

Heagy J.F., Platt N., Hammel S.M. Characterization of on-off intermittency, Phys.Rev.E., 49 (2), 1140-1150, 1994.

Kozelov B. V., Titova E. E., Trakhtengerts V. Yu., Jiricek F., Triska P., Collective dynamics of chorus emissions inferred from MAGION 5 satellite data, Geomagnetizm and Aeronom., 41 (4), 477-481, 2001.

Kozelov B.V., Titova E.E., Lubchich A.A., Trakhtengerts V.Y., Manninen J. “On-off” intermittency as a dynamical analogy of VLF chorus generation. Proc. XXV Annual Seminar "Physics of Auroral Phenomena", 73-76, 2002.

Trakhtengerts, V.Y., Magnetosphere cyclotron maser: Backward wave oscillator generation regime, J.Geophys. Res., 100 (A9), 17205-17210, 1995.

Trakhtengerts, V.Y., A generation mechanism for chorus emission, Annales Geophysicae, 17 (1), 95–100, 1999.

Trakhtengerts V.Y., Rycroft M.J., Demekhov A.G. Inter-relation of noise-like and discrete ELF-VLF emissions generated by cyclotron interactions. *J. Geophys. Res.*, 101 (A6), 13293-13303, 1996.