

## NONLINEAR DYNAMICS OF THE IONOSPHERIC ALFVEN RESONATOR

O.A. Pokhotelov and O.G. Onishchenko (*Institute of Physics of the Earth, 123995 Moscow, 10 B. Gruzinskaya Str., Russia*)

**Abstract.** A new mechanism of the Alfvén waves interaction with convective motions in the ionospheric Alfvén resonator is proposed. The model is based on the parametric excitation of convective cells by the finite amplitude Alfvén waves. A set of coupled equations describing the nonlinear interaction of Alfvén waves and electrostatic convective mode is derived. Our equations are then Fourier transformed to obtain a nonlinear dispersion relation, which admits the excitation of electrostatic cells. The generation of such cells is due to the Reynolds stresses of short-scale Alfvén waves which are nonzero only when the Alfvén perpendicular wavelengths are of the order of the collisionless electron skin depth or shorter. It is shown that classical shear Alfvén waves, when dispersion effects are neglected, do not generate the electrostatic convective perturbations. It is found that the wave vector of the convective mode is perpendicular to that of the pump Alfvén wave. The instability growth rate for the most growing modes is obtained. The results of the theory are applied to the satellite observations of the inertial Alfvén waves in the auroral ionosphere. It is shown that convective cells produced by the parametric instability can form the fine structure of the turbulent Alfvén boundary layer and play an important role in the ionospheric plasma turbulence.

### Introduction

The mechanism of parametric excitation of electrostatic convective cells by finite amplitude Alfvén waves was first suggested by *Sagdeev et al.* [1978]. Closed convective cells in this scenario are produced by the  $\mathbf{E} \times \mathbf{B}$  particle drift and are similar to those for two-dimensional vortex structures in an incompressible fluid. This mechanism may provide an efficient channel for the energy transfer from the small-scale Alfvénic perturbations to the large-scale convective motions (inverse cascade).

The most natural way of launching Alfvén waves, which is usually realized in space plasmas, is through a sheared plasma flow during plasma expansion into magnetic field. A typical example of such a process is the plasma expansion to the inner regions of the Earth's magnetosphere during magnetic storms. As the waves propagate towards the ionosphere, they generate filamentary structures extending along the magnetic field lines that connect the spatial gradients in the magnetosphere and ionosphere and result in an efficient magnetosphere-ionosphere coupling. The satellite observations of the small-scale Alfvén waves and analysis of existing theoretical models were recently reviewed by *Stasiewicz et al.* [2000].

Classical shear Alfvén waves are described by the simplest dispersion relation  $\omega_{\mathbf{k}} = k_z v_A$ , where  $\omega_{\mathbf{k}}$  is the wave frequency,  $k_z$  is the component of the wave vector  $\mathbf{k}$ , along the ambient magnetic field  $\mathbf{B}_0$ ,  $v_A = B_0 / (\mu_0 n m_i)^{1/2}$  is the Alfvén speed,  $n$  is the plasma number density,  $\mu_0$  is the permeability of free space, and  $m_i$  is the ion mass. These waves do not carry the parallel electric field and thus cannot provide parallel energizing of particles, which is a known feature of the coupling between the ionosphere and remote regions of the magnetosphere. In low- $\beta$  plasma such as the Earth's ionosphere this field is produced solely by the inertial Alfvén waves (IAWs) in which the wave vector perpendicular to the ambient magnetic field is comparable to collisionless electron skin depth  $\lambda_e$  [Goertz and Boswell, 1979]. These waves are described by the dispersion relation

$$\omega_{\mathbf{k}} = k_z v_A \Lambda_{\mathbf{k}}^{-1/2} \quad (1)$$

Here  $\Lambda_{\mathbf{k}} = 1 + \lambda_e^2 k^2$  accounts for the effect of the electron inertial length, and  $k$  is the component of the wave vector perpendicular to  $\mathbf{B}_0$ .

Space observations of the IAWs were recently provided by *Grzesiak* [2000] and *Chaston et al.* [1999, 2002] who used the data collected by Freja and FAST satellites. It was shown that the appearance of these waves in the Earth's ionosphere may be connected with operation of the ionospheric Alfvén resonator (IAR). However, the comparison of these observations with one-dimensional simulations of the IAR given by *Chaston et al.* [2002] shows that the properties of waves observed in the ionosphere are far from being in complete accord with those predicted from the linear theories. This disagreement becomes particularly evident since the observed energies of electrons accelerated by the IAWs turn out to be larger than those found in numerical simulations based on the linear inertial effects.

Apart from space observations there was a great deal of research devoted to the study of large amplitude Alfvén waves in the laboratory plasmas, the most recent among them are those provided by the LAPD (Large Plasma Device) experiments [for a review see *Gekelman, 1999* and references therein]. These experiments have examined, in great detail, shear Alfvén waves generated by filamentary currents in both spatially uniform and striated plasmas. A special emphasis was given to structures of the order of the skin depth. It was clearly demonstrated that the phenomena observed in laboratory experiments show striking similarities to what has been observed in space plasmas.

The main aim of the present paper is to provide a theory of the IAWs, which accounts for the effects of nonlinear coupling to convective motions. It will be shown that such a coupling is controlled by the Reynolds stresses of short-scale Alfvén waves with perpendicular wavelengths of the order of electron skin depth. This can give us a strong background for better understanding the existing observations of the IAWs in the near Earth environment.

### Model nonlinear equations

The IAWs are important in a low- $\beta$  plasma such as the Earth's ionosphere. It is convenient to introduce the two-potential representation

$$E_z = \mathbf{E} \cdot \mathbf{z} = -\partial_z \varphi - \partial_t A, \quad \mathbf{E}_\perp = \nabla_\perp \varphi, \quad \mathbf{B}_\perp = \nabla A \times \mathbf{z}, \quad (2)$$

where  $\mathbf{E}$  and  $\mathbf{B}$  are the perturbations of the wave electric and magnetic fields, respectively,  $\mathbf{z}$  is the unit vector along the ambient magnetic field  $\mathbf{B}_0$ , the subscripts  $z$  and  $\perp$  denote the components along and perpendicular to  $\mathbf{z}$ ,  $\varphi$  is the scalar potential of the electric field,  $A$  is the  $z$ -component of the vector potential. For pure-Alfvénic perturbations the perpendicular components of the vector potential are small and thus are neglected.

The field-aligned (parallel) electric field in a nonlinear IAW has both inductive and potential contributions, that is,

$$E_{||} = E_z + \frac{\mathbf{B}_\perp \cdot \mathbf{E}_\perp}{B_0} = -d_t A - \partial_z \varphi, \quad (3)$$

where  $d_t \equiv \partial_t + B_0^{-1} \{ \varphi, \dots \}$  and  $\{A, B\} \equiv (\partial_x A) \partial_y B - (\partial_y A) \partial_x B$  denotes Poisson brackets.

The field-aligned current  $j_{||} = -en v_{||e}$ , with  $v_{||e}$  being the field-aligned electron velocity, is carried mainly by electrons. Substituting (3) into the equation for parallel electron motion,  $E_{||} = (m_e / e) d_t v_{||e}$ , we obtain the gauge condition that relates the electrostatic potential  $\varphi$  and the vector potential  $A$ .

$$(4) \quad d_t (1 - \lambda_e^2 \nabla_\perp^2) A + \partial_z \varphi = 0.$$

On the other hand, the perpendicular current results solely from the inertia of the ions, so that  $\mathbf{j}_\perp = -\mu_0^{-1} v_A^{-2} d_t \nabla_\perp \varphi$ . Since the current is divergence free, one obtains after substituting the expressions for the perpendicular and parallel electric currents into the current closure condition  $\nabla \cdot \mathbf{j} = 0$  the relation

$$d_t \nabla_\perp^2 \varphi + v_A^2 d_z \nabla_\perp^2 A = 0. \quad (5)$$

Here we defined  $d_z \equiv \partial_z - B_0^{-1} \{ A, \dots \}$ , where the nonlinear term arises from the bending of the magnetic field line caused by the inertial Alfvén wave. The system (4) and (5) constitutes a closed set of equations that describes the dynamics of nonlinear IAWs and represents a generalization of the well-known reduced magnetohydrodynamic equations [e.g., *Strauss, 1976*] to the case when the effects due to the finite electron skin depth are taken into account.

### Generation of electrostatic convective cells

The simplest mechanism of generation of the convective cells refers to the parametric instability of a monochromatic Alfvén wave. Following the standard procedure to describe the evolution of the coupled system

(Alfvén waves plus convective modes), we decompose the scalar and the vector potentials into the low- and high-frequency parts, that is,

$$\phi = \phi + \psi, \quad A = a + A_h, \quad (6)$$

where  $\phi$  and  $a$  are the electro- and magnetostatic potentials of the convective mode which vary slowly with time and are independent of the  $z$  coordinate. Under the condition  $a \rightarrow 0$  and  $\phi \neq 0$ , the convective motions are purely electrostatic. In the opposite limit they represent a magnetostatic mode.

Averaging Eqs. (4) and (5) over the fast time scales  $\sim 1/\omega_k$  and short spatial scales  $\sim 1/k_z$  of the Alfvén wave, we obtain the equations for generation of the convective cells

$$\partial_t \nabla_{\perp}^2 \phi = -B_0^{-1} \langle \{\psi, \nabla_{\perp}^2 \psi\} - v_A^2 \{A, \nabla_{\perp}^2 A\} \rangle \quad (7)$$

and

$$\partial_t (1 - \lambda_e^2 \nabla_{\perp}^2) a = -B_0^{-1} \langle \{\psi, (1 - \lambda_e^2 \nabla_{\perp}^2) A\} \rangle \quad (8)$$

In these expressions, the brackets denote the averaging process. The terms on the right-hand side of Eqs. (7)-(8) represent the averaged (over the Alfvén wave period) Reynolds stresses of the rapidly oscillating Alfvén waves. For the sake of convenience, the subscript  $h$  in the definition of the high-frequency part of the vector potential is omitted here as well as in all the following expressions.

The nonlinear coupling between the Alfvén waves and convective cells is governed by

$$\partial_t \nabla_{\perp}^2 \psi + v_A^2 \partial_z \nabla_{\perp}^2 A = -B_0^{-1} [\langle \{\phi, \nabla_{\perp}^2 \psi\} + \langle \{\psi, \nabla_{\perp}^2 \phi\} \rangle] - v_A^2 (\langle \{a, \nabla_{\perp}^2 A\} + \langle \{A, \nabla_{\perp}^2 a\} \rangle), \quad (9)$$

and

$$\partial_z \psi + \partial_t (1 - \lambda_e^2 \nabla_{\perp}^2) A = -B_0^{-1} (\langle \{\phi, (1 - \lambda_e^2 \nabla_{\perp}^2) A\} + \langle \{\psi, (1 - \lambda_e^2 \nabla_{\perp}^2) a\} \rangle), \quad (10)$$

We choose the potentials of the convective mode in the form

$$\phi = \phi_q \exp[i(\mathbf{q} \cdot \mathbf{r} - \Omega t)] + c.c., \quad a = a_q \exp[i(\mathbf{q} \cdot \mathbf{r} - \Omega t)] + c.c. \quad (11)$$

Here  $\phi_q$  and  $a_q$  are the Fourier amplitudes of the convective potentials,  $\mathbf{q} \equiv \mathbf{q}_{\perp}$  and  $\Omega$  are the wave vector and the wave frequency of the convective mode, respectively, and *c.c.* denotes the complex conjugate.

The Alfvén waves are described by the combination of the scalar electrostatic potential  $\psi$  and the  $z$ -component of the vector potential  $A$ , which are taken as the superposition of the pump wave and the two sidebands, that is,

$$\psi = \psi_0 + \psi_+ + \psi_- \quad \text{and} \quad A = A_0 + A_+ + A_-, \quad (12)$$

where for the pump wave we have

$$\psi_0 = \psi_k \exp[i(\mathbf{k} \cdot \mathbf{r} - \omega_k t)] + c.c. \quad \text{and} \quad A_0 = A_k \exp[i(\mathbf{k} \cdot \mathbf{r} - \omega_k t)] + c.c., \quad (13)$$

with the frequency  $\omega_k$  given by (1).

The scalar and vector potentials for the Alfvén side-bands assume the form

$$\psi_{\pm} = \psi_{\pm} \exp[i(\mathbf{k}_{\pm} \cdot \mathbf{r} - \omega_{k_{\pm}} t)] + c.c. \quad \text{and} \quad A_{\pm} = A_{k_{\pm}} \exp[i(\mathbf{k}_{\pm} \cdot \mathbf{r} - \omega_{k_{\pm}} t)] + c.c., \quad (14)$$

where  $\omega_{k_{\pm}} = k_z v_A \Lambda_k^{-1/2} \pm \Omega$ ,  $\mathbf{k}_{\pm} = \mathbf{k} \pm \mathbf{q}$  are the frequencies and wave vectors of the Alfvén side-bands and  $\mathbf{k} \equiv \mathbf{k}_{\perp}$  is the wave vector of the pump wave perpendicular to the ambient magnetic field.

Substituting (11)-(14) into (7)-(10) we can obtain the desired dispersion relation for the parametric instability of the Alfvén waves. The rigorous analysis of this dispersion relation [see Pokhotelov *et al.*, 2003 for details] shows that perturbation of the magnetostatic potential  $a$  is small relative to the electrostatic  $\phi$ . Thus, the convective cells are purely electrostatic. After the lengthy but straightforward calculations one obtains

$$\Omega_{\pm} = \mathbf{q} \cdot \mathbf{v}_g \pm i \left[ b \frac{(\mathbf{q} \times \mathbf{k})_z^2}{B_0^2} |\psi_0|^2 - \delta\omega^2 \right]^{1/2}, \quad (15)$$

where  $\mathbf{v}_g = \partial\omega_{\mathbf{k}} / \partial\mathbf{k}$  is the pump Alfvén wave group velocity,  $b = \lambda_e^2 k^2 / (1 + \lambda_e^2 k^2)$  and  $\delta\omega = b\omega_{\mathbf{k}}(q^2 / 2k^2)$ . In the course of the derivation of Eq. (15) it was assumed that the typical wavelength of the convective cell is much larger than that for the pump Alfvén wave, i.e.  $|\mathbf{q}| \ll |\mathbf{k}|$ .

It is the upper sign in the expression (15) that yields an instability. A closer inspection of Eq. (15) shows that the fastest growing mode corresponds to  $\mathbf{q} \perp \mathbf{k}$ . Hence, the convective cells are non-propagating, zero-frequency modes in this case, the growth rate of which is given by

$$\gamma = \text{Im}\Omega_{+} = (bq^2 v_E^2 - \delta\omega^2)^{1/2}, \quad (16)$$

where  $v_E = k|\psi_0| / B_0$ .

The applying of Faraday's law enables us to rewrite the expression for  $v_E$  as

$$v_E = \frac{\omega_{\mathbf{k}}}{k_z} (1 + \lambda_e^2 k^2) \frac{|\mathbf{B}_{\mathbf{k}}|}{B_0}, \quad (17)$$

with  $\mathbf{B}_{\mathbf{k}}$  being the Fourier amplitude of the magnetic field of the pump Alfvén wave.

It follows from Eqs. (15) and (16) that the wave numbers  $q$  of the growing modes are localized in the range

$$0 < \left(\frac{q}{k}\right)^2 < \frac{4(1 + \lambda_e^2 k^2)^3}{k_z^2 \lambda_e^2} \frac{|\mathbf{B}_{\mathbf{k}}|^2}{B_0^2}. \quad (18)$$

For fixed  $k$ , the highest wave growth is attained at the value of  $(q/k)^2$  given by

$$\left(\frac{q}{k}\right)_{\max}^2 = \frac{2(1 + \lambda_e^2 k^2)^3}{k_z^2 \lambda_e^2} \frac{|\mathbf{B}_{\mathbf{k}}|^2}{B_0^2}. \quad (19)$$

This maximum growth rate becomes

$$\gamma_{\text{cell}} = \omega_{\mathbf{k}} (1 + \lambda_e^2 k^2) \frac{k^2}{k_z^2} \frac{|\mathbf{B}_{\mathbf{k}}|^2}{B_0^2}, \quad (20)$$

an expression which shows that  $\gamma_{\text{cell}}$  vanishes in the long wavelength limit,  $k \rightarrow 0$ , and increases as  $\propto k^5$  in the short wavelength limit,  $\lambda_e k \gg 1$ . Physically, this instability is a manifestation of an inverse cascade. It demonstrates that the spectral energy of the short-scale Alfvén wave turbulence is transferred in the interaction into the long scales occupied by the electrostatic cells, i.e. the Alfvén wave energy is converted into the energy of slow convective motion.

## Discussion and Conclusions

The present investigation shows that finite amplitude short-scale Alfvén waves in the low- $\beta$  plasmas decay in time transferring their energy to the large-scale electrostatic convective cells. These perturbations are driven at nonthermal levels due to the Reynolds stresses of the short-scale Alfvén waves. The latter are nonzero only

in the presence of the dispersion effects related to the finite electron skin depth. We have been able to derive a general dispersion relation for the parametric instability and to reduce it to the form suitable for the simple analytical analysis. The corresponding expression for the highest instability growth rate is obtained. It is shown that the most growing convective cell corresponds to the purely growing, nonpropagating zero-frequency mode with the wave vector that lies in the plane perpendicular to the external magnetic field and to the wave vector of the pump Alfvén wave.

There are a number of reasons for the vivid interest in the parametric instability of the Alfvén waves. This instability can serve as a basic mechanism for nonlinear saturation of the wave amplitude in space and laboratory plasmas. It may provide an essential nonlinear process that controls the energy cascading from the short- to the long-scale wave perturbations. The electrostatic convective cells, resulted from the parametric instability, may form the structural elements of the plasma turbulence in the Earth's ionosphere such as turbulent Alfvén boundary layer (TABL) [e.g., *Trakhtengertz and Feldstein*, 1987, 1991]. It is worth mentioning that the study of a weak turbulence of the IAWs provided by *Onishchenko et al.* [2003] shows that Kolmogorov spectra of these waves are nonlocal. Such a peculiarity of the IAWs indirectly points out to the possibility of generation of the large-scale structures. This instability is generic to a wide variety of plasma wave systems, such as drift waves in laboratory plasmas and Rossby waves in rotating fluids [e.g., *Smolyakov et al.*, 2000; *Guzdar et al.*, 2001; *Shukla and Stenflo*, 2003].

An obvious application of the developed theory to space plasmas refers to nonlinear dynamics of the ionospheric Alfvén resonator (IAR). The existence of the IAR in the topside ionosphere was well documented by ground-based observations both in middle [*Polyakov and Rapoport*, 1981; *Belyaev et al.*, 1987, 1990; *Hickey et al.*, 1996; *Bosinger et al.*, 2002] and in high latitudes [*Belyaev et al.*, 1999; *Demekhov et al.*, 2000]. In situ observations of the IAR eigenmodes were recently provided by *Grzesiak* [2000] and *Chaston et al.* [1999, 2002] who used the data collected by Freja and FAST satellites when crossing the auroral region. The IAR excitation in high latitudes is usually attributed to the feedback instability that is considered as the mechanism for the establishing of narrow-scale auroral arcs [e.g., *Lysak*, 1990, 1991; *Trakhtengertz and Feldstein*, 1987, 1991; *Pokhotelov et al.*, 2000, 2001; *Chaston et al.*, 2002; *Pilipenko et al.*, 2002]. The energetics of this instability was recently reviewed by *Lysak and Song*, 2002].

The measurements in the Earth's ionosphere demonstrate that Alfvén waves excited in the IAR practically never appear as a small amplitude linear disturbances [e.g., *Chmyrev et al.*, 1988; *Stasiewicz et al.*, 2000, *Chaston et al.*, 1999, 2002]. They always are of fairly large amplitude, and thus the IAR eigenmodes are in a highly nonlinear state. Comparison of the linear theory based on one-dimensional simulations with the case study example provided by *Chaston et al.* [2002] shows that the observed energies of accelerated by the parallel wave electric field electrons (which exceed 10 keV) suggest that this field should be larger than possible from electron inertial effects in the linear approximation particularly if this acceleration occurs at IAR altitudes. This can be explained by the fact that the value of the field-aligned electric field in the nonlinear Alfvén wave is greater than that for the linear wave. Indeed, using the expression (3), we can estimate the value of the parallel electric field  $E_{||}$  to

$$\frac{|E_{||}|}{E_{||}^l} = 1 + \frac{(1 + \lambda_e^2 k^2)^2 k^2 |\mathbf{B}_k|^2}{k_z^2 B_0^2}. \quad (21)$$

Here  $E_{||}^l = k_z k \lambda_e^2 E_{\perp} / (1 + \lambda_e^2 k^2)$  is the field-aligned electric field calculated in the linear approximation. This expression shows a substantial increase in  $E_{||}$  which arises from the bending of the magnetic field in the nonlinear Alfvén wave that forces the electrons to move along the perturbed magnetic field. All these facts point out to the important role of nonlinear effects in the IAWs.

It should be noted that the decay channel for the dispersive Alfvén waves studied in the present paper does not provide the sole possibility for the decay of IAWs. Other decay mechanisms that might compete with it and under certain conditions even might dominate it have been studied for instance by *Dubinín et al.* [1988], *Volokitin and Dubinín* [1989], *Voitenko* [1998] and *Voitenko and Goossens* [2000]. In particular, the decay instability IAW=IAW+IAW due to three-wave coupling, with two secondary excited Alfvén waves propagating in opposite directions, often turns out to have a larger growth rate and might thus be a more efficient mechanism. The growth rate in the inertial limit is given by *Voitenko and Goossens* [2000].

$$\gamma_{AA} = 0.15 \lambda_e v_A k_{\perp}^2 \left| \frac{\mathbf{B}_k}{B_0} \right|. \quad (22)$$

In order to investigate the relative importance of the two mechanisms we form the ratio

$$\frac{\gamma_{AA}}{\gamma_{cell}} = 0.15 \frac{k_z \lambda_e}{(1 + \lambda_e^2 k^2)^{\frac{3}{2}}} \left| \frac{\mathbf{B}_k}{B_0} \right|^{-1}. \quad (23)$$

The parametric generation of convective cells starts dominating if

$$\frac{k_z}{k(1 + \lambda_e^2 k^2)} > \left| \frac{\mathbf{B}_k}{B_0} \right| > 0.15 \frac{k_z \lambda_e}{(1 + \lambda_e^2 k^2)^{\frac{3}{2}}}. \quad (24)$$

The left inequality defines the condition of applicability of our theory,  $\gamma_{cell} < \omega_k$ , which has been obtained from Eq. (20). It follows from these inequalities that the left margin exceeds the right one for an arbitrary value of  $\lambda_e k$ . The decay instability due to the three-wave interaction of counterstreaming Alfvén waves prevails for a restricted range of pump wave amplitudes, which is bounded by the condition

$$|\mathbf{B}_k| < |\mathbf{B}_k^{AA}| \equiv 0.15 B_0 \frac{k_z \lambda_e}{(1 + \lambda_e^2 k^2)^{\frac{3}{2}}}. \quad (25)$$

For the lowest IAR eigenmode one has  $\omega_k L / v_A = \pi$  and  $k_z = \pi(1 + \lambda_e^2 k^2)^{\frac{1}{2}} / L$ , where  $L$  is the characteristic size of the IAR. (A more accurate calculation of the lowest IAR eigenmode-frequency that accounts for the exponential profile of the Alfvén velocity in the topside ionosphere yields the value  $\omega_k = 2.4 v_A / L$  [e.g., Pokhotelov *et al.*, 2001]). Typical values for parameters in the IAR region are:  $L \approx 1000$  km,  $B_0 = 0.3$  G and the electron skin depth  $\lambda_e = 100$  m. Assuming  $\lambda_e k \approx 1$  and substituting these values into Eq. (25), we obtain  $|\mathbf{B}_k^{AA}| \approx 0.5$  nT. For pump Alfvén wave amplitudes of  $|\mathbf{B}_k^{AA}| > 0.5$  nT the dominant nonlinear mechanism of IAWs decay is the parametric instability involving the generation of the convective cells. We note that the satellite observations in the auroral region provides the evidence of the Alfvén waves with the amplitudes up to 50 nT [Chmyrev *et al.*, 1988; Chaston *et al.*, 2002]. Such large amplitude waves certainly cannot rigorously be described in the framework of our simplified approach which according to Eq. (24) is limited by the amplitudes of the order 6 nT for the selected plasma parameters. Thus, the mechanism considered in the present paper may frequently be the most efficient. With the plasma parameters given above Eq. (19) implies a characteristic spatial scale  $\lambda = 2\pi / q$  from several kilometers to a few tenth of kilometers for the convective cells, depending on the amplitude of the pump Alfvén wave.

We note that Dubinin *et al.* [1988] and Volokitin and Dubinin [1989] also studied the similar case of the low- $\beta$  plasma when IAWs play a dominant role. These authors concluded that parametric instability of the dispersive Alfvén waves that lead to generation of convective cells does not exist in this limit. We note that in the course of the derivation of corresponding dispersion relation these authors erroneously assumed that the Fourier amplitudes of the electrostatic  $\phi_q$  and magnetostatic  $a_q$  convective modes are interconnected through the relation that is valid only for the linear IAWs. The rigorous analysis presented above shows that this is not the case, convective motions are substantially nonlinear and do not correspond to the Alfvén type perturbations. Moreover, our analysis shows that IAWs parametric instability does not lead to the growth of magnetostatic perturbations.

The parametric instability considered in the present paper may provide a substantial damping of the IAR eigenmodes that can transfer their energy into the turbulent convective cells in the topside ionosphere. We note that both the ground based [Maggs and Davis, 1968] and satellite [Chaston *et al.*, 2002] measurements revealed strong evidence for the presence of perpendicular short scales of the order of  $\lambda_e$  in the Earth's ionosphere. For the waves with shorter wavelengths,  $\lambda_e k \gg 1$  the damping becomes even stronger due to the strong dependence of the instability growth rate on  $k$ . It should be mentioned that the model developed in this paper yet remains oversimplified. So far it does not include, for example, the effects which are caused by the presence of the external convective flows, nor does it take into account the field-aligned plasma density inhomogeneity, nor the finite Larmor radius corrections which come into play at higher than the IAR altitudes. However, our analysis provides an essential nonlinear mechanism for the transfer of energy from the short-scale Alfvén waves to the long-scale enhanced convective motions which may result in the observable damping of the IAR eigenmodes. The latter can

represent the result of self-consistent interaction of the convective motions induced by turbulent Reynolds stresses and their back-reaction on Alfvén wave growth as an important mechanism contributing to the IAR eigenmode saturation. The assumption of a weakly turbulent state may become questionable as the convective cells become stronger and the coupled IAWs-cells wave turbulence enters a new regime. The Alfvén wave driven convective cells in the ionosphere can interact with the background medium and develop two-dimensional nonlinear motions in the form of Kelvin-Stuart vortex street [e.g., *Petviashvili and Pokhotelov, 1992*] which can be constructed using the same method as that described by *Shukla and Stenflo [2003]* in application to nonlinear dynamics of Rossby waves. Such vortices can form the fine structure of the TABL and may constitute a dynamical paradigm for intermittency in the ionospheric turbulence containing nonlinearly coupled IAWs and convective motions. The detailed comparison of theoretical results with satellite observations is, however, outside the scope of the present study and further analysis is required to elucidate the competition between the Alfvén wave growth caused by the feedback instability and subsequent damping due to the parametric instability. Thus, the intention of the present study is to provide a deeper insight into physics of nonlinear dynamics of the Alfvén waves in the ionospheric plasma. Hence the present paper can be considered as an extension of our previous analysis of the IAR [*Pokhotelov et al., 2000, 2001*], which was limited solely by the linear approximation.

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