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CELLULAR MODEL ANALOGY OF THE MAGNETOSPHERE-IONOSPHERE SUBSTORM ACTIVITY DRIVEN BY SOLAR WIND WITH FINITE VELOCITY OF PENETRATION INTO MAGNETOSPHERE

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Abstract. The cellular model as an analogy of the dynamical magnetosphere-ionosphere system related with the substorm activity is presented. Each cell in the model contains two connected parts, one of which may be associated with the magnetosphere current sheet piece, and other – with the ionosphere region at the same magnetic field tube. The local positive feedback between magnetospheric and ionospheric parts of each cell has been included in the model.

The magnetospheric part of the model system is organised as a rectangular cellular automation with local redistribution of the stored energy from the cells where the threshold value is exceeded. We suppose that the threshold value in each cell depends on external control parameter which influences the long boundaries of the rectangular array. The finite velocity is assumed for the influence penetration into the array and along boundaries. Internal rules in the model are fully deterministic: there is not used any random number.

As an external control parameter of the model we consider the z-component of interplanetary magnetic field (B_z IMF) which is «freezed» in the solar wind. Dynamics of the model for constant control parameter and for driving by real B_z IMF is discussed. The model demonstrates two types of transients. The characteristics of the transients are discussed.

Introduction

The magnetosphere of the Earth immersed in the solar wind, is an open non-linear dynamical system far from the equilibrium state. The dynamics of systems of such a class generally can be rather complex.

Recently the self-organizing processes in the magnetosphere have been connected to the possible appearance of so-called «self-organized critical» (SOC) state in the magnetospheric plasma (Bak, 1997; Jensen, 1998; Vespignani and Zapperi, 1998). The system in such SOC state should demonstrate properties, that are characteristic of plasma in the magnetic field: the power law spectra of fluctuations; small (and fractal) dimension of spatial and temporary features (Voros, 1991; Takalo et al., 1993; Sharma, 1997; Uritsky and Pudovkin, 1998).

In previous papers (Kozelov and Kozelova, 2000, 2002a) we have considered the classification of spontaneous and stimulated transients in the SOC-system controlled by B_z IMF. Developing this model, in paper (Kozelov and Kozelova, 2002b) we have included in the model the positive feedback as an analogy of the feedback between the Earth's magnetosphere and ionosphere, which is activated during an explosive phase of substorm and does not work in quiet time. Such model has allowed us to explain the obtained in (Lui et al., 2000) differences of distribution functions of the power dissipated by auroral spots for quiet time and during substorms. It is interesting that the necessary condition has appeared a system control just B_z ММП (that is a chaotic signal).

Here we make the following step from qualitative model of a sand pile (Bak, 1997) to the physically more real model. Saving internal local connections from our previous model, we organize an external influence on the model as a flow of the magnetospheric tail by the solar wind. Thus, we consider, that the effect from solar wind is spread inside the system with finite (Alfven) velocity. It has appeared, that in the model it is possible to completely refuse of a random variable, that is the model is completely deterministic. Thus, the transients in the magnetosphere-ionosphere system have analogies within the framework of the deterministic model with the chaotic external driving.

We have found that the model dynamics depend on the value of external control parameter. With increase of the absolute value of the control parameter the consequence of transitions from absence of transients to periodic generation and from periodic to the chaotic mode of generation takes place. It is possible to consider the series of transitions as a typical series of bifurcations.

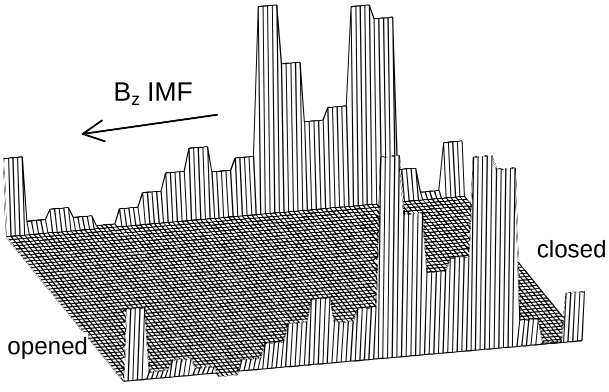

Fig.1. Scheme of the «magnetospheric» array and boundary conditions.

Model

The numerical model, presented here, is developing an analogy used in (Kozelov and Kozelova, 2000, 2002a,b). Now the current sheet of the magnetospheric tail is represented as the rectangular 50×100 array of cells. One short boundary of the array is supposed to be closed (this is the Earth-side edge of the current sheet), other boundaries are open (Fig.1). The state of a cell with coordinates (i,j) at a moment t is characterized by the stored energy $E_t(i,j)$. In addition to this, we shall assume, that some value $C_t(i,j)$ is connected to each cell. It is possible to consider this value as an analogy of ionosphere conductivity in the given magnetic tube.

The time is considered to be discrete, however for illustration we use the relation of the discrete time to minutes and seconds. At every step in time in each cell of the array the small portion of energy $dE(i,j)$ is added. As long as the value of the stored energy $E_t(i,j)$ does not exceed some critical level $E_{\max}(i,j)$, the cell remains steady. Here we set $dE(i,j)=0.02\ E_{\max}(i,j)$ for $E_{\max}(i,j)>0$. As against models discussed in (Kozelov and Kozelova, 2002a,b) now we suppose that the threshold value in each cell is individual and depends on an external control parameter which influences the long boundaries of the rectangular array.

The finite velocity is assumed for the influence penetration into the array and along boundaries. The values at the boundaries of the array $E_{\max}(1,j)$ and $E_{\max}(50,j)$ were determined by value of (-B$_z$) IMF and were shifted along boundaries with the velocity of 1 cell/ min. The velocity of propagation of the disturbance inside the array was also assumed 1 cell/min, and the value of the disturbance decreased proportionally to the distance from the boundary.

When the threshold level $E_{\max}(i,j)$ is exceeded the cell passes in an active state, and a certain part of the stored energy, $\Delta E=E_t(i,j)-E_{\min}(i,j)$, is distributed between four adjacent cells:

$$\begin{aligned}
&E_{t+1}(i,j)=E_{\min}(i,j), \qquad (1)\\
&E_{t+1}(i+1,j)=E_t(i+1,j)+0.25\Delta E,\\
&E_{t+1}(i-1,j)=E_t(i-1,j)+0.25\Delta E,\\
&E_{t+1}(i,j+1)=E_t(i,j+1)+0.4\Delta E\ ,\\
&E_{t+1}(i,j-1)=E_t(i,j-1)+0.1\Delta E
\end{aligned}$$

After that the energy of adjacent cells can exceed $E_{\max}$. Then these cells at the following time-step transmit, in turn, the energy to their neighbours. It is supposed, that the speed of internal transients in a current sheet is higher, than the speed of propagation of an external disturbance. In our calculations 1 time-step for internal processes of reallocation in the system was 10 seconds.

Observational studies of magnetospheric activity suggest that the magnetosphere-ionosphere coupling plays a critical role in the physical processes leading up to a substorm onset. The local reallocation of energy in the magnetosphere causes a local change of conductivity of the ionosphere in the same magnetic tube (the particles, diffused by pitch-angle, are precipitated in the loss-cone along the magnetic field, and ionize atmospheric gases). Let's take into account this effect in the model as:

$$C_{t+1}(i,j) = a\ C_t(i,j) + b, \qquad (2)$$

Here $a = 0.2$ means a «recombination coefficient», therefore conductivity of the ionospheric part of a cell depends on the cell history. The second term is determined as

$$b=\begin{cases} 0, & \text{for } E_t(i,j)<E_{\max}(i,j)\\ \Delta E, & \text{for } E_t(i,j)\geq E_{\max}(i,j)\end{cases} \qquad (3)$$

In turn, if the ionospheric conductivity exceeds some level $C_{\max}$ then the conditions of energy reallocation in the current sheet are changed (at the expense of formation of field-alined currents). In the model we shall take into account this influence as relation of $E_{\min}(i,j)$ from $C_t(i,j)$. This $E_{\min}(i,j)$ value determines, what part of energy may be reallocated in an active cell at the following time step:

$$E_{\min,t+1}(i,j)=\begin{cases} k\,E_{\max}(i,j), & \text{for } C_t(i,j)<C_{\max}\\ 0, & \text{for } C_t(i,j)\geq C_{\max}\end{cases} \qquad (4)$$

here $k<1$ ($k=0.75$) and $C_{\max}=5$ are parameters. The positive feedback arising between magnetospheric and ionospheric parts of a cell is shown schematically in Figure 2.

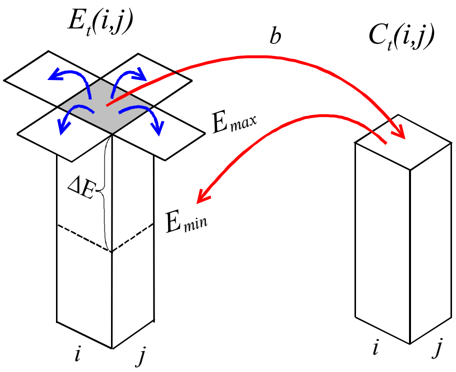

Fig.2. Scheme of the local positive feedback between «magnetospheric» and «ionospheric» parts of a cell: $E_t(i,j)$-stored energy, $C_t(i,j)$-ionospheric conductivity.

Each cell (except the boundary cells) has four adjacent cells, therefore the occurrence of the chain response of energy transmission is possible, and it proceeds as long as there are no active cells in the array. Such a chain response is usually named an "avalanche" by analogy with the formation of a slope for a sand pile (Bak, 1997). In our calculations we consider, that for one time-step each cell can change its state only once, so the avalanche may have different duration. Besides in the considered model there is no additional internal source of stochasticity (random variable), therefore in this sense the model is strictly deterministic.

Driving by B$_z$ IMF

We used the B$_z$ values measured by IMP-8 satellite as a control parameter of the model. We found that the system demonstrates two types of transients: 1) transients with active front and smooth «tail», see Fig.3a; 2) «turbulent» transients which contain a number of peaks, see Fig.3b. Transients of first type usually observed when B$_z$ has small negative value (-10 nT<B$_z$<0). Transients of second type appear at large negative values of B$_z$ and especially after fast change of B$_z$ from large negative to positive value.

The origin and motion of transients may be tracked in keograms, that are constructed by the method used in the analysis of television (TV) data of aurora registration. Fig.4 illustrates the dynamics of the set of the first type transients. The 25-th column in the array (analogy of north-south cross section of TV frame) and rows of 80, 50 and 20 (analogy of west-east cross sections at different latitudes) used for construction of the keograms. The grey gradation is figured the energy distributed by cells, for which the threshold of activation of feedback is exceeded. In this sense the keograms are the analogy of keograms of aurora. One can see that onsets of transients appear at different positions in array and they have no visible triggering by B_z variation.

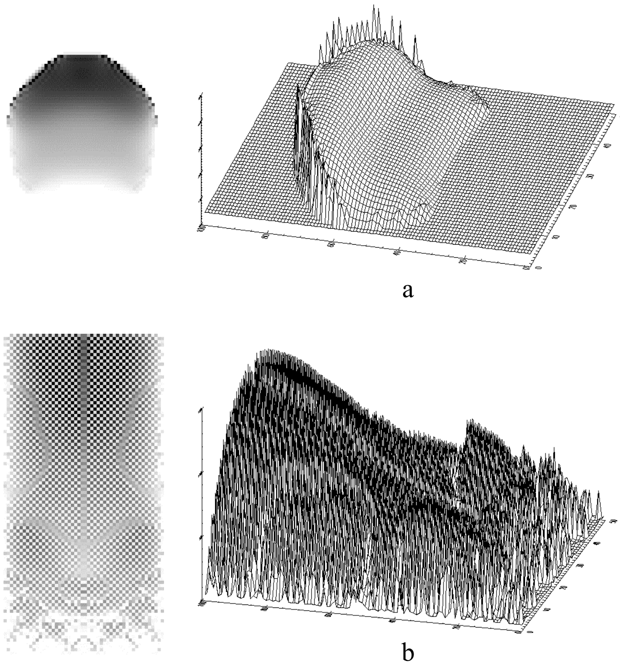

Fig.3. Examples of transients observed at varied B_z driving: a) - first type transient; b) - second type transient. Left panels - energy in grey gradation, right panels - energy as a surface.

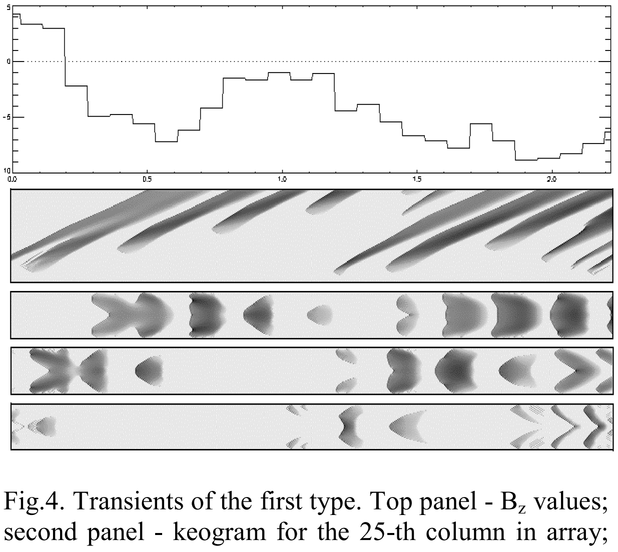

Fig.4. Transients of the first type. Top panel - B_z values; second panel - keogram for the 25-th column in array; other panels - keograms for 80, 50 and 20 rows.

Fig.5 shows the transient of second type. One can see that the onset of the transient apears near the closed boundary of the array and it is obviously stimulated by a fast change of B_z signum.

Dynamics for constant external parameter

It is found, that the following modes are possible in the model at the constant external parameter.

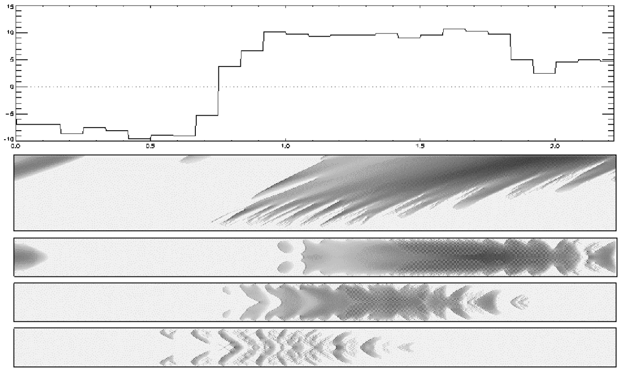

Fig.5. The same as Fig.4, but for transient of second type.

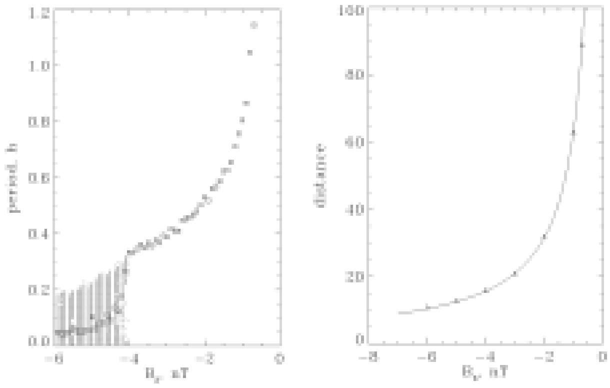

Fig.6. Characteristics of pseudo-periodical regime for constant B_z as a function of B_z value: a) points are time between onsets, squares are average periods; b) distance of onset point from the closed boundary.

1) At $B_z>0$ the energy in the system is not retained , the processes of reallocation in the «magnetospheric» part do not happen, the conductivity of «ionosphere» is equal to zero.

2) At $0>B_z>-0.6$ the energy in a system can be retained, but processes of reallocation in the «magnetospheric» part carry out excesses of energy. The stationary state is established. However the activity in the «magnetospheric» part is not sufficient for the excess of the threshold of ionospheric activation. Therefore the positive feedback is not working.

3) At $-0.6>B_z>-4.0$ at some moments the threshold of ionospheric activation is exceeded, after which the reallocation in the «magnetospheric» part becomes more intensive, that leads to the increase of energy which flows out, then the activity is falling, etc. This is a mode of (pseudo-) periodical generation. It is possible to obtain two obvious characteristics of the periodic mode observed in the system: the mean period and distance of

onset point of transient from the closed boundary of the array. In figure 6 the relations of these characteristics from B_z are shown. The smaller periods between transients (activations) correspond to smaller B_z, besides the onset of activations is observed closer to closed boundary (to the «Earth»).

4) At B_z<-4.0 the periodicity of generation is obviously broken, which leads to a sharp decrease of the mean period at -5.0>B_z>-4.0. There is a chaotic mode, passing at B_z<-10.0 in a «turbulent» mode at which the feedback will be activated at one time step from reallocation within one cell.

Thus, the dynamics of the system at the constant control parameter depends on the value of control parameter. It is possible to consider the series of transitions from the absence of transients to a periodic and, further, to a chaotic mode of generation as a typical series of bifurcations.

Discussion

In spite of the fact that the durable periods of constant B_z are rather rare, there are some indirect experimental confirmations of relations obtained in the previous section. In (Zverev et al., 1979), there was obtained the minimum latitude, up to which the auroral oval is spread, as a function of B_z IMF. The oval is descended at lower latitudes with the increase $|B_z|$. This is in agreement with the relation in Fig.6b.

The periods of constant B_z value are usually associated with periods of steady magnetospheric convection, during which substorms are not observed or are extremely rare (Sergeev et al., 1996). The steady magnetospheric convection events arise at rather large $|B_z|$ values (B_z<-3), the active mesoscale transients are observed in the magnetosphere, but large-scale transients (substorms) do not develop. The chaotic mode in the model has some qualitatively similar features. It arises indeed at large B_z values, when instead of large-scale periodic activations there appear some smaller-scale chaotic ones.

It is necessary to note, that in this paper we set a goel of observing the relation of model dynamics to external parameter. At qualitative preservation of relations from Fig.6, the absolute value of the characteristics of periodical mode may be adjusted by change of internal parameters of model.

Conclusions

New cellular model as an analogy of the dynamical magnetosphere-ionosphere system related with the substorm activity is presented. Saving internal local connections from our previous model, we organize an external influence on the model as a flow of the magnetospheric tail by the solar wind. Besides the model is strictly deterministic.

At the control by real sequence of B_z IMF values, we found that the system demonstrates two types of transients: 1) transients with active front and smooth «tail»; 2) «turbulent» transients containing a number of peaks. Transients of the first type usually observed when B_z has a small negative value (-10 nT<B_z<0). Transients of the second type appear at large negative values of B_z and especially after the fast change of B_z from large negative to positive value.

At the constant control parameter the model dynamics is determined by the value of southern components ($B_s = B_z$ for B_z<0). The periodic transients (activation) occur in the system at values of -4.0 < B_z < -0.6. Smaller periods correspond to smaller B_z, besides the position of activation onset in the array is shifting to the «Earth» with the falling B_z value. With the increase of the absolute value of southern components the transition (bifurcation) from periodic to a chaotic mode of generation happens.

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