«Physics of the Auroral Phenomena», Proc. XXV Annual Seminar, Apatity, pp.73-76, 2002

Polar Geophysical Institute

«ON-OFF» INTERMITTENCY AS A DYNAMICAL ANALOGY OF VLF CHORUS GENERATION

B.V. Kozelov, E.E. Titova, A.A. Lubchich *(Polar Geophysical Institute, Apatity, Russia)*
V.Y. Trakhtengerts *(Institute of Applied Physics, Nizhny Novgorod, Russia)*
J. Manninen *(Sodankyla Geophysical Observatory, Finland)*

Abstract. We consider a new numerical model to explain the dynamics of the VLF/ULF chorus elements detected in satellite and ground-based observation data. We suppose that generation mechanism of the chorus element is the same as in backward wave oscillator (BWO). However, now we assume that the BWO-like generator is driven by some other dynamic system which have noise component. In such case the so-called «on-off» intermittency regime is possible to occur if the BWO-like generator is near the generation threshold. A power-law distribution of time intervals between bursts of generation is a manifestation of the intermittency regime. This type of distribution is revealed for time intervals between chorus elements. By this model it is possible to explain also such features of VLF/ELF emission as: 1) transition from discrete chorus elements to the hiss then the magnetosphere disturbance is increasing; 2) modulation of the chorus groups by low frequency (<0.5 Hz) oscillations. Connection between the model dynamics and observations is discussed.

Introduction

The dissipation of plasma energy by the cyclotron interaction of energetic particles with low frequency waves is a widespread phenomenon and is actively studied theoretically, and from the satellite and ground based observations. This interaction often results in generation of separate discrete elements, separated by rather durable intervals, during which the generation is absent.

Examples of such a phenomenon are the ELF-VLF chorus emissions, which represent a sequence of elements in the frequency band 10^2 - 10^4 Hz and with duration 0.1-1 seconds, which are boosted on frequency. The mechanism of chorus generation is based on cyclotron interaction of (10-100 keV) electrons of radiation belts with low frequency waves near equatorial plane [Bespalov and Trakhtengerts, 1986]. However, the basic problem of chorus generation, as the sequence of discrete chorus elements is formed is still not solved. Recently, V.Yu. Trakhtengerts [1999] has offered a solution of this problem on the basis of a new mode of cyclotron magnetospheric maser - mode of backward-wave oscillator (BWO). The BWO generation mode was found in electron devices many years ago and, in details, was considered in paper [Ginzburg and Kuznetsov, 1981].

In [Kozelov et al., 2001] by the MAGION-5 satellite data it is found , that the time intervals between chorus elements in morning sector at 30-40^o from the equatorial plane have a power-law distribution with index ≈1.5. Thus, the physical mechanism of chorus generation should ensure certain dynamic properties, which are not stacked in frameworks of laboratory research of BWO. In paper [Kozelov et al., 2001] an attempt was made to explain the experimental distribution of intervals between chorus elements on the basis of the idea of self-organizing of a chain of ducts connected between themselves by the effect of external noise. However, the further research of the constructed mathematical model has resulted in a conclusion, that the interaction between the ducts (if it exists) does not play an important role.

The mathematical models, which could ensure the power-law distribution of time intervals between discrete activations are rather intensively discussed in connection with X-ray bursts on the Sun [Wheatland etc., 1998]. The most important result in this direction is the detection of a mode of «on-off» intermittency [Heagy etc., 1994]. As against other known types of intermittency, the «on-off» intermittency is the only one really dynamical, that is, it appears at random (noised) change of control parameter. In paper [Heagy etc., 1994] it was shown, that the «on-off» intermittency arose in rather a broad class of dynamical systems. In the review [Landa, 1997] this dynamic mode is classified as a noise-induced phase transition in non-linear systems.

In this paper we offer an explanation of the dynamics of VLF chorus emissions as a regime of «on-off» intermittency in the BWO generator, controlled by noise. The numerical dynamical model is constructed and its predictions are compared with the results of ground based observations in Porojarvi (Finland).

Experimental data

We selected for the analysis an isolated VLF/ELF emission event observed during about one hour at Porojarvi station (69.2N, 21.5E, L = 6.15) January 19, 1993. The chorus emissions were registered in the day time 12:40-13:25 LT after SC which happened at 10.22 UT. During all this time interval the emission in the range of 1.3-2.8 kHz were observed: at the bottom of this frequency range there was located a ELF hiss, intensity of which was varied; and upwards from the hiss the emission was mainly discrete. The lowest frequency of chorus elements often coincides with the upper frequency of ELF hiss and these frequencies changed simultaneously during the development of the event. The frequency correlation testifies of the connection between generation of VLF chorus and ELF hiss.

In the beginning of the event during first 5 minutes there was a fast increase of intensity of ELF emission, then during ~40 minutes the intensity was smoothly dropping. The dependence of a number of discrete elements on ELF intensity was obvious. So near the maximum of intensity the separate chorus elements merged in a wide hiss (Fig.1-a), at smaller intensity the sequence of elements (Fig.1-b) with intervals between them of ~0.1-1.0 s can be seen. Rare separate chorus elements (Fig.1-d) are observed during the further decrease of the intensity. It is necessary to note, that during this event there was a modulation with the period of ~ 4 s. This modulation influenced the waves in the ELF range as a modulation of hiss intensity. The chorus elements were also grouped with period ~4 s, see Fig.1-c.

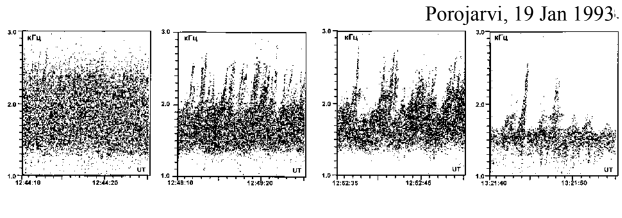

Fig.1. Examples of sonogram of VLF/ELF emission during the considered event.

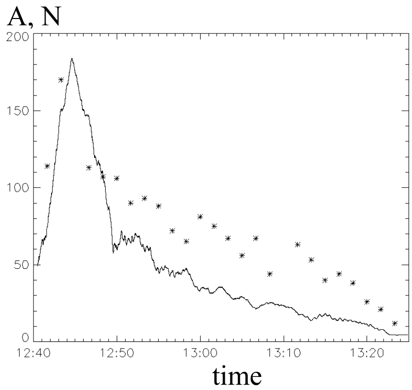

Fig.2. Solid line is the amplitude of ELF hiss (in relative units), Asterisks are the quantity of chorus elements in intervals for 100 s.

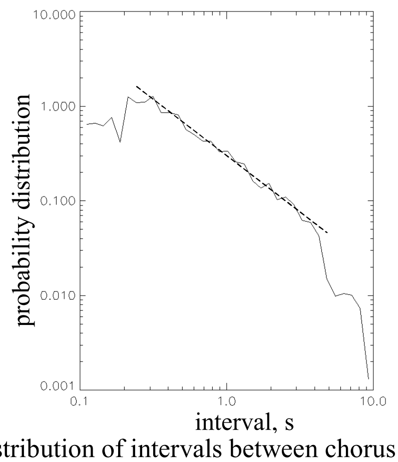

Fig.3. Distribution of intervals between chorus elements for the considered event.

Fig.2 shows the time dependence of the ELF emission amplitude smoothed in 1 min intervals. This dependence illustrates the above described change of ELF intensity. The characters in Fig.2 mark the quantity of chorus elements, observed during 100 s intervals. The chorus elements were selected manually in a sonogram.

The probability density of intervals between chorus elements calculated (for interval) from 12:48 till 13:20 UT is presented in Fig.3. The region of power law distribution is obvious. From one side this region is limited by resolution of the sonogram, in which a chorus element has a «width» of about 0.1 seconds. From the other side the limitation is explained by period of modulation (4 s). The power index (α~1.2) has appeared a little less, than obtained by satellite data, which , apparently, is explained by essential change of generation parameters during the event and is considered below .

Numerical model

The elementary model of the BWO generator according to paper [Ginzburg and Kuznetsov, 1981] is described by an equation:

$$A(t) = \lambda\,[A(t-T_0) - A^3(t-T_0)] + \delta, \qquad (1)$$

where T_0 is the feedback time, λ is the gain of a disconnected chain in linear (A<<1) mode. Dynamics of discrete model circumscribed by Eq.2 at constant λ parameter, was investigated in paper [Ginzburg and Kuznetsov, 1981]. At λ<1 the generation in the model is absent. At λ>1, depending on λ value, the model demonstrates different modes of generation.

We shall consider the behaviour of the model in a case, when the value is not constant, but depends on time generally as:

$$\lambda(t) = C_0 + C_1\,\xi(T_1,t) + C_2 \sin(2\pi t/T_2) + C_3 t \qquad (2)$$

Here C_0, C_1, C_2, C_3, T_1, T_2 are constants, $\xi(T_1,t)$ is a (pseudo-) random number, uniformly distributed from 0 to 1, and let us consider the value of $\xi(T_1,t)$ as a constant during some random period with average value of T_1. Thus, the stochastic or periodical term can predominate in $\lambda(t)$ depending on values of constants of C_1 and C_2. Physically, the periodical term can reflect a modulation of plasma properties in BWO by waves of other origin. At $C_3 \neq 0$ relations $\lambda(t)$ will have a general trend.

«On-off» intermittency

In a system described by Eq.1-2 the origin of the mode of «on-off» intermittency is possible near the points of bifurcations in Eq.1 at nonzero noise term in Eq.2. The threshold of the intermittency is determined from a condition $\langle \ln \lambda \rangle = 0$. While limiting ourselves only in two terms in Eq.2 and uniform distribution of ξ from 0 to 1, it is possible to obtain this condition as a relation of $C_1(C_0)$, see Fig.4. The «on-off» intermittency arises at parameter values lying above this curve. In this mode the peaks of activity of rather a large amplitude are formed in the background of continuous intervals, during which the amplitude is small. Examples of relation of the amplitude with time in this mode of generation is shown in Fig.5. The time scale for visualization is related to characteristic periods observed for chorus emissions.

According to paper [Heagy etc., 1994] near the threshold of intermittency (λ_{thr}) the intervals between peaks have the power-law distribution with power index ~1.5. Besides the mean interval between peaks in the «on-off» intermittency is inversely proportional to the distance of the mean value of λ from the λ_{thr} point:

$$<t_{ст}> \sim (<\lambda> - \lambda_{thr})^{-1} \quad (3)$$

In other words, with the increase of $<\lambda>$ the quantity of peaks is increased. In a case, when both noises are present, and periodical term in Eq.2, the output signal, to a greater or lesser extent reflects the presence of these two components depending on values of the parameters.

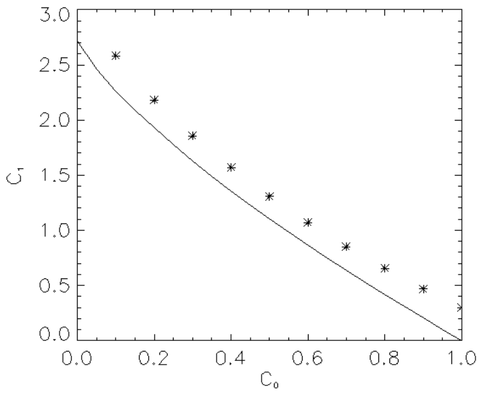

Fig.4. Solid line - values of C_1 and C_0, which satisfied the condition of intermittency threshold. Asterisks - pairs of C_1 and C_0 values, at which the mean interval between peaks with amplitude > 0.4 is equal 1.5 s.

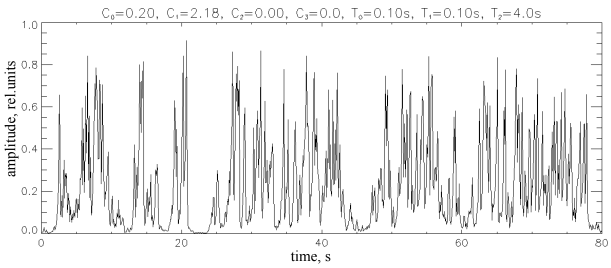

Fig.5. Example of output amplitude in «on-off» intermittency mode.

Discussion of C_0 and C_1 values

What values of parameters in the model can correspond to chorus generation? Let us go back again to a simpler case, when in Eq.2 there are only the first two terms. For transition in BWO mode it is necessary to exceed the threshold, which can take place both at the expense of increase of C_0, and at the increase of C_1. At some values of C_0 and C_1 parameters (see Fig.4) the «on-off» intermittency arises. Observation of chorus emissions give us two additional conditions: firstly, the quantity of discrete units (see Fig.2); secondly, the amplitude of peaks should be rather large. At fixed C_0 it is possible to supply the necessary quantity of peaks by selection of C_1 value, taking into account Eq.3.

In Fig.4 the asterisks mark the pairs of C_0 and C_1 values, at which in output signal of the model the mean interval between peaks with amplitude A>0.4 is equal 1.5 s, which corresponds to the mean period of the following chorus elements in a middle of the interval, shown in Fig.2. Despite the identical quantity of peaks, the output signal in these cases varies essentially depending on C_0. This is clearly seen in Fig.6, where the probability density functions of the amplitude values for the considered cases are adduced.

One can see that at C_0<0.4 the peak mode is really realized: the peaks have an amplitude that is exceeds essentially (in some times) its average value. The probability density of amplitude values drops in all ranges and has no maxima at large values.

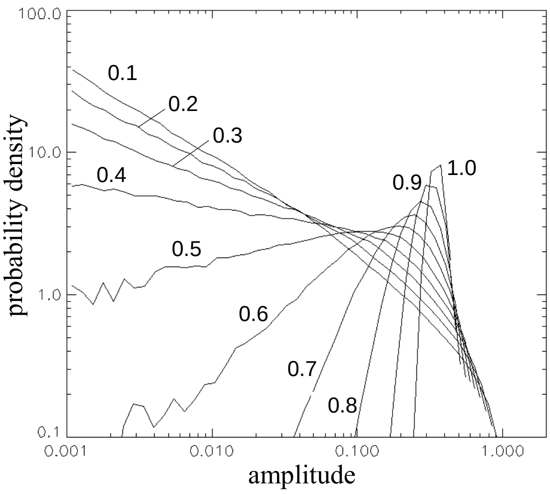

Fig.6. Probability density functions of the amplitude values for various values of C_0.

At C_0 > 0.5 the probability density of amplitude values has the maximum appropriated to average value, that means that the mode of continuous generation of a noise near the average value of the amplitude is achieved.

According to [Trakhtengerts, 1995, 1999], the step-like distribution function of energetic electrons is necessary for generation of chorus emissions. Such kind of a distribution function can be shaped at interaction of energetic electrons with ELF hiss. The step-like distribution function is a source of a fast increase of the increment of cyclotron instability and it determines the development of the BWO generation mode. It is possible to expect, that the increase of hiss intensity results in an increase of height and slope of the «step», to which the increment of instability γ_{st} is connected. The increase of the parameter λ in Eq.1 is also connected to the increase of instability increment. Usually, the hiss bandwidth is equal to ~ 100 Hz, from here the characteristic time for the hiss amplitude variations is ~ 0.01 << γ_{st}^{-1}. Therefore these variations should not have any effect on the BWO generation, and the main role is played by an average amplitude determining the mean parameters of the «step». In our discrete model the C_0 in Eq.2 corresponds to this.

In [Trakhtengerts et al., 1996] it was shown, that the gain at the interaction of whistler waves with step-like distribution function of energetic electrons has a narrow peak near a certain frequency. At the formation of the «step» by ELF hiss, this frequency corresponds to the high boundary of the hiss. The width of the peak in frequency relation of the gain is 3-10 Hz, and the change of the gain value ~100 times. Thus, it is possible to expect, that the fluctuations in the band of ~ 10 Hz near the hiss high boundary can result in significant fluctuations of the gain. These fluctuations can be connected with fluctuations of «step» slope [Hobara etc., 1998]. In discrete model these random fluctuations are described by second term in Eq.2. Therefore, the large fluctuation of C_1 discussed here is possible

Dependencies of the power index α

As it was noted in section 5, identical quantity of peaks can be obtained at different values of parameters C_0 and C_1. However for these cases, the power index in distribution of intervals between peaks does not remain constant. For pairs of values of C_0 and C_1, which were considered in section 5, this change of the power index is shown in Fig.7 depending on C_0.

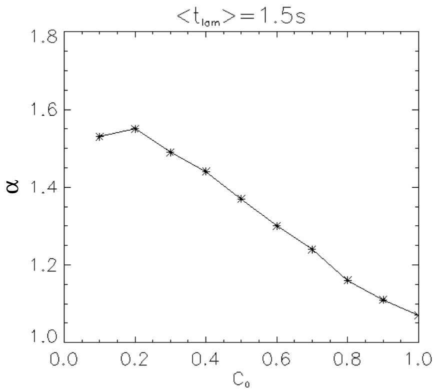

Fig.7. Dependence of power index on C_0. The appropriate pairs of C_0 and C_1 values are marked in Fig.4.

Thus, from Fig.7 one can see, that in the «peak» mode of intermittency (at small C_0) when moving from the threshold it is possible to expect the power index which is equal to, or a little larger than 1.5. At the same quantity of peaks in the region of noise generation the power index can be less than 1.5.

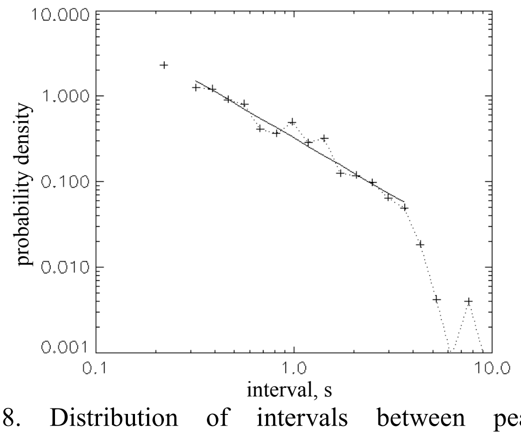

Fig.8. Distribution of intervals between peaks. Calculation at the presence of the λ value trend.

The distribution of intervals between peaks in the case, when in Eq.2 the periodical term and slow trend are present, is shown in Fig.8. The model parameters: T_0=T_1=0.1 s, T_2=4 s, C_0=0.47, C_1=2.0, C_2=0.25, C_3= -10^{-5}. We consider the interval of 50 min, i.e. 30000 periods of T_0. The peaks were selected at the excess of the amplitude level A=0.25. It is found that power index α~1.36. The obtained distribution is practically identical to distribution in Fig.3.

Conclusions

The «on-off» intermittency regime of VLF chorus generation ensure:

1) the power-law distribution of the intervals between the chorus elements, Fig.3, and see also [Kozelov et al., 2001];
2) the increase of mean quantity of chorus elements with the increase of hiss intensity at lower frequency, Fig.2;
3) the transition from generation of discrete elements to continuous generation at the greatest values of intensity of low frequency hiss, see Fig.1-a.
4) at external periodic modulation a grouping of discrete elements in groups with external period, see Fig.1-c.

The offered numerical model allows us to reproduce observable features of temporary dynamics of VLF - chorus and it supplements the modern theory of processes of discrete emission generation in the magnetosphere.

Acknowledgements. This work is supported by grants INTAS 99-0502 and RFBR 01-05-64382.

References

Bespalov, P.A., and V.Y Trakhtengerts, The cyclotron instability in the Earth radiation belts, in: Reviews of Plasma Physics, Ed. M.A. Leontovich, vol.10, 155-192, Plenum, New York, 1986.

Ginzburg, N.S., and S.P. Kuznetsov, Periodic and stochastic regimes in electron generators with distributed interaction, in Relativistic HF Electronics, Institute of Applied Physics, Gorky, USSR, 101-104, 1981, (in Russian).

Heagy J.F., Platt N., Hammel S.M. Characterization of on-off intermittency, Phys.Rev.E., 49 (2), 1140-1150, 1994.

Hobara Y., Trakhtengerts V. Y., Demekhov A. G., Hayakawa M. Cyclotron amplification of whistler waves by electron beams in an inhomogeneous magnetic field, *J. Geophys. Res.,* 103 (A9), 20449-20458, 1998.

Kozelov B. V., Titova E. E., Trakhtengerts V. Yu., Jiricek F., Triska P., Collective dynamics of chorus emissions inferred from MAGION 5 satellite data, Geomagnetizm and Aeronom., 41 (4), 477-481, 2001.

Landa P.S., Nonlinear oscillations and waves, Moscow, «Nauka», 1997.

Trakhtengerts, V.Y., Magnetosphere cyclotron maser: Backward wave oscillator generation regime, J.Geophys. Res., 100 (A9), 17205-17210, 1995.

Trakhtengerts, V.Y., A generation mechanism for chorus emission, Annales Geophysicae, 17 (1), 95–100, 1999.

Trakhtengerts V.Y., Rycroft M.J., Demekhov A.G. Interrelation of noise-like and discrete ELF-VLF emissions generated by cyclotron interactions. *J. Geophys. Res.*, 101 (A6), 13293-13303, 1996.

Wheatland M.S., Sturrock P.A., McTiernan J.N. The waiting-time distribution of solar flare hard x-ray bursts, Astrophysical J., 509, 448-455, 1998.