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# SELF-CONSISTENT MODELING OF PC 1 EMISSIONS IN THE EARTH'S MAGNETOSPHERE

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## Introduction

Despite many years of studies of magnetospheric Pc 1 pearl emissions, there is still no any detailed theory describing formation of their spectral forms. Even the basic question concerning the influence of the ionospheric reflection on Pc 1 generation remains open and causes controversial opinions. At present, there is an increased interest in this problem [e.g., *Trakhtengerts et al.*, 2000; *Prikner et al.*, 2000; *Demekhov et al.*, 2000; *Mursula et al.*, 2001], related to accumulation of much more precise and detailed ground-based and satellite data of Pc 1 waves as well as related precipitation of energetic particles.

The conventional scheme of Pc 1 formation is based on the concept of bouncing Alfven-wave packets which are amplified near the equatorial plane due to the cyclotron resonant interaction with energetic protons and lose a part of their energy due to refraction and nonideal reflection from the ionosphere. A detailed study of this concept and relevant references can be found in *Gendrin et al.* [1971]. An important element introduced into this scheme by *Polyakov et al.* [1983] is the magnetosphere-ionosphere feedback related to the influence of precipitated energetic protons on the ionospheric reflection. This influence can be significant due to the resonant frequency dependence of the ionospheric reflection, related to the ionospheric Alfven resonator. *Belyaev et al.* [1984; 1987] developed an analytic theory of this feedback and showed that a single pearl-type wave packet should be formed in the system if the feedback is negative i.e., if the precipitation of energetic protons leads to a decrease in the ionospheric reflection coefficient. This model, called an Alfven sweep maser (ASM), is schematically shown in Fig. 1. Only one wave packet survives in this scheme. The proton precipitation it causes disturbs the opposite ionosphere and suppresses reflection of the symmetric wave packet. Since the symmetric wave packet is absent, the existing wave packet arrives at a reflection point when the corresponding ionosphere has relaxed from the disturbance caused by the previous precipitation burst. *Trakhtengerts et al.* [2000] obtained an analytic nonlinear soliton-like solution of the ASM equations in the form of a single wave packet oscillating between the reflection points, and *Demekhov et al.* [2001] demonstrated using numerical simulations that such a pearl-like solution is formed naturally in the system.

The ASM scheme is not commonly accepted, and one argument against it is that the ionospheres have to have very similar properties for this scheme to operate. Some new satellite and ground-based observations are interpreted in terms of a scheme in which the ionospheric reflection does not play a significant role in wave generation. This scheme attributes formation of Pc 1 pearl elements only to the external modulation of the growth rate by large-period (Pc 3–4) hydromagnetic waves.

In this paper, we do not provide the final solution to the problem., We rather present some new results of self-consistent modeling of the generation of Pc 1 emissions in the Earth's magnetosphere. We address two questions pointed in the previous paragraph, namely, how the Alfven maser operates if the conjugate ionosphere regions have strongly different properties, and how does it operate if the magnetosphere–ionosphere feedback is absent but the growth rate is modulated by an external factor.

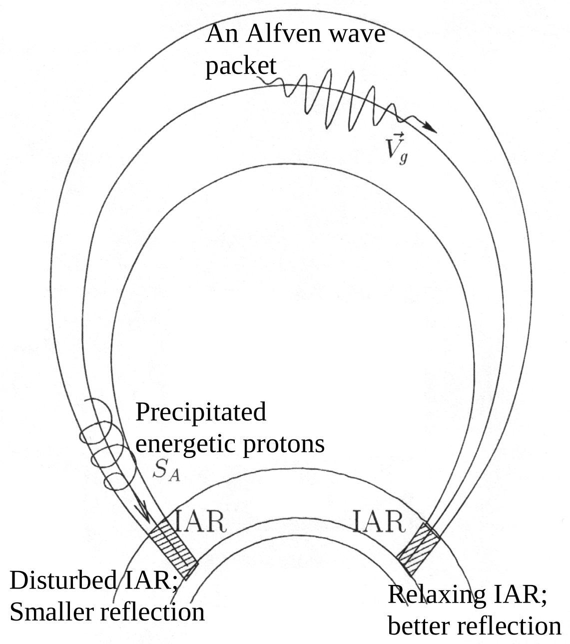


Fig.1. Qualitative model of ASM.

## Basic equations

Our basic model is the same as developed by *Belyaev et al.* [1984]. It is based on the quasi-linear plasma theory.

The wave energy transfer is described by the equation

$$\frac{\partial \varepsilon_\omega^\pm}{\partial t} \pm v_{g\omega} \frac{\partial \varepsilon_\omega^\pm}{\partial z} = N\gamma(\omega, z)\varepsilon_\omega^\pm + a_\omega \, . \tag{1}$$

Here $\varepsilon_\omega^\pm$ is the spectral energy density of ion-cyclotron waves propagating in $\pm z$ directions, $z\in[-l, l]$ is the coordinate along the magnetic field, $v_{g\omega}$ is the Alfven wave group velocity, $a_\omega$ is the background source of Alfven waves, the function $\gamma(\omega,z)$ describes the frequency and space dependence of the growth rate, and $N$ is the number of energetic protons in a magnetic flux tube with the unit cross-section at the ionospheric level.

The evolution of energetic protons is described using the balance approximation valid for the weak pitch-angle diffusion [*Bespalov and Trakhtengerts*, 1986]:

$$\frac{dN}{dt} = -(S^+ + S^-) + J(t) - \frac{N}{T_N} \qquad (2)$$

Here, $S^\pm$ are the fluxes of energetic protons precipitated into the conjugate ionospheres, $J(t)$ is the source of energetic-protons, and $T_N$ is their loss time due to the pair processes (mostly charge exchange). The precipitated fluxes are determined by the relation

$$S^\pm = DN\int \gamma(\omega,z)\varepsilon_\omega^\pm dz\, d\omega \qquad (3)$$

where $D$ is the known coefficient providing the energy conservation for wave-particle interactions.

Reflection of waves from the ionosphere is described by the boundary conditions

$$\varepsilon_\omega^\pm(\mp l) = R^\pm \varepsilon_\omega^\mp(\mp l)\,, \qquad (4)$$

where $R^\pm = R^\pm(n^\pm,\omega,t)$ is the reflection coefficient from the ionosphere. We consider $R^\pm$ as known functions of $\omega$, time and $n^\pm$. The dependence on $n^\pm$ is the most significant part of the ASM scheme. Since protons with energies of 30–100 keV modify mainly the $E$–layer, $n^\pm$ represents here the $E$–layer electron density. Its evolution is described by the ionization balance equation

$$\frac{dn^\pm}{dt} = \eta(S_0 + S^\mp) - \alpha^\pm (n^\pm)^2\,, \qquad (5)$$

where $n^\pm$ is the electron plasma densities in the conjugate ionospheres, $S_0$ is the effective proton flux providing the background ionization, $\alpha^\pm$ are the recombination coefficients, and $\eta$ is the ionization efficiency.

## Strongly different properties of conjugate ionospheres and their diurnal variation

In this section, we take into account the diurnal variations of the magnetically conjugate parts of the ionosphere. The frequency dependences of reflection coefficients were calculated using the IRI model for the ionosphere height profile and the full-wave code for Alfven wave propagation kindly provided by A.A.Ostapenko (PGI). The nonlinearity of the ionospheric reflection was chosen in a simplest parametric way consistently with calculations by *Ostapenko and Polyakov* [1990]. Specifically, we use the shift of the $R(f)$ profile along the frequency axis with the rate $\partial f_m/\partial n^+ = -10^{-6}$ and $\partial f_m/\partial n^- = -9\cdot 10^{-7}$.

Here we simulate the morning-time pearl formation. At this time the properties of conjugate ionospheres can be strongly different, especially at solstice (fig. 2). As a result we can see the following.

1. The pearl-type pulsation can appear even if the reflection properties of conjugate parts of ionosphere are strongly different (Figs. 2 and 3).
2. The time scales of pearl formation and diurnal variation are close to each other.
3. The formation of pearl-type pulsation takes place if the condition $R^+ R^- e^{2\Gamma} > 1$ is realized and doesn't occur in the opposite case, where $R^+$ and $R^-$ are the reflection coefficients and $\Gamma$ is the Alfven-wave one- hop amplification.
4. The pearl-type pulsation can be generated even if the reflection from the ionosphere is very low.

## The influence of modulation

It is easy to include in the model additional factors such as external modulation of the Alfven-wave growth rate.

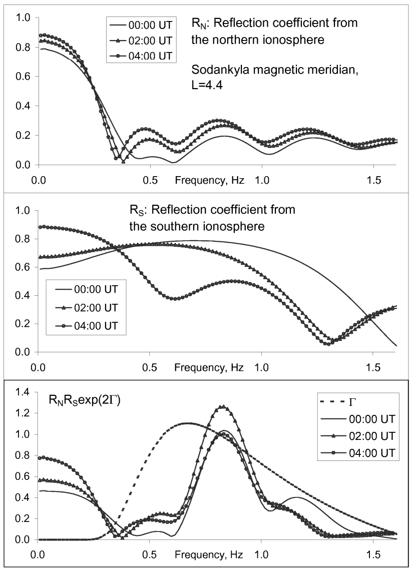


Fig. 2. Diurnal variation of the reflection coefficients from conjugate ionospheres and the one-hop amplification $\Gamma$. $R_N$ and $R_S$ stand here for $R^+$ and $R^-$, respectively.

The cause of such a modulation can be a hydromagnetic wave with the period close to that of Alfven wave bouncing between the reflection points.

Here we consider the fundamental mode of field-line oscillations. To model the finite spectral width of the oscillation, we assume that several harmonic components with close frequencies are present. In this case, the modified function of amplification can be written as

$$\tilde\gamma(\omega,z,t) = \gamma(\omega,z)\cdot\left(1+\sin\left(\frac{\pi}{l}z\right)\cdot\sum_{i=1}^{m}\alpha_i \sin(\Omega_h^i t)\right) \qquad (6)$$

where $\Omega_h^i$ are the frequencies of the spectral components and $\alpha_i$ are weight coefficients satisfying the normalization

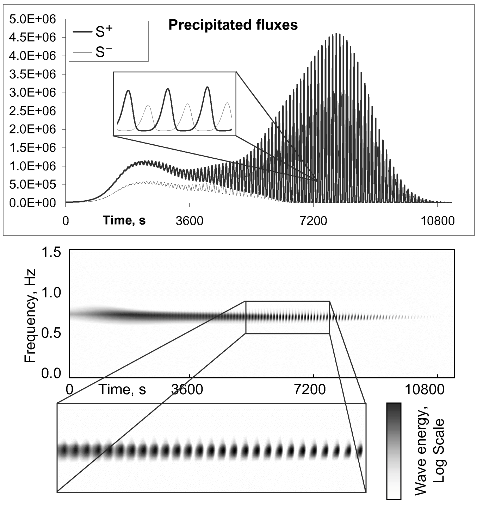


Fig. 3. The morning-time pearl formation. Properties of the conjugate parts of the ionosphere are calculated for morning time on June 22, 1999 (Fig. 2). *Upper panel:* precipitated energetic-proton fluxes to conjugate parts of the ionosphere (in $cm^{-2}\ s^{-1}$). The fluxes being in antiphase means that only one wave packet is generated. *Lower panel:* the spectrogram for wave energy of Alfven waves in the northern ionosphere. The development of Pc 1 pearl is determined here by diurnal variation of the ionospheric reflection.

condition $\sum_{i=1}^{m}\alpha_i = a$; $a$ has the meaning of the effective amplitude of oscillation. All frequencies $\Omega_h^i$ are close to the frequency $\Omega_h \approx 2\pi/T_g$ where $T_g$ is the Alfven-wave bounce period for a reference frequency. Since the group velocity of Alfven waves depends on the frequency, each modulation period corresponds to a certain Alfven-wave frequency $f$ for which

$$\Omega_h^i = 2\pi/T_g(f) \qquad (7)$$

and at which the influence of this modulation component is the strongest.

In this section, we neglect the presence of the ionospheric Alfven resonator and consider the model where reflection coefficients are independent of frequency, daytime and electron density of ionospheres, $R^{\pm}(\omega)\equiv 0.3$. It allows us to study the pure influence of the modulation to the processes in the Alfven maser and, therefore, to consider this influence as an alternative mechanism of pearl formation. The active proton concentration in the magnetic flux tube is chosen according to the condition

$$R_+(\omega_m)\,R_-(\omega_m)\exp[2\Gamma(\omega_m)] = 1, \qquad (8)$$

where $\omega_m$ the frequency of the maximum amplification.

We use the value of 20% for the modulation depth; this corresponds to the amplitude ~70 nT of the magnetic-field oscillations at $L = 4.4$. Other magnetospheric parameters are the same as above.

First, we add a purely monochromatic modulation. We study the possibility of wave packet formation and its characteristics time, influence of the reflection value and the period mismatch between the modulation and wave bounce oscillations. Note that for smaller reflection, we need to use larger active proton concentration to satisfy the condition (8).

Results of numerical simulations are as follows.

1. Single wave packet is generated even if the ionospheric reflection is independent of frequency and electron density (Fig. 4).
2. However, the spectral width of Pc 1 is extremely narrow (Fig. 4).
3. Rather large compressional component is required for the effect to be significant (Fig. 5).
4. If the period of the hydromagnetic oscillation differs from one-hop wave propagation time by more than 1.5%, the wave packet generation is not observed (Fig. 6).

Now we consider a finite spectrum of modulation, assuming the presence of several equidistant harmonics with frequencies $\Omega_h^i = 2\pi\left(1/T_g + \Delta F_m\cdot(i-0.5\cdot m)/m\right)$, where $\Delta F_m$ is the modulation spectrum width, and the Gaussian distribution $\alpha_i = \exp\left(-\left(2\cdot(i-0.5\cdot m)/m\right)^2\right)$ of amplitudes. The results shown in Fig. 7 correspond to $\Delta F_m = 0.01$ Hz. Note that this value corresponds to the range [0.705; 0.794] Hz of «synchronous» Pc 1 waves (see Eq.(7)). The modulation influence decreases here by about 60%, and the wave packet has only a slightly wider spectrum (Fig. 7).

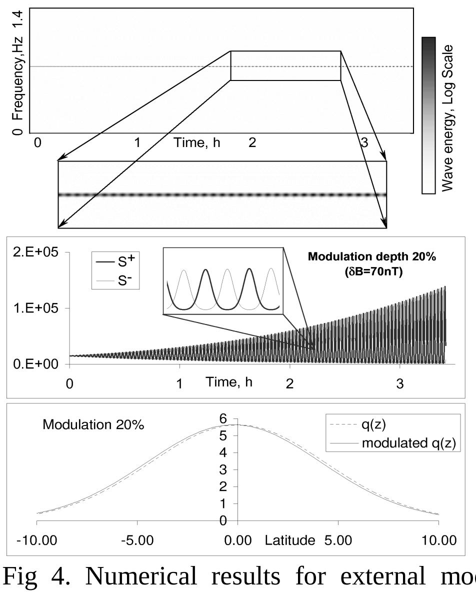


Fig 4. Numerical results for external modulation of amplification. Reflection coefficients are independent of frequency ($R$=0.3). Note that the wave spectrum is extremely narrow. $q(z)$ is the wave-particle interaction efficiency function. The wave-particle interaction takes place in the equatorial zone where the node of the field-line oscillation is located and the modulation disturbance is small.

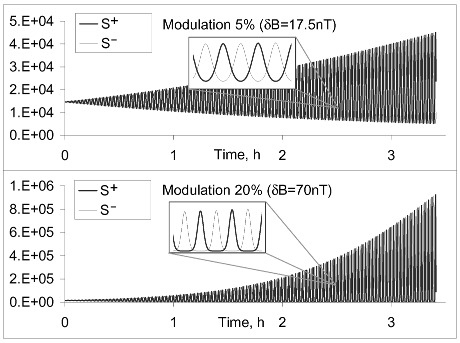


Fig 5. Sensitivity of the pulsating regime to the modulation depth. Note different vertical scales of upper and lower panels.

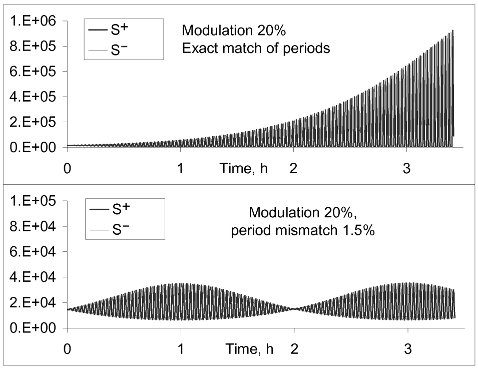


Fig. 6. Sensitivity of pulsating regime to the period mismatch between the modulation and wave-packet oscillations. Very high sensitivity to the period mismatch results from the long formation time of the pulsating regime.

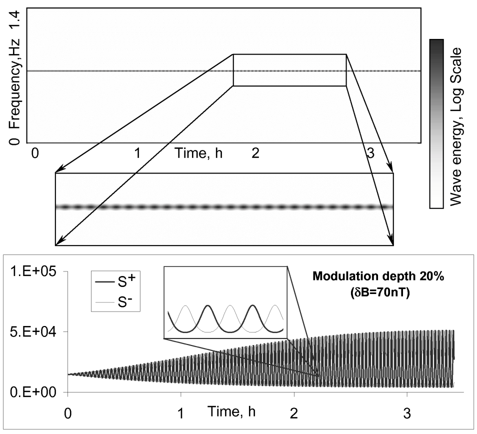


Fig. 7. Modulation with finite width of spectrum.

## Conclusions

The main results of this study can be summarized as follows.

1.The pearl-type pulsation is formed in tne ASM model even if the reflection properties of conjugate parts of ionosphere are strongly different and vary in time.

2.The influence of external modulation of amplification can also result in formation of a single Pc 1 wave packet if the modulation is antisymmetric in space with respect to the equator (see Eq. (6)). However, within the present model, the antisymmetric modulation must be of unrealistically high value for its effect to be significant. Moreover, it is very sensitive to the mismatch of the modulation period and the bounce period of Alfven-wave packet. Taking into account finite spectral width of the modulation signal also leads to a significant decrease of the modulation effect.

Further studies should clarify the effect of symmetric-type external modulation on the Pc 1 pearl dynamics. For a really quantitative comparison of the model with observations, most of which are ground based, one needs also to calculate the Pc 1 amplitude transmitted to the ground.

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