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VLF CHORUS CHARACTERISTICS AND PREDICTIONS FROM BACKWARD WAVE OSCILLATOR MODEL: A COMPARISON

E. E. Titova, B. V. Kozelov *(Polar Geophysical Institute, Apatity, Russia)*
F. Jiriček, J. Smilauer *(Institute of Atmospheric Physics, Prague, Czech Republic)*
A. G. Demekhov, V. Yu. Trakhtengerts *(Institute of Applied Physics, Nizhny Novgorod, Russia)*

Abstract. We present a study of chorus emissions in the magnetosphere detected onboard Magion 5, when the satellite was not far from the magnetic equator. We determined the frequency sweep rate of more than 8500 electromagnetic VLF chorus elements. The comparison of the observed chorus characteristics with the backward wave oscillator regime (BWO) of the chorus generation mechanism shows both qualitative and quantitative agreement with BWO model.

Introduction

Generation of chorus emissions is one of the most puzzling problems of VLF waves in the Earth magnetosphere. These emissions are the most intense of all natural VLF waves in the frequency range from a few hundred Hz to several kHz. They are observed as a succession of repeating discrete elements with rising frequency. It is generally accepted that the chorus is generated in a near-equatorial region by the cyclotron instability of radiation belt electrons (Helliwell, 1965). However, mechanisms responsible for the origin of chorus succession and formation of spectrum of separate chorus elements both are still unclear.

Recently, Trakhtengerts (1999) suggested a mechanism of chorus generation based on the backward wave oscillator (BWO) regime of magnetospheric cyclotron maser (Trakhtengerts, 1995). The BWO regime of chorus generation gives us a hope to explain such features of chorus as appearance of a succession of discrete elements and their spectrum, relation of chorus to ELF hiss, large growth rates of chorus, and different generation regimes such as quasi-periodic and stochastic ones. The suggested BWO model of chorus generation allows for a number of predictions that can be checked experimentally. In this paper we compare spectral and amplitude characteristics of chorus observed onboard the MAGION 5 satellite with those predicted by the backward wave oscillator model of chorus generation.

The BWO regime of chorus generation in the magnetosphere

In this section, we briefly review the BWO regime of whistler wave generation in the magnetosphere (Trakhtengerts 1995, 1999) to select the parameters that can be compared with experimental data. This regime is similar to the backward wave oscillator in laboratory electronic devices where the wave propagates opposite to the motion of an interacting electron (Ginzburg and Kuznetsov, 1981). Similar wave–particle interactions take place in the magnetospheric cyclotron maser. The generation of chorus is based on the cyclotron resonance of radiation belt electrons with whistler waves

$$\omega - \omega_H = k_{\|} v_{\|}, \qquad (1)$$

where ω is the wave frequency, ω_H is the electron gyrofrequency, $k_{\|}$ and $v_{\|}$ are the magnetic field-aligned components of the wave vector and electron velocity.

Certain conditions have to be satisfied for a generator to operate in the BWO regime. The first condition requires that the phase velocity component along the magnetic field should be opposite to the electron motion. According to (1), this condition is satisfied if $\omega < \omega_H$. The second condition is the existence of a well-organized electron beam with small velocity dispersion in the region of chorus generation. This condition poses a significant problem, since there is no obvious reason for such a beam to be formed. The solution of this problem can be related to the fact that cyclotron interaction of band-limited natural ELF/VLF noise-like emissions with energetic electrons results in formation of a specific step-like feature of the distribution function (Bespalov, 1986, Nunn and Sazhin, 1991). This step-like deformation of energetic electron distribution function ensures large growth rate γ_{HD} of whistler waves and transition to the BWO regime. Trakhtengerts (1995) showed that the step-like deformation of the distribution function, caused by interactions of natural ELF/VLF noise-like emissions and energetic electrons, acts in the magnetosphere as a well-organized beam in laboratory devices. The magnetospheric BWO has no fixed boundaries, and its interaction length l is determined by the inhomogeneity of the geomagnetic field. According to Helliwell (1967) and Trakhtengerts (1995), the interaction length l of whistler waves and energetic electrons can be written for the dipole magnetic field as follows:

$$l = (R_0^2 L^2/k)^{1/3}, \qquad (2)$$

where R_0 is the Earth's radius, L is the geomagnetic shell, and k is the whistler wave number.

The BWO generation starts when the density of energetic electrons exceeds some threshold value. According to Trakhtengerts (1995), this threshold condition can be written as

$$p = 2\,\gamma_{HD}\, l\,/\,[\pi (V_{\|} V_g)^{1/2}] = 1, \qquad (3)$$

where l is the working length of the magnetospheric generator, V_g is the group velocity of the whistler waves, $\gamma_{HD} \sim (0.1\ \Delta n_h/n_c)^{1/2}\omega_H$ is the hydrodynamic growth rate in the case of the distribution function with a step-like deformation, Δn_h is the height of a step, and n_c is the cold plasma density. The BWO (chorus generation) growth rate can be written as function of p in the form (Trakhtengerts, 1999):

$$\gamma_{BWO} = 2p\,(p - 1)\,T^{-1}, \quad (4)$$

where

$$T = 1.6\,l\,(V_g^{-1} + V_\parallel^{-1}) \quad (5)$$

If the step height increases, the BWO generation regime changes from the stationary generation that takes place for $1 < p < p_2$ to the periodic one with the period T for $p_2 < p < p_3$. Further increase of the parameter $p > p_3$ leads to the stochastic generation regime with random variation of the wave amplitude in time. The bifurcation values $p_{2,3}$ are equal to: $p_2 = 2$, and $p_3 = 4.5$ for laboratory devices (Ginzburg and Kuznetsov, 1981). To get the bifurcation values $p_{2,3}$ for chorus, it is necessary to develop the strict nonlinear theory for the magnetospheric BWO, which is absent now. Therefore, taking into account a big similarity of laboratory and magnetospheric BWO generators, we will take for further estimations the laboratory values of $p_{2,3}$. After transition to the BWO regime, the dynamical spectrum of a separate chorus element is formed similar to discrete signals triggered by VLF transmitters (Nunn, 1974). In this case, the frequency sweep rate df/dt ($f \equiv \omega/2\pi$) at the exit from BWO generation region can be written as

$$df/dt = [\Omega_{tr}^2 + 3V_\parallel\,(d\omega_H/dz)\,]*0.15\omega/(\omega_H + 2\omega) \quad (6)$$

where the trapping frequency Ω_{tr} is determined by the expression

$$\Omega_{tr} = (ku\,\omega_H\,b)^{1/2} \quad (7)$$

Here $b = B_\sim/B_L$, $B_\sim$ is the whistler wave magnetic field amplitude, B_L is the geomagnetic field, and u is the electron velocity component across the geomagnetic field. There is the additional relation between the chorus amplitude and the BWO growth rate (Trakhtengerts, 1999):

$$\gamma_{BWO}/\Omega_{tr} \approx 3\pi/32, \quad (8)$$

where γ_{BWO} is determined by Eq. (4). Taking Eq. (8) into account, we can rewrite (6) in the form

$$df/dt = (\gamma^2_{BWO} + S_1)\,1.5\omega/(\omega_H + 2\omega), \quad (9)$$

where $S_1 = 0.3\,V_\parallel\,(d\omega_H/dz)$ characterizes the magnetic field inhomogeneity effect. One can see from Eq. (9) that, if $S_1 << \gamma^2_{BWO}$, chorus elements are formed mainly inside the BWO generator, and the corresponding frequency variation is determined by nonlinear effects. In the opposite case, the frequency shift is determined by the magnetic field inhomogeneity factor S_1.

Since the frequencies (ω/ω_H) of the observed chorus elements are spreading rather wide, it is convenient to group the experimentally known values in (9) in a new function

$$G^2 \equiv df/dt\,(\omega_H + 2\omega)/\,1.5\omega = \gamma^2_{BWO} + S_1, \quad (10)$$

which we call thereafter the "reduced" frequency sweep rate. Note that G is equal to the BWO growth rate if the frequency shift is determined by nonlinear processes ($\gamma^2_{BWO} >> S_1$). As one can see from (6) and (7), the BWO model predicts an increase in frequency sweep rate df/dt and G^2 with chorus amplitude. These relationships are analyzed experimentally below on the basis of Magion 5 data.

Results of observations and their comparison with the BWO model

To obtain the frequency sweep rate df/dt, we have analyzed the spectrum of chorus detected by the magnetic antenna along 10 orbits of Magion 5 in the morning sector (MLT < 08) during the period from October 1998 to January 1999. The analysis was carried out for more than 8500 chorus elements, which were identified on sonograms. During that period Magion 5 was crossing the region of chorus registration at latitudes of 30°–40°. The VLF chorus was detected in the plasmapause region at L = 2.5 - 6.

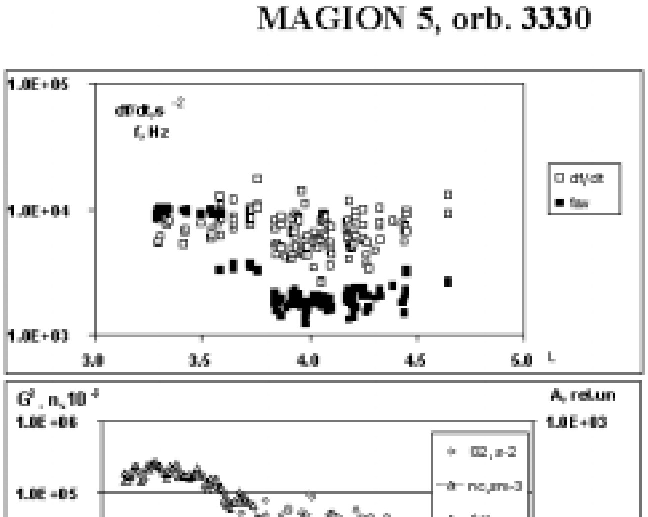

Fig. 1. Relationships of the frequency sweep rate df/dt and G^2 with chorus amplitudes on orbit 3330 of MAGION-5. Top panel. The frequency sweep rate df/dt and the center frequency of the chorus elements f_{av} depending on L-shell for orbit 3330, Magion-5. Bottom panel. The chorus amplitude, G^2 and the cold plasma density n_c depending on L shell for orbit 3330, Magion-5.

According to the BWO model, the frequency sweep rate in a chorus element is a function of the characteristic chorus amplitude. As it follows from (10), the quantity G^2, that we named the reduced df/dt is more appropriate for comparison of observations with the BWO model, since, first, it is directly related to the BWO growth rate and, second , it does not include the term depending on a highly variable ratio of f/f_H. The importance of using the reduced frequency sweep rate G^2 instead of df/dt is demonstrated by the sample data detected on orbit 3330 of Magion 5 and shown in Fig. 1. The top panel shows the frequency sweep rate df/dt and the center frequency of the chorus elements f_{av} depending on L-shell. Chorus emissions were detected at $L = 3.2$–4.5. Bottom panel of Fig. 1 shows the chorus amplitudes along this orbit. Unfortunately, we had some technical problems with measurements of absolute amplitude for the chorus detected by Magion 5, so we analyzed variations only of the chorus relative amplitude of. The specific feature of this orbit is that the mean df/dt almost did not vary along the entire satellite pass, while both the chorus frequency and amplitude varied considerably, the frequency decreasing from $f_{av} = 10$ kHz at $L = 3.3$ to $f_{av} = 2$ kHz at $L = 3.8$–4.5 and the amplitude increasing approximately 4 times in the same range of L. Note that the change in both amplitude and frequency occurs near the density drop, which is seen from the data on the cold plasma density (bottom panel). Due to the fast variations in frequency and amplitude, the direct correlation between df/dt and the amplitude is not seen in this case, but a good correlation exists between the amplitude and G^2, which is in accord with the BWO model (Fig.1). The correlation between the reduced frequency sweep rates, G, and the chorus amplitudes was also confirmed on the other orbits of the Magion 5 satellite.

Figure 2 shows the L dependence of the reduced frequency shift G^2 (10). The solid line shows the running average over 100 points of G^2. It is seen that G^2 decreases in both directions from the maximum reached about $L = 3.2$. The decrease at smaller L can be related to the fact that these points correspond to the inner plasmasphere region, where the chorus amplitude is known to decrease inwards. At $L > 3.2$, G^2 decreases as L^{-3}.

Now we compare the experimentally obtained values of G and their dependence on L with estimations based on the BWO model. According to (10), two terms give an additive contribution to the value of G, the first one is a nonlinear term, which can be expressed in terms of the BWO growth rate γ_{BWO}, and the second one is a linear term S_1 related to the magnetic field inhomogeneity.

According to (6), (7), and (9),

$$S_1 = V_\parallel/3\ (\mathrm{d}\omega_H/\mathrm{d}z) = 1.5\,(V_\parallel/l)^2\,(1 - f/f_H)^{-1} \qquad (11)$$

Taking into account (4) and (11), expression (10) can be rewritten as

$$G^2 = \gamma^2_{BWO} + S_1 = \gamma^2_{BWO}\,\{1+$$

$$+1.2\,(1 + f_H/2f)^2\,(1 - f/f_H)^{-1}/[p^2\,(p-1)^2]\} \qquad (12)$$

It is seen from Eq.12 that the relation between S_1 and γ^2_{BWO} depends on f/f_H. and p. For Magion 5 chorus, $f/f_H \approx 0.25$ is a typical value. Furthermore, we suggest that most chorus emissions in Magion 5 data are related to the periodic and stochastic regimes of BWO generation. This suggestion is supported by the types of chorus spectra. In this case, γ_{BWO} is determined by relation (4), where $p > 2$. Taking into account the relation $V_g/V_\parallel = 2 f/f_H$, we obtain:

$$\gamma_{BWO} = 2p(p-1)\,V_g/l\,(1 + 2f/f_H) \qquad (13)$$

Using the typical group velocity of chorus $V_g = 2.5\cdot 10^4$ km s^{-1}, $l = 10^3$ km, and $2f/f_H = 0.5$, we obtain $\gamma_{BWO} = 70$–200 s^{-1} for $p = 2$–3. Note that $S_1 \le \gamma^2_{BWO}$ if $p \ge 2$, i.e., near the threshold of the periodic BWO regime. Therefore, $G \approx \gamma_{BWO}$, and we can compare the above estimate for γ_{BWO} with the experimentally obtained values of G. According to Fig. 2, G lies between 100 and 300 s^{-1}, which yields a good agreement between two independent estimates of γ_{BWO}.

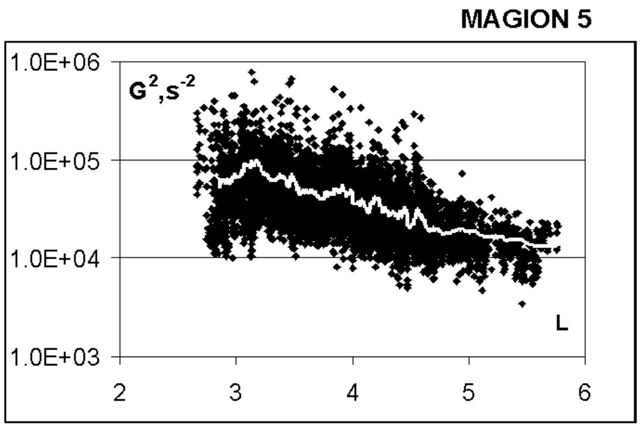

Fig. 2. The reduced sweep rate G^2 as a function of L shell. The solid line shows the running average over 100 points of G^2. The mean $G(L)$ varies within a small interval 100–300 s^{-1}.

Taking into account these estimates and relations (7) and (8) and assuming that $ku \sim \omega_H$, the dependence of G^2 on L can be obtained in the form

$$G^2 \propto \Omega^2_{tr} \propto \omega_H B_\sim \qquad (14)$$

For the 7 orbits where the relative wave amplitude was measured, we can plot the ratio of $G^2/B_\sim$ as a function of L. According to (14), this dependence should be the same as $\omega_H(L)$. As seen from Fig. 3, $G^2/B_\sim \propto L^{-3}$. Therefore, the experiment again displays an agreement with the BWO model. Note that taking different orbits separately, we obtained slightly different indices ranging from 2.2 to 3.6 for the power-law fit of $G^2/B_\sim$.

Another possible test of the BWO model validity is based on the dependence of γ_{BWO} and G on the cold plasma density n_c. Indeed, it follows from (12) and (13) that $G^2 \propto \gamma^2_{BWO} \propto (V_g/l)^2 \propto n_c^{-2/3}$; therefore, strong variations in n_c should correlate with variations in G^2. Such correlation is clearly seen in the Magion 5 data, e.g., i n the lower panel in Fig. 1 showing the satellite motion towards higher latitudes. When the drop of density was crossed near $L = 3.5$, the cold plasma density n_c decreased more than an order of magnitude. At the same time, G^2 increased about 3–4 times, which corresponds to the BWO model both qualitatively and quantitatively.

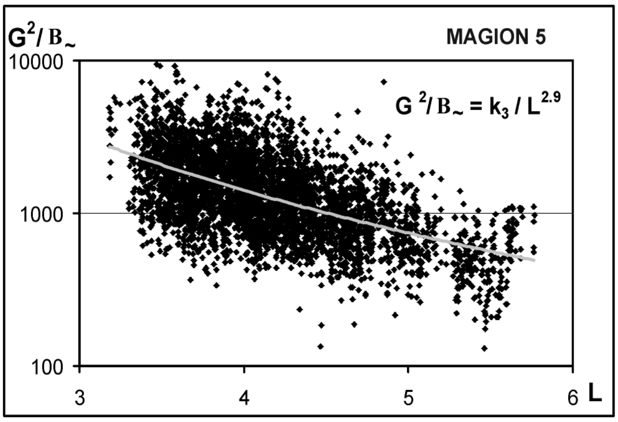

Fig. 3. The value of $G^2/B_{\sim}$ depending on L shell for 7 orbits of MAGION 5. $B_{\sim}$ is the whistler-wave magnetic field amplitude. Note that $G^2/B_{\sim}(L)$ decreases indeed as $L^{-2.9}$ (grey line) in a good agreement with BWO model.

Conclusions

We have analyzed the frequency sweep rates df/dt for more than 8500 chorus elements recorded by MAGION 5 satellite at latitudes of 30–40 degrees from the geomagnetic equator at $L = 2.5$–6 in the morning sector near the plasmapause. We have performed the first direct comparison of the scalings obtained experimentally with those following from the backward wave oscillator (BWO) theory for chorus generation. Observations showed both qualitative and quantitative agreement with the theory in the following tests:

(i) There is a correlation between the reduced frequency sweep rates and the chorus amplitudes. The reduced frequency sweep rate increases with chorus amplitude, in accordance with expectations from the BWO model.

(ii) The chorus growth rate, estimated from the measured frequency sweep rate, is close to that calculated from the BWO theory and to that obtained in other studies.

(iii) The BWO regime of chorus generation ensures the observed dependence of the reduced frequency sweep rate on L shell.

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