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A NUMERICAL GLOBAL MODEL OF THE HORIZONTAL AND VERTICAL WIND IN THE LOWER AND MIDDLE ATMOSPHERE

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Abstract. The new mathematical model is described. The model enables one to calculate the three-dimensional global distributions of the zonal, meridional , and vertical components of the neutral wind and neutral gas density at levels of the troposphere, stratosphere, mesosphere, and lower thermosphere. The principal distinction of the described model from existing global circulation models of the atmosphere consists in that the former model does not include the pressure coordinate equations of atmospheric dynamic meteorology, in particular, the hydrostatic equation. Instead, the vertical component of the neutral wind velocity is obtained by means of a numerical solution of the appropriate momentum equation, with whatever simplifications of this equation being absent. Thus, three components of the neutral wind velocity are obtained by means of a numerical solution of the Navier- Stokes equation. For obtaining the neutral gas density, the continuity equation is utilized. The global temperature distribution is assumed to be the input parameter of the model and obtained from the MSISE-90 empirical model.

1. Introduction

It is generally understood now that the global distribution of the horizontal atmospheric wind depends on the spatial structure of the pressure field. The existing global circulation models (for references, see *Cess et al.,* 1996) produce the horizontal atmospheric wind with a satisfactory accuracy. Unfortunately, these models can not produce the vertical atmospheric wind with an acceptable accuracy. Usually, a global circulation model is based on the pressure coordinate equations of atmospheric dynamic meteorology including the hydrostatic equation. As a consequence, the vertical component of the wind velocity produced by the existing global circulation models has the values of several centimeters per second. However, the observed vertical component of the wind velocity can achieve some tens m/s at levels of the lower thermosphere [*Wardill and Jacka,* 1986; *Crickmore et al.,* 1991; *Leontyev and Bogdanov,* 2001]. It is easy to see that the existing global circulation models can not describe the behaviour of the vertical atmospheric winds in all regimes, in particular, under disturbed conditions.

This paper is intended to present a new mathematical model which enables to calculate the three-dimensional global distributions of the zonal, meridional, and vertical components of the neutral wind at levels of the troposphere, stratosphere, mesosphere, and lower thermosphere, with whatever restrictions on the vertical transport of the neutral gas being absent.

2. The mathematical model

2.1. Basis principles

It has been well established that the temperature regime influences significantly the global distributions of the horizontal and vertical atmospheric winds. It is known that the atmospheric temperature distributions, calculated by using the global circulation models, as a rule, differ from the observed distributions of the atmospheric temperature. These differences are conditioned by an unsatisfactory accuracy of the global circulation models as for the solution of the internal energy equation for the neutral gas. Due to complexities and uncertainties in various chemical-radiational heating and cooling rates, there is no reason to expect an exact correspondence between the calculated and measured atmospheric temperatures. Despite considerable research, the problem of the atmospheric temperature simulation is not completely solved. On the other hand, over the last years empirical models of the global atmospheric temperature field have been developed, for example, the MSISE-90 model [*Hedin,* 1991]. In the presented mathematical model, the internal energy equation for the neutral gas is not solved. Instead, the global temperature field is assumed to be a given distribution, i.e. the input parameter of the model. The global atmospheric temperature distribution is obtained from the MSISE-90 empirical model.

To avoid any restrictions on the vertical transport of the neutral gas we do not utilize the pressure coordinate equations of atmospheric dynamic meteorology, in particular, the hydrostatic equation. Instead, the vertical component of the neutral wind velocity is obtained by means of a numerical solution of the appropriate momentum equation, with whatever simplifications of this equation being absent. The horizontal components of the neutral wind velocity are obtained by means of a numerical solution of the appropriate momentum equations, too. Thus, the Navier-Stokes equation is utilized for obtaining all components of the neutral wind velocity which is the fundamental equation of the dynamics of the compressible and Newtonian (deformation proportional to the velocity gradients) fluid. For obtaining the neutral gas density, the continuity equation is used.

2.2. Governing equations

The global circulation of the atmosphere is described in a spherical coordinate system, (r, β, φ), with r, β, and φ being the radius, latitude, and longitude, respectively. The vertical , meridional, and zonal components of the neutral wind velocity (v_r, v_β, and v_φ , respectively) obey the Navier-Stokes equations,

$$\frac{\partial(\rho v_r)}{\partial t}+\frac{1}{r^2}\frac{\partial(\rho v_r^2 r^2)}{\partial r}+\frac{1}{r\cos\beta}\left(\frac{\partial(\cos\beta\cdot\rho v_r v_\beta)}{\partial\beta}+\frac{\partial(\rho v_r v_\varphi)}{\partial\varphi}\right)-$$
$$-\rho\frac{(v_\beta^2+v_\varphi^2)}{r}=-\frac{\partial p}{\partial r}-\rho g_0\left(\frac{R_E}{r}\right)^2+\rho\cdot r\cdot\Omega^2\cos^2\beta+2\rho v_\varphi\Omega\cos\beta+ \tag{1}$$
$$+\frac{1}{r^2}\frac{\partial(2\cdot\mu\varepsilon_{rr}\cdot r^2)}{\partial r}-\frac{2\mu(\varepsilon_{\beta\beta}+\varepsilon_{\varphi\varphi})}{r}+\frac{1}{r\cos\beta}\left(\frac{\partial(\cos\beta\cdot 2\mu\varepsilon_{r\beta})}{\partial\beta}+\frac{\partial(2\mu\varepsilon_{r\varphi})}{\partial\varphi}\right),$$

$$\frac{\partial(\rho v_\beta)}{\partial t}+\frac{1}{r^3}\frac{\partial(r^3\rho v_r v_\beta)}{\partial r}+\frac{1}{r\cos\beta}\left(\frac{\partial(\rho v_\beta^2\cos\beta)}{\partial\beta}+\frac{\partial(\rho v_\beta v_\varphi)}{\partial\varphi}\right)+$$
$$+\frac{tg\beta}{r}\rho v_\varphi^2=-\frac{1}{r}\frac{\partial p}{\partial\beta}-\rho r\cdot\Omega^2\cos\beta\sin\beta-2\rho v_\varphi\Omega\sin\beta+ \tag{2}$$
$$+\frac{1}{r^3}\frac{\partial(2\mu\varepsilon_{r\beta}\cdot r^3)}{\partial r}+\frac{2\mu\varepsilon_{\varphi\varphi}}{r}tg\beta+\frac{1}{r\cos\beta}\left(\frac{\partial(2\cdot\mu\varepsilon_{\beta\beta}\cos\beta}{\partial\beta}+\frac{\partial(2\mu\varepsilon_{\beta\varphi})}{\partial\varphi}\right),$$

$$\frac{\partial(\rho v_\varphi)}{\partial t}+\frac{1}{r^3}\frac{\partial(r^3\rho v_r v_\varphi)}{\partial r}+\frac{1}{r\cos^2\beta}\frac{\partial(\rho v_\varphi v_\beta\cos^2\beta)}{\partial\beta}+\frac{1}{r\cos\beta}\frac{\partial(\rho v_\varphi^2)}{\partial\varphi}$$
$$=-\frac{1}{r\cos\beta}\frac{\partial p}{\partial\varphi}-2\rho\Omega\left(v_r\cos\beta-v_\beta\sin\beta\right)+\frac{1}{r^3}\frac{\partial(2\mu\varepsilon_{r\varphi}\cdot r^3)}{\partial r}+ \tag{3}$$
$$+\frac{1}{r\cos^2\beta}\frac{\partial(2\mu\varepsilon_{\varphi\beta}\cos^2\beta)}{\partial\beta}+\frac{1}{r\cos\beta}\frac{\partial(2\mu\varepsilon_{\varphi\varphi})}{\partial\varphi},$$

where ρ is the neutral gas density, g_0 is the gravity acceleration on the ground, R_E is the Earth's radius, μ is the coefficient of viscosity, Ω is the Earth's angular velocity, p is the pressure, and ε_{rr}, $\varepsilon_{\varphi\varphi}$, $\varepsilon_{\beta\beta}$, $\varepsilon_{r\varphi}$, $\varepsilon_{r\beta}$, and $\varepsilon_{\varphi\beta}$ are the components of the tensor $\hat{\varepsilon}$, defined as

$$\hat{\varepsilon}=\hat{D}-\tfrac{1}{3}\hat{I}\,div\vec{v}$$

where $\hat{D}$ is the strain rate tensor, and $\hat{I}$ is the unit tensor. The coefficient of viscosity, μ, is assumed to vary with temperature according to Sutherland's law. The continuity equation can be written as

$$\frac{\partial\rho}{\partial t}+\frac{1}{r^2}\frac{\partial(r^2\rho v_r)}{\partial r}+\frac{1}{r\cos\beta}\left(\frac{\partial(\cos\beta\cdot\rho v_\beta}{\partial\beta}+\frac{\partial(\rho v_\varphi)}{\partial\varphi}\right)=0. \tag{4}$$

In addition, the state equation for an ideal gas is utilized in the presented model.

2.3. Solution of equations

We find the solution of the equations in the spherical layer surrounding the Earth globally. This layer stretches from the ground up to the altitude of 120 km. The Navier-Stokes equations and the continuity equation presented in Sect. 2.2 form the system of four coupled differential equations which are partial and non- linear. When solving this system, the main difficulties arise from coupling between the equations and from the non-linearity of each equation. We cope with both complexities at the same time by solving the differential equations not simultaneously but one after other and using the principle of frozen coefficients. In such a way we reduce our problem to solving of a system of linear partial differential equations. The latter equations are solved using the combination of finite-difference and splitting-up methods.

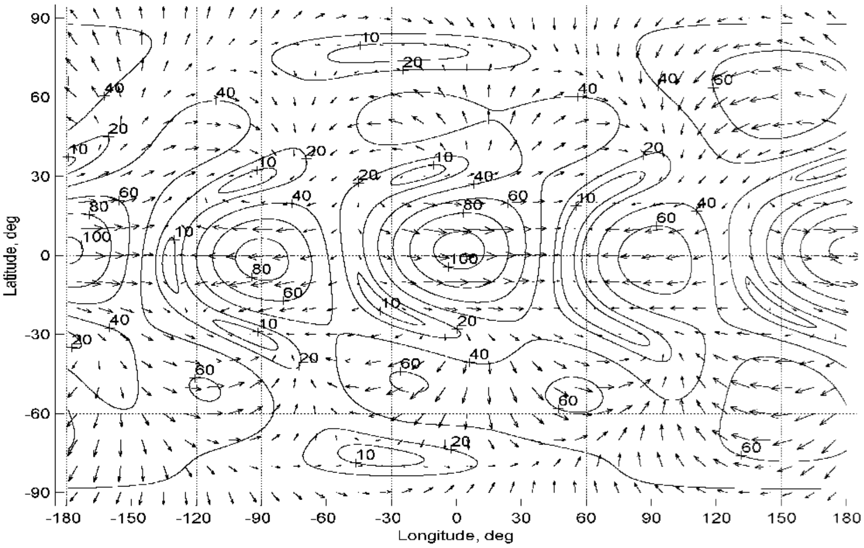

Fig.1. The distribution of the vector of the horizontal component of the neutral wind velocity and isolines of its absolute value (m/s) as a function of longitude and latitude at the altitude of 110 km for the initial moment t=0.

At the lower boundary the velocity vector is determined from the no-slip conditions on the ground. At the upper boundary the components of the neutral wind velocity are taken from the HWM-90 empirical model [*Hedin et.al.,* 1991], and the neutral gas density is determined from the MSISE-90 empirical model [*Hedin,* 1991].

The presented mathematical model allows us to calculate the three-dimensional global distributions of the zonal, meridional, and vertical components of the neutral wind and neutral gas density. The calculated parameters are determined on a 1° grid in both latitude and longitude. The discretization in altitude is non-uniform. The height step is constant and equal to 2 km above 12 km. Below this altitude a condensing grid is used, and the height step decreases in accordance with the geometric progression so that it has a value of about 200 m near the ground.

3. Probation of the model

The mathematical model described above has the potential of taking into account different combinations of solar cycle, geomagnetic activity level and seasons. For examination of the model, the calculations were performed for the late autumn (5 November) and not high solar activity ($F_{10.7}$=101) conditions under low geomagnetic activity (K_p=1).Initially, we make an attempt to obtain steady-state distributions of the atmospheric parameters at 10.30 UT. For this purpose, we solve the system (1)-(4) on condition that the input parameters of the model and the boundary conditions are time-independent.

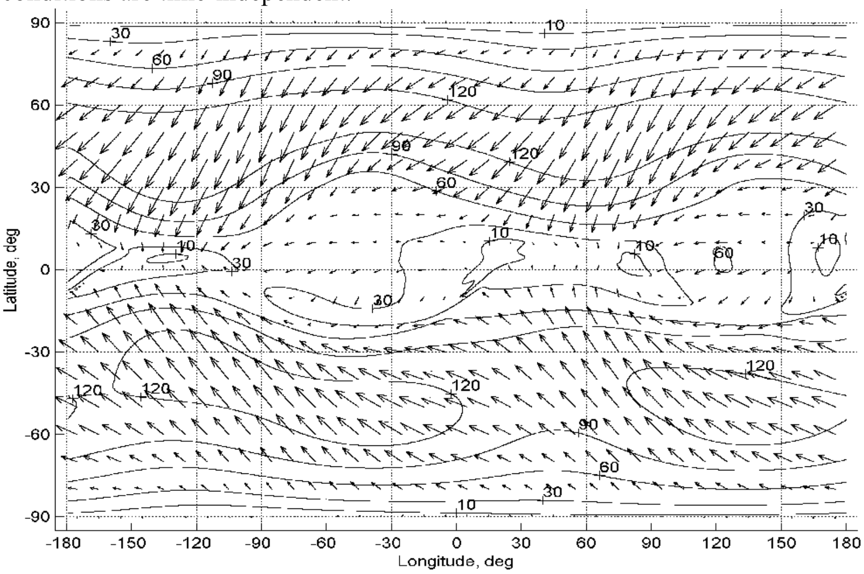

Fig.2. The same as in Fig.1 but for the moment t=17 hours.

The initial conditions are defined as following. The neutral gas density is taken from the MSISE-90 empirical model. Spatial distributions of the zonal and meridional components of the neutral wind are assumed to depend on the altitude, latitude, and longitude. On the ground all components of the neutral wind velocity are equal to zero. For any point located on the ground, the zonal and meridional components of the neutral wind increase with the altitude according to a parabolic law and achieve the maximal values at the upper boundary. These maximal values as functions of latitude and longitude are taken from the HWM-90 empirical model. The vertical component of the neutral wind is assumed to be zero at all points of the spherical layer under consideration at the initial moment.

We calculate variations of the atmospheric parameters with time until they become stationary. The calculated distributions of the atmospheric parameters at various moments are presented in Figs.1-4. It can be seen that the new mathematical model allows us to calculate the global circulation of the lower and middle atmosphere.

4. Conclusions

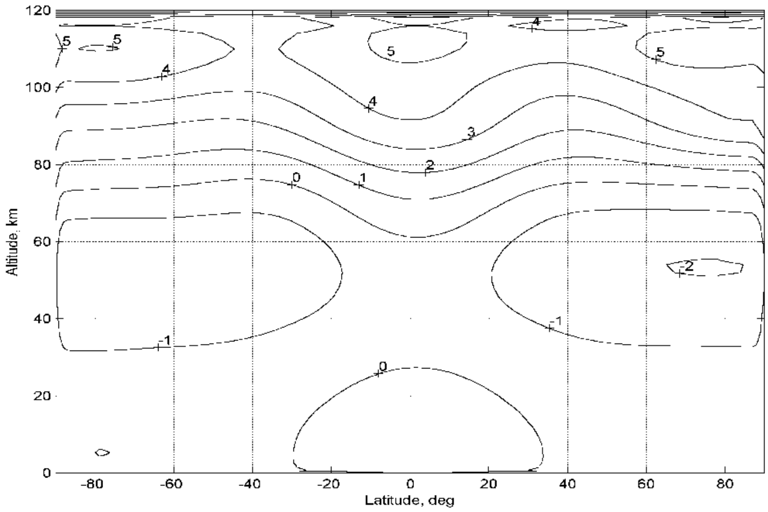

Fig.3. The calculated isolines of the vertical component of the neutral wind velocity (m/s) as a function of latitude and altitude at the longitude of 0° for the moment t=4 min.

The new mathematical model has been briefly described which enables us to calculate three-dimensional global distributions of the zonal, meridional, and vertical components of the neutral wind and neutral gas density at levels of the troposphere, stratosphere, mesosphere, and lower thermosphere. The characteristic feature of the developed model is that the vertical component of the neutral wind velocity is obtained from the Navier-Stokes equation rather then from the hydrostatic equation which is usually utilized by the existing global circulation models of the atmosphere.

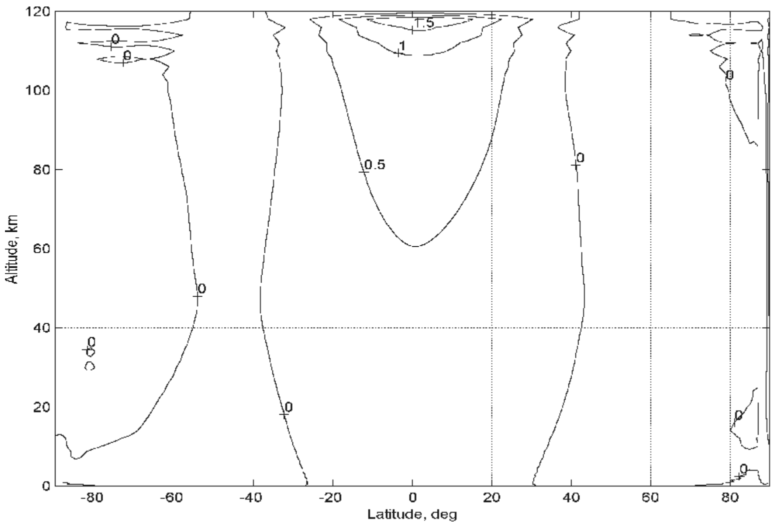

Fig.4. The same as in Fig.3 but for the moment t= 30 min.

Therefore, the developed model has the potential to describe the behaviour of the lower and middle atmosphere under disturbed conditions. Another feature of the developed model is that the internal energy equation for the neutral gas is not solved. Instead, the global temperature field is assumed to be the input parameter of the model obtained from the MSISE-90 empirical model. The probation of the model has been performed which has demonstrated the possibility of the model to calculate the global circulation of the lower and middle atmosphere.

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