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NUMERICAL ANALYSIS OF THE ANOMALOUS IONIZED ATMOSPHERE

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Abstract. The subject of the paper is the quantitative analysis of VLF powerful disturbances (PwDs) caused by ultrarelativistic electron precipitations (UrREPs) into the atmosphere [1]. The main purpose of the paper is to get of the atmosphere effective electron concentration profile $N_e(z)$ corresponding to an experimental fact of the first ionosphere radio ray full compensation by the ground ray in the cases of PwDs. [2]

An anomalous decrease of VLF signals for the high latitude radio trace Aldra-Apatity was registered many times [3]. For so called PwDs the amplitudes of one or more radio signals of the 10 - 14 kHz range become equal to apparatus zero (with a 20 Hz channel for amplitude registration) and during a few minutes to some hours. These events are caused by the UrREPs (energy E > 10 MeV) in the middle polar atmosphere [2,3] and they often take place in geophysical "calm" conditions when there are no significant Earth magnetic field variations, when there is none anomalous absorption of cosmic radio noise, there are no solar proton precipitations and no anomalous X-rays from the Sun.

Physics statement of the problem. The problem we formulate as the inverse VLF problem for PwDs when the radio waveguide Earth-ionized atmosphere is considered homogeneous and isotropic along the path at every moment of the time. The VLF emitter was RNS "OMEGA" in Aldra (Norway) with the frequencies ω = 10.2, 12.1, 13.6 kHz. Two variants of the inverse VLF problem have been realized. In the first case variant a complex reflection coefficient of the radio wave from the upper boundary neutral atmosphere - ionized atmosphere with a vertical coordinate (the height from the ground) z=h, where h is an effective waveguide height [2], were to be found[1]. In the second variant - the profile of the ionized atmosphere $N_e(z)$ practically from the ground up to low undisturbed ionosphere has been the subject of consideration, the effective electron collision frequency $\nu_{eff}(z)$ has been given. The demand of full compensation of the ground ray by the first ionosphere ray for the radiotrace Aldra-Apatity for the frequency ω= 12.1 kHz. has been an input data of the inverse problem. The length of the radiotrace was 885 km, the trace was situated completely in the auroral zone, the ground surface is rocky with the electric conductivity $\sigma=10^{-3}$ - 10^{-4} Sm/m and the relative dielectric tolerance ε = 10 - 5.

The RNS was a source of the vertically polarized electromagnetic waves so the inverse problem was stated for this polarization.

The first kind of the mathematical statement of inverse problem. A homogeneous spherical waveguide with ε_m= 1 and with the impedance boundary conditions at the bottom and ceiling boundaries. The height h and a real (uncomplex, the time dependence of $e^{-i\omega t}$ was used) impedance of the top conventional boundary were the parameters to be determined from the experimental variations of VLF signals. Practically it was more convenient to look for the modula R of negative reflection coefficient R_i of the electromagnetic field from the top instead of the impedance. Due to the spherical coordinate system, with the polar axis passing through the VLF source on ground, the boundary conditions are as follows:

$E_\theta = - ZH_\varphi$ for r=a; $E_\theta = Z_i H_\varphi$ for r=c.

where a and c are the radial coordinates of the bottom and ceiling boundaries and h = c-a. When we say that h is an effective height of the waveguide we consider that due to its value the impedance Z_i is real and R_i is real (negative) too. The problem is to find the values of R and h due to which the ground wave is compensated by the first ionosphere ray.

Second kind of the mathematical statement of inverse problem. In the new statement the problem is considered for the ionized atmosphere inhomogeneous in one dimension (along coordinate) and the regular ionosphere waveguide with the impedance boundary conditions on the ground bottom r = a and on the conventional ceiling r = g of the waveguide as follows:

$E_\theta = - ZH_\varphi$ for r=a; $E_\theta = Z_{ii} H_\varphi$ for r=g.

The "boundary" g is fixed in the ionosphere high enough that the approximate analytic expression for the impedance value Z_{ii} depends on fraction parameter ν of the initial three dimension problem very slightly. The multilayer waveguide consists of the parts as follows: a homogeneous neutral atmosphere spherical layer with ε_m = 1 and h = c-a; a h is small

[1] Relative to this *h* the argument of the complex reflection coefficient for the electromagnetic wave is equal to π.

(100 m) and is not an effective height in the case; an ionized atmosphere in the r range from c to r = a+z_0; the electric parameters of it are the subject of the inverse problem; the regular polar daytime ionosphere from r = a+z_0 up to r = g; its electric parameters are considered as known and are given [2].

The profile of the effective electron collision frequency is given for all heights and is regulated by the atmosphere pressure. It was necessary to find an increment of effective electron concentration profile β [2], demanding of compensation condition fulfillment.

An attempt to solve the problem according to the first kind statement. According to the paper [4] the electric field decrease function W in the case of the source and are receiver on ground, the second ionosphere ray being neglected, is represented representation as follows:

$$W = W_0 + \sqrt{\frac{\theta}{\sin\theta}}\sqrt{\frac{\cos\psi_1}{\cos\varphi_1}}\left[1+R_g\right]^2 |R_i| \cdot \sin^2\varphi_1 \cdot \frac{e^{ik(2D-d)+i\pi}}{2D} \cdot d \qquad (1)$$

where d is the radiotrace length, 2D is the length of geometric-optical path of the first ionosphere ray, W_0 is the decrease function for the ground diffraction ray tabulated in [5]. The angle φ_1 is an output angle and ψ_1 is an angle of reflection from the ceiling boundary r = a+h. The angles in the case have the mentioned value if the value of h is not less then 25-30 km and so called reflection formula (1) is valid.

For calculations two values of rocky ground conductivity $\sigma = 10^{-3}$ Sm/m $W_0=0{,}652e^{i0.83}$ and $\sigma = 10^{-4}$ Sm/m $W_0=0{,}579e^{i2.0}$ have been used. The decrease function (1) has been calculated in dependence of h = 25-50 km for the values of R = 0.1, 0.2, ... 0.9. In all cases the interference minimum was either not deeper then 25% from the undisturbed value of W or the minimum trended into the intolerable (for expression (1)) region with r < (a+25) km. So the problem does not have the correct solution without to considering the diffraction nature of the first ionosphere ray in the case.

The problem solution according to its second statement. If we take h very small then it is possible to take into consideration practically all atmosphere ionized and at the same time to use the ray theory but in order to lose the possible diffraction nature of the first iosphere ray it is necessary in eq. (1) to change an approximate second item on its precise expression with the contour integral on the complex plane of the fraction parameter ν.[1]

$$W_1 = \frac{2\mathrm{d}}{k^3a^2}\frac{\sqrt{2}\cdot e^{i\frac{\pi}{4}}}{\sqrt{\pi\cdot\sin\theta}}e^{-ikd}\int\sqrt{\nu}\,\frac{r_i}{[\zeta^{(1)\prime}_{\nu-\frac{1}{2}}(ka)+i\delta\zeta^{(1)}_{\nu-\frac{1}{2}}(ka)]^2}e^{i\nu\theta}\,\frac{\nu^2}{a^2}d\nu \qquad (2)$$

$$r_i = -\frac{\zeta^{(1)\prime}_{\nu-\frac{1}{2}}(kc)-i\delta_i\zeta^{(1)}_{\nu-\frac{1}{2}}(kc)}{\zeta^{(2)\prime}_{\nu-\frac{1}{2}}(kc)-i\delta_i\zeta^{(2)}_{\nu-\frac{1}{2}}(kc)} \qquad \text{where} \quad \zeta^{(1),(2)}_{\nu-\frac{1}{2}}$$

- modified are Bessel's functions of first and second kind,

k is a wave vacuum number, δ and δ_i are the normalized (divided by a value of vacuum impedance) ground (r = a) and ionized atmosphere and ionosphere (r = a+h) impedances.

Due to the small value of h the parameter δ_i is an essential function of the parameter ν. In order to get this function it is necessary to solve by numeral integration the following nonlinear differential Ricatty's equation [6].

$$\frac{dy}{dr} = \varepsilon'_m + \left\{k^2 - \frac{\nu^2-1/4}{r^2\varepsilon'_m}\right\}y^2 \qquad (3)$$

with the initial condition $y=\varepsilon'^{1/2}_m/ik$ at the point r = g;

[1] The approximate expression is gotten from the strict one by approximate analytic calculation of the contour integral by the method of saddle point [4].

$$\varepsilon'_m = 1 - \frac{4\pi e^2 N(r)\left[1 - i\frac{\nu_{eff}(r)}{\omega}\right]}{m\left[\omega^2 + \nu_{eff}^{\ 2}(r)\right]},$$

$$\nu_{eff}(r) = 0.87\cdot 10^7 \exp(b(z-a-70km)), rad./s$$

$$N_e(z) = A(z_0)\exp[\beta(z-z_0)] \qquad \text{for } c < z < z_0 \qquad (4)$$

$$N_e(z) = A(z_0)\exp[(0.407 + 0.134\frac{|z_0-58|}{25})(z-z_0)] \qquad \text{for } z_0 < z < g - a$$

$$A(z_0) = 80(1+\frac{19}{8}(z_0-40)/43)\exp\left[b\left(z_0-58\right)\right], \qquad \text{el./sm}^3,$$

where $y(r) = -i/k\delta_i(r)$, $z = r - a$ and $b = -0.14$ 1/km is the increment of atmosphere pressure.

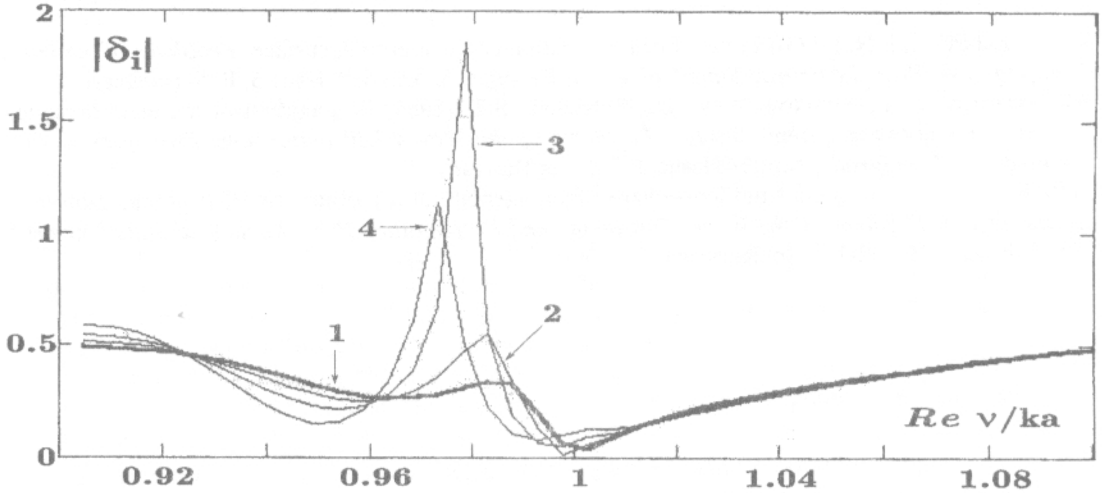
In the picture below four dependencies of $|\delta_i|$ on the normalized variable Re ν / ka are presented. The values of this parameter determine the contour integration points according to the definitions which are as follows:

$$\begin{aligned} Re\,\nu &= s\cdot\cos(\pi/6) + ka \\ Im\,\nu &= s\cdot\sin(\pi/6) \end{aligned} \qquad \text{for Re } \nu > ka, \qquad (5)$$

$$\begin{aligned} Re\,\nu &= ka - s\cdot\cos(\pi/4) \\ Im\,\nu &= s\cdot\sin(\pi/4) \end{aligned} \qquad \text{for Re } \nu < ka, \qquad (6)$$

Due to the definition (5) and (6) s is the distance between the "saddle point" (ν=ka) and the varible point of integration in two straight half lines.

Curve 1 presents the case of monotonic profile of electronic concentration (undisturbed daytime polar ionosphere). Curves 2, 3, 4 show the cases of non-monotonic profiles with the values of β= -0.05, -0.07, -0.09 in (4) respectively. One can conclude that the behavior of $|\delta_i|$ is strongly nonmonotonic along the contour of integration which indicates the necessity to calculate numerically an integral in the decrease function expression (2).



In the table the values of W (and of normalized function W* modulus) are given as a function of the electric properties of the ionized atmosphere, i. e. as a function of β in (4), for three values of σ. The first line of the Table corresponds to the undisturbed daytime ionosphere model with β = 0.1 and z_0= 62 km [2].

β км $^{-1}$	σ=10^{-3} Sm/m		σ=$5\cdot10^{-4}$ Sm/m		σ=10^{-4} Sm/m	
	W	|W*|	W	|W*|	W	|W*|
0.1	$2.62e^{0.3i}$	7.04	$2.24e^{0.5i}$	9.68	$1.52e^{1.0i}$	84.7
-0.050	$1.20e^{-0.2i}$	3.22	$0.87e^{0.1i}$	3.78	$0.61e^{0.8i}$	33.8
-0.060	$0.94e^{-0.3i}$	2.53	$0.63e^{-0.1i}$	2.72	$0.44e^{0.7i}$	24.7
-0.070	$0.67e^{-0.6i}$	1.80	$0.38e^{-0.4i}$	1.63	$0.27e^{0.7i}$	14.8
-0.075	$0.55e^{-0.7i}$	1.46	$0.28e^{-0.8i}$	1.19	$0.17e^{0.6i}$	9.67
-0.080	$0.44e^{-1.0i}$	1.17	$0.23e^{-1.4i}$	1.00	$0.08e^{0.4i}$	4.61
-0.083	$0.39e^{-1.3i}$	1.05	$0.25e^{-1.9i}$	1.08	$0.03e^{0.1i}$	1.72
-0.085	$0.37e^{-1.5i}$	1.00	$0.28e^{-2.0i}$	1.21	$0.02e^{-1.5i}$	1.00
-0.090	$0.38e^{-2.0i}$	1.01	$0.39e^{-2.4i}$	1.67	$0.10e^{-2.4i}$	5.50
-0.100	$0.54e^{-2.6i}$	1.46	$0.64e^{-2.8i}$	2.76	$0.26e^{-2.6i}$	14.4

It is seen from the Table that function W(β) reaches a minimum value in each case of the conductivity σ. The depth of this minimum varies from sevenfold decrease in the case of σ=10^{-3} Sm/m to eighty fivefold for σ=10^{-4} Sm/m. The presented solution of the inverse VLF problem for the cases of PwDs is the proof of sporadic ionization layer existence in the atmosphere in the 10 - 40 km range. The analogous proof prove was obtained by the normal wave method in paper [1].

References

1. Beloglazov M. I., Remenets G. F., Nemirov V. P. (2001) Review of all powerful VLF disturbances initiated by the ultrarelativistic electron precipitations while 1974-1992 years, *Physics of Auroral Phenomena. Proceedings of the 23nd Annual Apatity Seminar*. Препринт Полярного геофизического инст. РАН: PGI-00-01-108. Apatity. P. 97-100.
2. Remenets G. F. (1997) Unique ground VLF monitoring of relativistic electron precipitations. *Problems of Geospace*, Eds. M. I. Pudovkin, B. P. Besser, W. Riedler and A. M. Lyatskaya. Vienna (Austria). P. 273-278.
3. Remenets, G. F. and Beloglazov M. I. (1999) Ground VLF monitoring of the ultrarelativistic electron precipitations. *Problems of Geospace 2. Proceedings of the 2nd International Workshop held at St.Petersburg, Russia, June 29-July 3.* Eds. V.S. Semenov, H. K. Biernat, M.V. Kubyshkina, C.J. Farrugia, and Muhlbachler. Vienna (Austria). P.~275-281.
4. Gyunninen, E. M. and Zabavina, I. N. (1966) Long wave propagation above a ground surface. *Problems of the Waves Diffraction and Propagation. Proc. Leningrad State University, Leningrade, U.S.S.R.* Issue 5. P. 5. (in Russian).
5. Gyunninen, E. M., Makarov, G. I., Novikov, V. V. and Rybachek, S.T. (1964) Propagation of the electromagnetic pulses and their garmonics above a ground surface. *Problems of the Waves Diffraction and Propagation. Proc. Leningrad State University, Leningrade, U.S.S.R.* Issue 3. P. 5. (in Russian).
6. Rybachek, S.T. (1968) Taking into account the ionosphere inhomogeneity at a ploblem of VLF propagation in the Earth-ionospgere waveguide. *Problems of the Waves Diffraction and Propagation. Proc. Leningrad State University, Leningrade, U.S.S.R.* Issue 7. P. 152-163. (in Russian).