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# ION WEIBEL INSTABILITY IN THE EARTH'S NEUTRAL SHEET AND CURRENT DISRUPTION PROBLEM

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**Abstract.** The quasilinear equation for the ion Weibel instability is solved for waves propagating along the magnetic field. The moments of the ion distribution function in the saturation stage are determined, and the energy of the excited waves is estimated as a functions of the current velocity for parameters characteristic of the neutral sheet of the Earth's magnetotail.

## Introduction

Among plasma instabilities due to a cross-field current the purely growing Ion Weibel Instability (IWI) recently was intensively studied. It was identified by Chang et al. [3] who have shown that in general ion response would play a significant role in exiting electromagnetic waves directed almost parallel to the ambient magnetic field. Investigation of the dispersion equation for this instability [6, 10] showed that it exists in high beta regimes as in the Earth's neutral sheet. That is the reason why IWI was assumed to be helpful in the explanation of a mechanism of a current disruption [5, 6, 7, 10].

Several instabilities and mechanisms has been proposed to accomplish the current disruption (see for discussion and references [5,6]) such as the tearing instability, the ballooning instability, the thermal catastrophe model, the coupling between the magnetosphere and the ionosphere, and the model based on the cross-current instabilities. The preliminary analysis of the latest mechanism was done by Lui et al.[6]. They have introduced the model combining several types of instabilities driven by a cross-field current. Among these instabilities IWI plays the role of no small importance. The function of it is to provide anomalous resistivity in order to modify significantly the local current density and supply a collisionless dissipation necessary to initiate the fast magnetic reconnection or facilitate the development of other instability process in the magnetosphere tail [5].

The derivation and numerical analysis of the dispersion relation was done in [6, 10, 12]. Nonlinear evolution of IWI was discussed by Yoon [11] for a quasiperpendicular collisionless shock and Lui et al.[7] for the parameters related to the Earth's neutral sheet. The numerical solution of quasilinear equation of IWI was performed using moments of kinetic equation. However in both papers the ion distribution was taken to retain its original functional form in time and only temperatures and drift velocity change.

Below we perform the analytical treatment of the quasilinear kinetic equation for IWI to find how the ion distribution function changes and solve the dynamic equations for the moments to obtain the saturation level of this instability.

## Derivation of basic equations

Let us at first briefly describe the physical model and the geometrical configuration based on [10, 11]. The basic assumption following from [6, 7, 11] is that the ions are unmagnetized and allowed to drift with the initial velocity $\mathrm{v}_0 = \upsilon_0 \mathrm{y} \underset{\sim}{<} \upsilon_{Ti}$ perpendicular to the ambient magnetic field $\mathbf{B}_0 = \mathrm{B}_0\mathbf{z}$. The electrons are treated fully magnetized and stationary. To simplify the analysis we take electrons and ions to be isotropic and use the Maxwellian distribution function for the electrons and the drifting Maxswellian distribution for the ions:

$$f_i(\mathrm{v}) = \frac{n}{\pi^{3/2}\upsilon_{T_i}^3}\exp\left\{-\frac{\upsilon_x^2+(\upsilon_y-\upsilon_0)^2+\upsilon_z^2}{\upsilon_{T_i}^2}\right\} \qquad (1)$$

where $\upsilon_{Tj} = (2T_j/m_j)^{1/2}$ is a thermal velocity, $T_j$ is the temperature, $m_j$ is the mass of jth species.

The derivation of the dispersion tensor elements and the linear dispersion equation for the wave vector parallel to the ambient magnetic field ($\mathbf{k} = k\mathbf{z}$) can be found in [10]. Neglecting the displacement current we can write the linear dispersion equation in the form [10, 11]:

$$-1-\frac{k^2c^2}{\omega_{pi}^2}-\frac{T_{i\perp}}{T_{i\parallel}}\frac{Z'(\xi_i)}{2}+\frac{\omega^2}{1+\dfrac{k^2c^2}{\omega_{pi}^2}+\dfrac{T_{i\perp}}{T_{i\parallel}}\dfrac{Z'(\xi_i)}{2}}-\frac{\upsilon_0^2}{\upsilon_{Ti}^2}\frac{T_{i\parallel}Z'(\xi_i)Z'(\xi_e)}{T_eZ'(\xi_i)+T_{i\parallel}Z'(\xi_e)}=0, \qquad (2)$$

where $\omega_{pi}$ is the ion plasma frequency, $c$ is the speed of light and $Z(\xi_j)=\frac{\upsilon_{Tj}}{n}\int d^3\mathrm{v}\frac{f(\mathrm{v})}{\upsilon_z-\xi_j}$ and for the Maxwellian distribution coincides with the well-known plasma dispersion function. The "prime" defines the first derivative of this function over $\xi_j$. The argument of $Z$ are defined by: $\xi_j = \omega/k\upsilon_{Tj}$.

Similarly to [6, 7] we consider the IWI for two sets of plasma parameters relevant to the neutral sheet. The first set corresponds to the inner edge of the Earth's neutral sheet: $T_i/T_e = 4$, $T_i = 12$ keV, $n_e = n_i = n = 0.6$ cm$^{-3}$, $B_0 = 25$ nT and the second corresponds to the midtail region: $T_i/T_e \approx 10$, $T_i = 2$ keV, $n_e = n_i = n = 0.3$ cm$^{-3}$, $B_0 = 5$ nT [1, 6]. Here $n_j$ is the density of jth species. Since for the both regions $T_e/T_i$ and $\upsilon_{Ti}/\upsilon_{Te}$ are much less than unity, they can give only small corrections and below will be omitted.

The obtained dispersion equation supports the purely growth mode (Re $\omega = 0$) and making use of the asymptotic expansion for $Z(\xi_j)$ in the limit of $|\xi_j| << 1$ we find from (2) the resulting growth rate:

$$\gamma_k = \frac{k\upsilon_{T_{i\parallel}}}{\sqrt{\pi}}\left\{\frac{Z'_{0i}}{2}+T_{i\parallel}\frac{1+\frac{k^2c^2}{\omega_{pi}^2}}{m\upsilon_0^2+T_{i\perp}}\right\}\left(\frac{\upsilon_{T_{i\parallel}}^3\sqrt{\pi}}{2n}\int\frac{\partial^2 f(\mathrm{v})}{\partial\upsilon_z^2}\delta(\upsilon_z)d\mathrm{v}\right)^{-1}, \tag{3}$$

where $Z'_{0i}=\frac{\upsilon_{T_{i\parallel}}^2}{2n}\int\frac{\partial f(\mathrm{v})}{\upsilon_z\partial\upsilon_z}d^3\mathrm{v}$ is the first term of expansion of $Z'(\xi_i)$ in power series in $\xi_j$.

For a purely growing instability, as in our case, the quasilinear theory is applicable only if $k\upsilon_{Ti} >> \gamma$, so we can write that for the IWI quasilinear theory is valid under the condition

$$Y = 1+\frac{2T_{i\parallel}}{Z'_{0i}(m\upsilon_0^2+T_{i\perp})} << 1 \tag{4}$$

One can see that this condition is fulfilled for the parameters cited above. From (3) it can be easily found that the instability occurs while $Y > 0$. According to results obtained by Yoon [11] the saturation level of the unstable Ion Weibel modes is high enough when only the ions are allowed to drift. Therefore in quasilinear analysis we can neglect contribution of the magnetized electrons and write the quasilinear kinetic equation for the ion distribution function:

$$\frac{\partial f_i}{\partial t} = -\mathrm{i}\frac{e^2}{m^2c^2}\int\Bigg\langle\Bigg\{\left[\left(\frac{\omega^*}{k}-\upsilon_z\right)\frac{\partial}{\upsilon_\perp\partial\upsilon_\perp}\upsilon_\perp+\upsilon_\perp\frac{\partial}{\partial\upsilon_z}\right]\left(B_{yk}\cos\theta-B_{xk}\sin\theta\right)$$
$$-\left(\frac{\omega^*}{k}-\upsilon_z\right)\frac{\partial}{\upsilon_\perp\partial\theta}(B_{yk}\sin\theta+B_{xk}\cos\theta)+\upsilon_0 B_{xk}\frac{\partial}{\partial\upsilon_z}\Bigg\}\Bigg\{\frac{\upsilon_0 B_{xk}}{\omega-k\upsilon_z+i0}\frac{\partial f_i}{\partial\upsilon_z}$$
$$+\sum_{\pm}\frac{\left[\left(\frac{\omega}{k}-\upsilon_k\right)\frac{\partial}{\partial\upsilon_\perp}+\upsilon_\perp\frac{\partial}{\partial\upsilon_z}\right]f_i}{\omega-k\upsilon_z\pm\omega_{ci}+i0}(B_{yk}\cos\theta-B_{xk}\sin\theta)\Bigg\}\Bigg\rangle dk$$

Here the angular brackets denote the averaging over the random phases of the fluctuating Fourier components of electric and magnetic fields, * is the complex conjugation, $\upsilon_x(t)=\upsilon_\perp\sin(\theta-\omega_{ci}t)$, $\upsilon_y(t)=\upsilon_0+\upsilon_\perp\cos(\theta-\omega_{ci}t)$, $\upsilon_z(t)=\upsilon_z$, $\upsilon_\perp^2=\upsilon_x^2+\upsilon_y^2$, $\theta$ is the gyrophase angle and the small electrostatic wave energy was neglected.

The time evolution of the magnetic field fluctuations is described by the equation $\frac{\partial}{\partial t}|\mathbf{B}_k|^2 = 2\gamma_k|\mathbf{B}_k|^2$, where the growth rate $\gamma_k$ is determined by (3). When studying the linear dispersion relation Wu et al. [10] found that a polarization of the unstable mode is almost linear so $|\mathbf{B}_{yk}|^2$ can be omitted.

Above we have assumed that the ions are treated as unmagnetized, so using the limit $\omega_{ci} \to 0$ and after averaging over the gyrophase angle $\theta$ in equation (5) we arrive at:

$$\frac{\partial f_i}{\partial t}=\frac{ie^2}{2m^2c^2}\int dk\left\{\left[\left(\frac{\omega^*}{k}-\upsilon_z\right)\frac{\partial}{\upsilon_\perp\partial\upsilon_\perp}\upsilon_\perp+\upsilon_\perp\frac{\partial}{\partial\upsilon_z}\right]\frac{(i\gamma_k+k\upsilon_z)|B_{xk}|^2}{\gamma_k^2+k^2\upsilon_z^2}\right.$$
$$\left.\left[\left(\frac{\omega}{k}-\upsilon_z\right)\frac{\partial}{\partial\upsilon_\perp}\upsilon_\perp+\upsilon_\perp\frac{\partial}{\partial\upsilon_z}\right]f_i+\frac{\partial}{\partial\upsilon_z}\frac{i\gamma_k\upsilon_0^2|B_{xk}|^2}{\gamma_k^2+k^2\upsilon_z^2}\frac{\partial f_i}{\partial\upsilon_z}\right\} \tag{6}$$

Although the equation (6) describes the non-resonant wave interactions with all background ions, the efficiency of such interactions is different for different parts of the ion distribution. The main effect comes from the strong diffusion for $\upsilon_z << \upsilon_{Ti}$. $f_i$ can changes significantly in the region of small $\upsilon_z$ whereas the average values such as $T_\perp$, $T_\|$, $\upsilon_0$ vary slightly in the limit of $Y << 1$. Thus equation (6) may be rewritten keeping only two last terms in the right-hand side. Taking the integral over $\upsilon_\perp$ and introducing the new variable $h=\frac{e^2}{2m_ic^2}\int\frac{|B_{xk}|^2}{k^2}dk$ we reduce (6) to the simple form [2, 4]:

$$\frac{\partial f_i}{\partial h}=\frac{\partial}{\partial\upsilon_z}\frac{\upsilon_0^2+\upsilon_{T_{i\perp}}^2/2}{m_i\upsilon_z^2}\frac{\partial f_i}{\partial\upsilon_z} \tag{7}$$

The equation (7) has an analytical solution [2, 4] in terms of the initial reduced ion distribution function (1):

$$f(h)=n\frac{\Gamma(3/4)|\upsilon_z|^{3/2}\,2^{1/4}}{\pi\upsilon_{T_{i\|}}^3}\int_0^\infty\frac{e^{-4(\upsilon_0^2+\upsilon_{T_{i\perp}}/2)\lambda^2h/m}\lambda^{1/4}J_{-3/4}(\lambda\upsilon_z^2)}{\left(\lambda^2+\upsilon_{T_{i\|}}^{-4}\right)^{3/4}}d\lambda, \tag{8}$$

where $J_\upsilon(x)$ is Bessel function of the first kind. The evolution equations for the drift velocity and the perpendicular and parallel temperatures may be found directly from (5) by taking the appropriate moments [8, 11]. In the limit of small $|\xi_i| << 1$ we arrive to the system of differential equations describing the evolution of moments of the ion distribution function in terms of $h$:

$$\frac{dT_{i\perp}}{dh}=1+\frac{T_{i\perp}}{T_{i\|}}Z'_{oi}(1-Y)$$
$$\frac{dT_{i\|}}{dh}=\frac{T_{i\perp}+2K}{T_{i\|}}Z'_{0i}(1-Y)$$
$$\frac{dK}{dh}=\frac{2K}{T_{i\|}}Z'_{0i}(1-Y).$$

where $K=m_i\upsilon_0^2/2$. For the obtained distribution function (8) $Z'(0,h)\approx-2\left[1-\alpha\left(h\frac{T_{i\perp}+2K}{T_{i\|}^2}\right)^{1/4}\right]$, where α is a constant ≈ 1.

On the saturation stage of the instability $\gamma_k\to 0$ and from (4) we obtain:

$$\frac{-Z_{0i}^{\prime f}(h)}{2}=\frac{T_{i\|}^f}{2K^f+T_{i\perp}^f}, \tag{10}$$

where the superscript $f$ denotes the final values of parameters. Let us assume $T_{i\|}=T_{i\perp}=T_i$ for the sake of simplicity and set $T_{i\|}^f=T_i+\delta T_{i\|}$, $T_{i\perp}^f=T_i+\delta T_{i\perp}$, $K^f=K_0+\delta K$. It is evidently that the contributions from $\delta T_{i\|}$, $\delta T_{i\perp}$, $\delta K$ in (10) are of the order of $h/T << 1$ and may be neglected in comparison with the contribution from $Z'_{0i}\sim h^{1/4}/T^{1/4}$. As a result we find: $h=\frac{Y^4}{\alpha^4}\frac{T_i^2}{T_i+2K_0}$.

Neglecting terms of higher order of $Y$ and replacing $T_{i\perp}$, $T_{i\|}$, $K$ by their initial values in the right-hand side of equations (9) that also corresponds to omitting terms of the higher order of $Y$, we obtain:

$$\delta T_{i\perp}=\frac{2K_0-T_i}{2K_0+T_i}h$$
$$\delta T_{i\|}=2h$$

$$\delta K = -\frac{4K_0}{2K_0 + T_i} h.$$

As can be seen from the definition of $h$ and equation (3) the value of wave energy has the next order in $Y$. Assuming that the main contribution to the wave energy comes from the value of the wave vector, when the growth rate has its maximum we can estimate the wave energy as:

$$\frac{\delta B^2}{B_0^2} = \int dk \frac{|B_k|^2}{B_0^2} \sim \beta \frac{h}{T_i} \frac{Y}{3}$$

Here we normalize $\delta B^2$ on the value of external magnetic field energy $B_0^2 / 8\pi$.

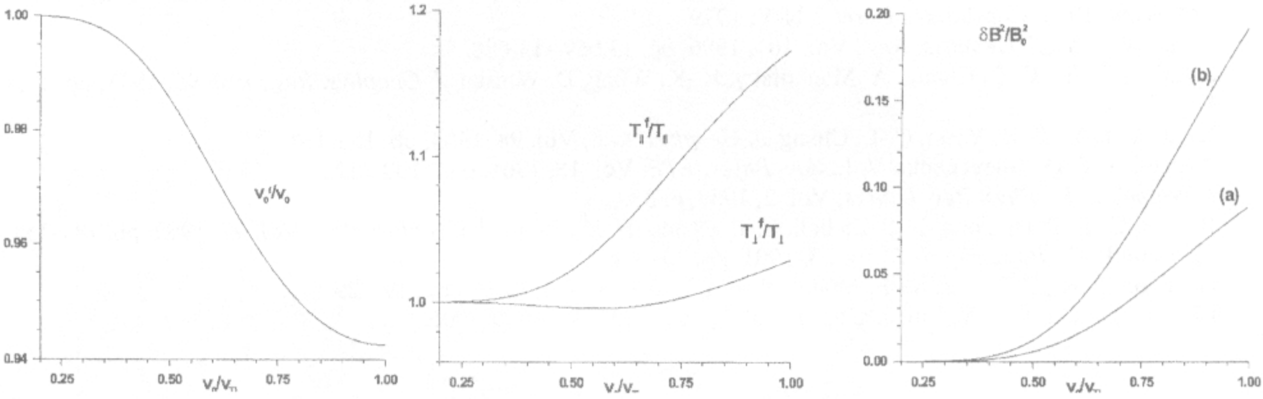


Figure 1. The behavior of the $T_{i\parallel}^f, T_{i\perp}^f, v_0^f$, normalized on their initial values and the total waves energy $\delta B^2 / B_0^2$ versus $v_0 / v_{T_i}$ (a) inner edge; (b) midtail

Fig. 1 demonstrate saturation values of the drift velocity $\upsilon_0^f$, parallel temperature $T_\parallel^f$ and perpendicular temperature $T_\perp^f$ normalized on its initial values as the functions of $\upsilon_0/\upsilon_{Ti}$. Within the accuracy of analysis the relative changes of the appropriate moments is equal both for the inner edge and for the midtail. As would be expected the excitation of the pure growing mode results in the growth of the parallel temperature due to the reduction of the cross-field current value. However the value of the perpendicular temperature remains almost unchanged that justifies assumption described above.

The wave energy for the two parts of the Earth's neutral sheet is shown in Fig. 1: (a) for the inner edge and (b) for the midtail.

The amplitude of the fluctuating magnetic field for the case (b) is greater than for the case (a) but one should take into account that the value of β is greater for the midtail case.

## Conclusion

The obtained results have shown that the saturation level of this instability is reached due to the formation of the plateau on the reduced ion distribution function. The criterion of validity of the quasilinear theory is reduced to the smallness of deviation of the plasma parameters from the critical values corresponding to the instability threshold. These critical values may be easily obtained from (10).

The saturation level of IWI depends on the values of the drift velocity and plasma parameters. For the set of parameters typical for the neutral sheet the values of the moments of the ion distribution on saturation stage is much less then the values obtained earlier in papers [7] where the equations for the evolution of the moments was solved numerically. Even for $\upsilon_0 = \upsilon_{Ti}$, when the changes of plasma parameters are the highest, the drift velocity and parallel temperature vary in magnitude only on ~ 5,8 % and ~ 17 % respectively. It is about 4.5 times less then the values found by Lui et al. [7]. Such discrepancy can be explained by the fact that in numerical solution in [7] it was assumed that the ion distribution was assumed to be Maxwellian for all time, and only the global plasma parameters was changed.

The calculated value of the drift velocity is less than the obtained from the experimental data from IMP 6 and ISEE 1 which is estimate as ~ 25 % [7]. Thus we can conclude that in used approximation IWI can't provide necessary ion heating in current disruption during the substorm onset. However it could serve as a trigger for another type of instability.

Nevertheless the investigation of IWI has preliminary character. For example, in the analysis of IWI the magnetic field inhomogeneities must be taken into account. Analysis of IWI in the cold plasma approximation was done in [13] for the Harris neutral sheet. However the authors didn't take into consideration any kinetic effects which can change the dispersion equation.

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