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NONLINEAR EVOLUTION STAGE OF GRADIENT DRIFT INSTABILITY IN THE F-REGION OF THE POLAR IONOSPHERE

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Abstract. The coherent evolution of gradient drift instability in the ionospheric F-region is considered when the effects of Coulomb collisions and enhanced electric field are taken into account. In certain geophysical conditions the saturated amplitudes and spectra of relative density fluctuations are calculated, and both analytical and numerical results are given. The effect of an external electric field shear is considered. It is shown that its influence on stabilization of instability under existing conditions is not great.

Introduction

A basic direction of the theory of generation of ionospheric irregularities is the search for mechanisms of plasma instabilities [Gershman et al., 1984; Tsunoda, 1988]. An important instability associated with the structuring of ionospheric F-region is the gradient drift instability, also know as diamagnetic and Pedersen current instability. The physical mechanism of this instability is bound up with separation of charges. In the geomagnetic field direction it is called into being by practically free electron drift (because of a difference in mass of electrons and ions), and in the transverse direction it is provided by Pedersen drift of the ions (because of the highly magnetized electrons). The polarization electric field, originating at it, (along with geomagnetic field) will lead to the electrodynamics transposition of denser plasma into regions containing smaller density and vice versa. As a result of the original density perturbations the amplitude of drift waves is magnified. If dissipative processes do not compensate these modifications, the amplitude of the drift wave increases in time, i.e. the plasma appears unstable concerning this perturbation.

The subject has been reviewed in articles and sections of books Rognlien and Weinstock (1974); Keskinen and Ossakow (1983); Tsunoda (1988); Tereshchenko and Tereshchenko (1998). Our goal is to include the effects of enhanced electric field and Coulomb collisions in the analysis of the gradient drift instability. In the first section we present a theoretical model for a long wavelength of gradient drift instability. The next section deals with the nonlinear saturation of this instability, and analytical and numerical solutions are obtained for the saturation amplitude of the fluctuations. Applications to the high-latitude ionosphere are discussed.

Linear Fluid Theory

We consider a coordinate system appropriate for the auroral F-region in which the geomagnetic field $\mathbf{B}_0$ is in the *z* direction, the ambient electric field $\mathbf{E}_0$ lies in the *xy* plane, and a weak density gradient $\mathbf{E}_0$ is oriented in the *x* direction. The electric field in the *y* direction is constant, while electric field along the *x* direction is allowed to be a function of *x*. Temperature effects are ignored.

We assume that the perturbed quantities vary as a plane wave $\left(\tilde{N} = N_1 \exp(ik_x x + ik_y y + ik_z z - \omega t)\right)$, where $N_1 >> N_0$; k_x, k_y and k_z are the wave numbers along the *x*, *y*, and *z* directions, respectively; $\omega >> c_s k_z$ is the complex angular frequency, implying the growth for $\omega >> c_s k_z$. We shall consider the magnitude k_x to be small. We suppose that the perturbations will be the geomagnetic field aligned $\omega >> c_s k_z$, so the displacement of ions along the field can be neglected.

In this case the total velocity of the charged particles along axis *x* (in the direction of the density gradient) is defined by the expression

$$u_x = \left\{ i \frac{k_y v_{Te}^2}{\omega_{Be}} \left[1 - \frac{1}{\nu_i N} \frac{\partial \tilde{N}}{\partial t} \right] + \frac{\nu_{in}}{\nu_i} \frac{c E_{0y}}{B_0} \right\} \frac{\tilde{N}}{N} + \frac{c E_{0x}(x)}{B_0} \left\{ \frac{\nu_{in}}{\omega_{Bi}} + \frac{c E_{0y}}{\omega_{Bi} L_E B_0} \right\} \quad (1),$$

where $v_{Te} = \sqrt{T_e / m_e}$ is the thermal speed for electrons; $\omega_{Be} = eB_0 / m_e c$ is the gyrofrequency for particles of α species (electrons or ions); ν_i is the total ion – ion and ion – neutral collision frequency; ν_{in} is the ion – neutral collision frequency; $L_E = E_{0x}(x)/E'_{0x}(x)$ is the scale length of the inhomogeneity in $L_E = E_{0x}(x)/E'_{0x}(x)$; e, m_e и T_e are the charge, mass and temperatures (in energy units) for particles of α species; c is the light velocity in vacuum.

The first right-hand term of (1) determines the drift velocity due to the periodical potential electric field, which originates from electron motion in the direction of $\mathbf{B}_0$. The second term describes the resultant velocity caused by the polarization of electric fields, which are generated by the ion Pedersen, and inertial drifts. Finally, the last term represents the velocity, which is driven by the field $L_E = E_{0x}(x)/E'_{0x}(x)$.

From the requirement of quasi-neutrality, $\tilde{N}_e = \tilde{N}_i$, the relation $N(x)E_{0x}(x) = const$, and the continuity equation for ions

$$\frac{\partial \tilde{N}}{\partial t} = -\nabla\left(N u_x \mathbf{e}_x\right), \tag{2}$$

we find the oscillation frequency of the perturbation $\tilde{N}$

$$\omega = \omega_{de} + i\left(\frac{\omega_{de}^2}{\nu_i}\frac{\tilde{N}}{N} - \frac{\nu_{in}}{\nu_i}\frac{cE_{0y}}{B_0 L_N}\left[1 - \frac{\nu_i}{\omega_{Bi}}\frac{E_{0x}}{E_{0y}}\right]\right), \tag{3}$$

where $\omega_{de} = k_y v_{de} = k_y T_e / m_e \omega_{Be} L_N$ is the characteristic frequency of drift oscillations for electrons; $L_N = N(x)/N'(x)$ is the density scale length.

The phase velocity of these waves is equal to the diamagnetic velocity of electrons, bound up with the density gradient. From (3) we obtain for the growth rate of perturbations

$$\gamma = \frac{\omega_{de}^2}{\nu_i}\frac{\tilde{N}}{N} - \frac{\nu_{in}}{\nu_i}\frac{cE_{0y}}{B_0 L_N}\left[1 - \frac{\nu_i}{\omega_{Bi}}\frac{E_{0x}}{E_{0y}}\right]. \tag{4}$$

Hence it follows, that an inhomogeneous electrical field can stabilize the gradient drift instability, if it is parallel to the density gradient or relation $\frac{\nu_i}{\omega_{Bi}}$ not so little. For real conditions in the F-region ionosphere $\frac{\nu_i}{\omega_{Bi}}$, therefore the influence of the velocity shear, caused by the inhomogeneous electric field on the stabilization of instability is insignificant.

Let the arisen drift wave propagate in the xy plane perpendicular to the magnetic field and at some angle to the density gradient. In inhomogeneous plasma it can be reduced in the origin of a reflected wave, and the density perturbation can be presented as the sum of two plane waves propagating at equal but opposite angles to the y axis

$$\tilde{N}/N_0 = A\sin\left(k_x x + k_y y - \omega t\right) + B\sin\left(k_y y - k_x x - \omega t\right) = A_{11}\sin\left(k_y y - \omega t\right)\cos k_x x. \tag{5}$$

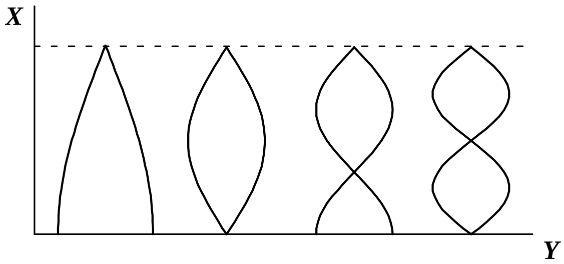

Fig.1. The first four spatial harmonics of standing waves in the x axis direction (along $\tilde{N}$).

The superposition of running crosswise waves is equivalent to the mixed wave, which has a stationary value component on the z axis, represents a standing wave on the x axis (along the gradient), and is the running wave on the y axis. The amplitude of the mixed wave varies in the x direction what is shown in Fig. 1 for the first mode.

Thus, in any direction, in which there are reflecting boundaries for the drift waves, there are standing waves just as it happens in case of a string, one extremity of which is free, and the other is anchored. The standing waves are normal modes. It is easy to excite the first mode. The sine shape of standing waves is the property of the homogeneous system. In inhomogeneous medium the normal modes have the shape of a distorted sinusoid.

The linear growth rate γ is given by expression (4) describing the temporal growth of the amplitude of the perturbation in the small-amplitude limit. This growth does not continue indefinitely, but it is rather modified by nonlinear processes that ultimately saturate the instability.

Nonlinear Stage

To extend the above theory to nonlinear stage, we need to consider the equations for the perturbed parts of the density $\tilde{N}$ and electrostatic potential $\tilde{\varphi}$ [Rognlien and Weinstock, 1974; Chaturvedi and Ossakow, 1979]

$$\frac{\partial}{\partial t}\tilde{N} = \gamma\,\tilde{N} + \frac{c}{B_0}\left[\nabla_{\perp}\tilde{\varphi}\times e_z\right]\nabla_{\perp}\tilde{N}\,; \tag{6}$$

$$N_0\,\nabla_{\perp}^2\,\tilde{\varphi} = \mathbf{E}_0\,\nabla_{\perp}\,N\,. \tag{7}$$

These equations are valid for F-region ionosphere altitudes $\dfrac{\nu_i}{\omega_{Bi}}$ and for long wavelengths of the order of several hundred meters. In (6) the second term on the right-hand side there is a nonlinear term representing a nonlinear flux of particles through mode interactions. The non-linearity of mode (5), as manifested by the nonlinear term in the continuity equation (6), produces the second mode. When the first and second modes interact through the nonlinear term, they produce the mixed wave with the third spatial harmonic in the x direction. The nonlinear interaction of two-mixed wave produces the fourth mode, et cetera (see Fig.1). Herewith the even modes of oscillations have other configuration than in case of the string, an extremity of which is anchored. Thus our general perturbation can be presented in the form

$$\tilde{N}/N_0 = \left[A_{11}\cos k_x x + A_{31}\cos 3k_x x + \cdots\right]\sin\left(k_y y - \omega t\right) + A_{20}\sin 2k_x x + A_{40}\sin 4\,k_x x + \cdots, \tag{8}$$

where A_{ik} is the unknown mode amplitude; the subscript i and k correspond to the special harmonics in the x and y directions respectively. The basic saturation process for our instability can be illustrated, considering the interaction of only two waves or modes.

Substituting (8) in (6), we obtain two coupled equations for the mode amplitudes:

$$\frac{\partial A_{11}}{\partial t} = \gamma_1 A_{11} - \alpha\,A_{11}\,A_{20}\,,\quad \frac{\partial A_{20}}{\partial t} = -\gamma_2\,A_{20} + \frac{\alpha}{2}A_{11}^2\,, \tag{9}$$

where $\gamma_1 = \dfrac{1}{2}\dfrac{k_y^2}{k_\perp^2}\left\{\left(\nu_i^2 - 4\nu_{in}\dfrac{cE_0}{B_0 L_N}\dfrac{T_{iz}}{T_{i\perp}}\right)^{1/2} - \nu_i\right\} - \dfrac{k_\perp^2 c_s^2\,\nu_{en}}{\omega_{Bi}\,\omega_{Be}}$ is the linear temporal growth rate of the first mode [Tereshchenko and Tereshchenko, 1998]; $\alpha = \left(k_y^2\,k_x/k_\perp^2\right)\left(cE_0/B_0\right)$ is the coupling coefficient; $\gamma_2 = 4k_x^2\,c_s^2\,\nu_{en}/\omega_{Bi}\,\omega_{Be}$ is the linear damping rate of the second special harmonic; $\gamma_2 = 4k_x^2\,c_s^2\,\nu_{en}/\omega_{Bi}\,\omega_{Be}$ is the ion sound speed. The equations (9) can be rewritten so

$$\begin{aligned}
&\frac{\mathbf{d}}{\mathbf{d}t}\left|A_{11}\right|^2 = 2\gamma_1\left|A_{11}\right|^2 - \frac{\alpha^2}{\gamma_2}\left|A_{11}\right|^4 \\
&\frac{\mathbf{d}}{\mathbf{d}t}\left|A_{20}\right|^2 = -2\gamma_2\left(\left|A_{20}\right|^2 - \frac{\gamma_1^2}{\alpha^2}\right)
\end{aligned}\quad ; \qquad (10)$$

and

$$\begin{aligned}
&\left|A_{11}\right|^2 = \left(\mathbf{const}\,e^{-2\gamma_1 t} + \frac{\alpha^2}{2\gamma_1\gamma_2}\right)^{-1} \\
&\left|A_{20}\right|^2 = \mathbf{const}\,e^{-2\gamma_2 t} + \frac{\gamma_1^2}{\alpha^2}
\end{aligned}\quad ;. \qquad (11)$$

In an asymptotically steady state (11) yields $A_{11} = \frac{\left(2\gamma_1\gamma_2\right)^{1/2}}{\alpha}$ and $A_{20} = \gamma_1/\alpha \approx \left(k_x L_N\right)^{-1}$. The saturation of gradient drift instability at the expense of nonlinear generation of even harmonics along the gradient of electron density corresponds to the soft mode of turbulence occurrence. In the typical for F-region conditions $A_{20} >> A_{11}$, then one obtains $A_{20} >> A_{11}$. Therefore the modes of even harmonics prevail in the power spectrum of density disturbances.

Numerical Results and Conclusions

Results of the numerical estimates of the average growth rate as a function of $A_{20} >> A_{11}$ for $\lambda = 0.1 L_N$, L_N and average parameters of polar ionosphere [Tereshchenko and Tereshchenko, 1998] are provided in Fig. 2. The dashed curves show the growth rate in the absent of Coulomb collisions, while the continuous curves show the growth rate when these collisions are included. The curves of Fig. 2 demonstrate the stabilizing influence of ion collisions.

Fig.3 shows the theoretical fluctuation level of long wavelength irregularities associated with gradient drift instability for the parameters used in Fig.2. We see that the collisions can noticeably reduce the amplitude of saturated electron density fluctuations. It is bound up with decrease of the growth rate $\lambda = 0.1 L_N$ under the influence of Coulomb collisions, and with increase of the coupling parameter α as $A_{20} >> A_{11}$ increases. The maximum fluctuation level occurs when $A_{20} >> A_{11}$ is about 50 mV/m. The peak amplitude of 1,4% agree well with experimental observations [Rognlien and Weinstock, 1974]. The power of irregularities, which is determined as $\left(\tilde{N}/N_0\right)^2$, depends on the scale $l_\perp = 2\pi/k_\perp$. In the considered case $A_{02} \approx k_x^{-1}$ and the saturated power spectrum of ionospheric irregularities obeys the law $\Phi\left(k_\perp\right) \approx k_\perp^{-2}$. This power law for the Fourier wave numbers was established for the ionospheric irregularities of different nature [Chaturvedi and Ossakow, 1979]. In order to determine the spectrum of irregularities, in reality, many modes should be considered. Thus, estimates given by (9) or (10) might represent overestimates.

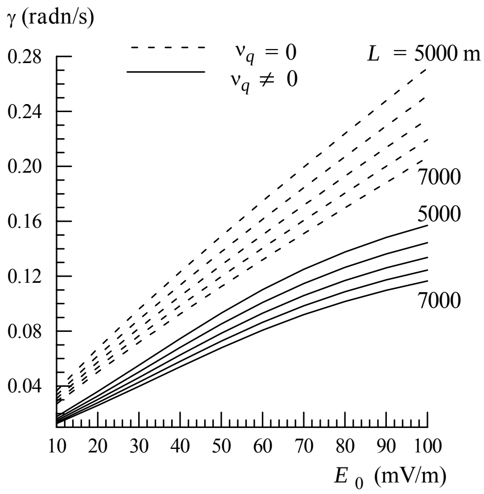

Fig.2. The average growth rate of the gradient drift instability.

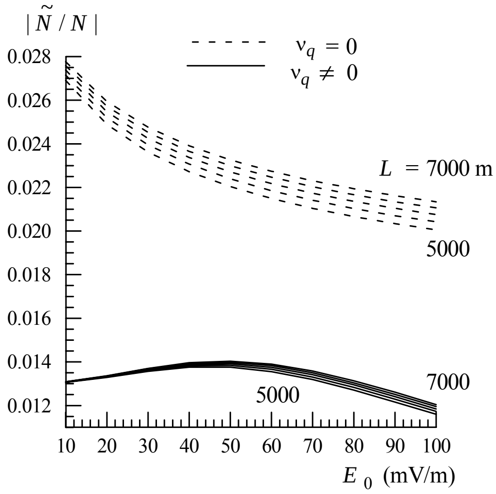

Fig.3. The level of density fluctuations.

Here we have considered the effects of Coulomb collisions and a strong electric field on the gradient drift instability in the high-latitude ionosphere. In the linear regime it is found that ion collisions have the stabilizing influence on the instability for modes with transverse scale sizes of several hundred meters or more. We show that the physical mechanism for saturation of the gradient drift instability is the nonlinear generation of even spatial harmonics in the direction of the zero-order density gradient. The saturated fluctuation level of these large-scale size structures (500-1000 m) could be of an order of few percent of the background values.

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