"Physics of Auroral Phenomena", Proc. XXIII Annual Seminar, Apatity, pp.25-28, 2000

Polar
Geophysical
Institute

SPONTANEOUS AND STIMULATED EVENTS IN SOC SYSTEM AND THEIR ANALOGY WITH SUBSTORM ONSETS

B.V.Kozelov and T.V.Kozelova *(Polar Geophysical Institute, Apatity, 184200 Russia, E-mail: kozelov@pgi.kolasc.net.ru)*

Abstract. The types of substorm onset stimulation have been classified by analogy with dynamic of the model system which has a self-organized critical (SOC) state. Spontaneous, locally stimulated, and globally stimulated events have been considered. We suppose that the strength of the IMF Bs determines the maximum energy, which the magnetosphere can hold in it and the rate of energy accumulation in the magnetosphere. In this model the energy which may be emitted during substorm depends on the history of energy accumulation. The statistical results about IMF B_z features near substorm onset have been explained, but it was found that such results cannot be used as a substantiation of spontaneous or stimulated nature of substorm onsets. It was found that the probability of onset stimulation must depend on value of IMF B_z deviation. It was noted that statistical investigation of SOC system may lead to erroneuos deductions.

Introduction

By analogy with simple model of dynamic system, which may be self-organized in critical state (Bak, 1997; Uritsky, Pudovkin, 1998), we try to classify the substorm onsets as transitions in such a system. Non-linear response of the system on different changes of external (relative to the system) parameters was analysed and dependence of this response to the system dynamic history was demonstrated. Moreover, supposing that the system is driven by natural B_z IMF, there was constructed the statistical distribution of B_z IMF near onset of substorm analogues. The main aim of the work is to reproduce by this analogy some features of magnetospheric substorms: spontaneous or stimulated nature of onsets, non-linear response of magnetosphere on external influence, and statistical features of B_z IMF near substorm onset.

SOC model

Detailed description of the idea of self-organized criticality (SOC) may be found in papers (Bak, 1997). Further we will imply the numerical model based on analogy, which mainly coincides with the one used in (Uritsky and Pudovkin, 1998). In this model the magnetospheric current sheet is represented as a rectangular array of cells. One can consider that each cell corresponds to the magnetospheric region of the scale about the characteristic scale of fluctuation in the magnetospheric plasma. Each cell with coordinates *i* and *j* at the moment *t* may store some energy $E_t(i,j)$. The time in the model is a discrete variable too, and at each time step the energy *dE* adds to the energy stored in a random cell of array. Following the usual SOC algorithm, we suppose that while the energy $E_t(i,j)$ of the cell is less then some critical level E_{max}, this cell is stable. When the critical level is exceeded, the cell turns into excited state, and the energy of the cell is redistributed among four neighbouring cells by rules:

$$\text{for } E_t(i,j)<E_{max} \quad E_{t+1}(i,j) = E_t(i,j)$$
$$\text{for } E_t(i,j)>E_{max} \quad E_{t+1}(i,j) = 0,$$
$$E_{t+1}(i\pm1,j\pm1) = E_t(i\pm1,j\pm1) + E_t(i,j)/4$$

Thus, the only neighbouring cells are directly connected during the energy transmission in the system, and this connection is non-linear. Boundary conditions supposed in the model are zero.

Prolonged evolution with constant *dE* и E_{max} brings the system into a SOC state. In this state the average energy of cells does not exceed the E_{max} due to redistribution of the energy among the cells and leaves the system boundary. This redistribution is usually named 'avalanches', by analogy with a sand pile formation (Bak, 1997). Dynamics of the system in a SOC state were not considered here, nevertheless we use the term a "SOC system" to stress that this system may lead to a SOC state. In this paper we examine some transition processes in the system. Namely, the response of the system to a different kind of variation of external parameters *dE* and E_{max} have been discussed. In these considerations we bear in mind that E_{max} at each time step is a global parameter, that is the same for all cells. In this sense the *dE* is a local parameter, because at each time step its value influences only one cell.

By analogy of the SOC system with the magnetosphere, we will consider that avalanche in the SOC system corresponds to magnetospheric substorm, and the avalanches onset is an analogy of substorm onset. For several precisions, we consider that the ($-E_{max}$) value corresponds to *Bz* IMF, and *dE* value corresponds to the solar wind pressure or ($v{\cdot}B_z$). The full energy of SOC system E_{tot} may be used as an analogy of the energy stored in the magnetosphere, and the free energy of a SOC system - as AE index. Also we will use the saturation $\eta = E_{tot}/(N{\cdot}E_{max})$ to measure the stored energy which is an analogy of the angle χ (angle between the magnetic field line and the plane of solar-magnetospheric equator. Here *N* is the number of cells in the system.

The analogy of ($-E_{max}$) with Bz IMF based on the result of (Iyemori T., 1998), where it was found that the magnitude of the southward component of IMF determined the energy which may be stored in the magnetosphere. Here we mean only the magnetospheric energy which may be further released during substorms (Akasofu and Kamide, 1976).

Global and local events in a SOC system

Let us see the possible kinds of avalanche onsets in the SOC system discussed above. Even originally an empty system with constant dE and E_{max} is inevitably overfull during not more then $N^* E_{max}/dE$ steps of time, here N is the number of cells in the system. In this case the beginning of an avalanche can be named **truely spontaneous**, as it is not caused by any change of external parameters, and is the result of limitation of the internal characteristic of the system (its power volume).

If the system is already in the critical state at given constant dE and E_{max}, under reduction E_{max} it will turn out, that energy of some part of cells exceeds new value of E_{max}. It will result in transition of the system into the supercritical state, i.e. the large part of the system is simultaneously covered by an avalanche, therefore such event (avalanche) is possible to be named **globally stimulated**. As was shown in (Uritsky and Pudovkin, 1998), the quickly returning E_{max} to initial value after the beginning of an avalanche cannot altogether suppress the development of the avalanche. The critical state, that corresponds to the new value of E_{max}, is established when the system gets rid of the energy surpluses.

If the system is in a critical state at given constant dE and E_{max}, then the increase of E_{max} will result in transition of system into subcritical state, i.e. the occurrence of an avalanche becomes less probable.

Another situation can arise at the short-term change dE. The case is the most interesting, when at the background constant dE and E_{max} at one time step dE has a higher value. This local increase of dE stimulates any but only one cell of the system. The response of the system to such influence was discussed in paper (Uritsky and Pudovkin, 1998) as an analogue of spontaneous substorm. It is shown, that SOC system intensifies such local influence, i.e the energy of an arising avalanche exceeds by far the initial influence. It is possible to consider such event as **locally stimulated**, or **globally spontaneous**, since for all cells of the system except one the parameters dE and E_{max} do not vary.

The reduction of dE slows down the increase of the system energy, therefore it reduces the probability of occurrence of an avalanche at the following step.

Thus, using the analogy between dynamics of transients in a SOC system and preliminary-explosive phase of substorm, possible types of substorm onset can be classified. We want to mark a basic opportunity of truly spontaneous events, that is these events are not caused by any change of external parameters, but they are the result of the limited "volume" of the system. Also there are the globally stimulated events, caused by external influence simultaneously in all cells of the system, and locally stimulated, which from global point of view can be considered as spontaneous ones.

Probability of event stimulation

We shall consider in details a transient, arising n the considered system after the jump E_{max} from 0 up to some positive value. It is possible to consider such process as an analogue of the preliminary phase of substorm, which begins after the fast southward turn of B_z IMF. We shall try to find out the probability of an avalanche occurrence depending on time after this jump.

At E_{max}=0 the system is empty and the energy begins to be accumulated in the system after the jump E_{max} up to some constant value. Process of accumulation passes as follows: in the originally empty system, consisting of N cells, at each time step the energy dE $<< E_{max}$ is added in one of cells. The probability of hit of this energy dE for all cells is identical. Then, after a large enough number of steps k, the distribution of energy in cells of the system will be close to normal with the average value $k{\cdot}dE/N$ and the relative variance $\sim k^{1/2}$. It is obvious, that the closer is the average energy in cells to E_{max} (i.e. the saturation $\eta = E_{tot}/(N{\cdot}E_{max})$ is higher) and the larger is the variance, then it is more probable that at the following time step the energy of a cell will exceed E_{max} and an avalanche will rise (Fig.1a). As in the considered situation after a jump the parameters E_{max} and dE did not vary, such avalanche can be attributed to truely spontaneous ones (see the previous section).

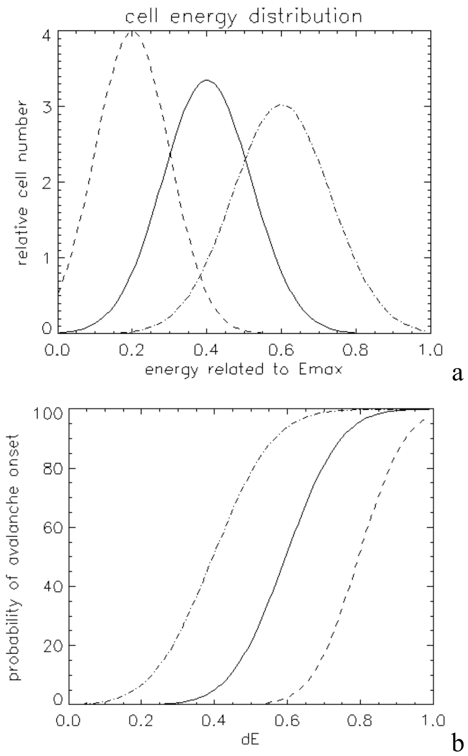

Fig.1. State of the SOC system for time steps $t_1<t_2<t_3$ (dashed, solid and dashed-dotted lines, respectively): a) energy distribution of cells; b) probability of avalanche onset depending on dE.

On the other hand, if the system is much far from being saturated (small η) and the distribution of the number of cells from the energy is described, for example, curve 1 on Fig.1a, then in such state the probability of avalanche occurrence is small. However this probability will be greater, if an increase of dE value will happen at the next step after moment t_1. In Fig.1b calculated probabilities of avalanche occurrence at the following step after moments of time $t_1 < t_2 < t_3$ are presented. It is seen, that stimulation of avalanche from the state with average energy farther from E_{max} can occur mainly at higher dE values, than from the state with the average energy closer to E_{max}. It is possible to note also, that the more is the system saturation (i.e. the more distribution in Fig.1a is shifted from left to right), than the more probable is the occurrence of an avalanche stimulated by same dE (Рис.1b).

The conclusion that the stimulation of an avalanche occurs at different size of external influence dE depending on the saturation of the system, agrees with the observation of real magnetospheric substorms. So, in paper (Kozelova et al., 1989) it was shown, that there was the dependence of size of external disturbance of B_z IMF, under which the substorm begins, on internal magnetosphere condition.

Statistics near onset

By the model of SOC system we shall try to study statistical behaviour of external parameter B_z IMF for onsets of typical insulated substorms. The typical substorm has a preliminary phase, duration of which T_1 is of the order of 30 minutes and which begins after the southward turn B_z (i.e. B_z<0). The insularity of substorm means, that during a sufficient long-duration T_2 time interval there was no disturbances (or they were insignificant, that is at B_z>0).

Now we believe, that both external parameters dE and E_{max} can vary and they both are operated by the real value of B_z IMF. For the sake of simplicity we believe, that

$$dE \sim \text{const} \times E_{max} \sim -Bz.$$

We shall consider conditions of occurrence of a large enough event (avalanche-substorm) in such a system. At $E_{max} \leq 0$ the energy of system $E_{tot} = 0$. At $E_{max} > 0$ system begins to be filled with energy. As $dE \sim$ const $\times E_{max}$, the condition of the beginning of an avalanche can be recorded in this way:

$$E_{tot}(t_0) \approx const \times \sum_{\tau=t_1}^{t_0} E_{\max}(\tau) \geq N\ E_{\max}(t_0) \qquad (1)$$

Here the summation will be carried out at "a preliminary phase", that is from the moment, when E_{max} became positive. Under the fulfilment of the condition (1) we consider, that an avalanche begins and the whole free energy of the system is dumped, therefore, at the following time step $E_{tot} = 0$ and the accumulation of energy begins at first.

Here we used five-minute values of B_z IMF from IMP-8 satellite during 1973-1974. From sequential temporary B_z data there were selected sets of B_z variations with specific durations of the preliminary phase T_1 and intervals T_2, when B_z was more than 0.

Fig.2a presents the averaged behaviour of B_z value (that is $-E_{max}$) about the moment of the onset. Cases with duration of the preliminary phase T_1>30 minutes were taken into account, and before the southward turn the B_z was positive not less then T_2=1.5 hours. Such "substorms" appeared 835. We shall note at once, that variation of T_1 and T_2 values does not change the qualitatively obtained curve, however the their introduction is necessary in order to locate an avalanche from transients at $E_{max} < 0$.

From Fig.2a one can see, that the averaged curve has a minimum, the value of which exceeds by far the statistical fluctuations, and this minimum occurs before the onset moment t_0. The averaged avalanche begins, when the growth rate B_z (fall E_{max}) is the greatest. We shall note, that the requirement (1), imposed when selecting events in this case is much weaker, than the requirement of reduction E_{max}, which obviously brings the system to a supercritical state. Nevertheless, the average behaviour E_{max} even for this condition demonstrates the fall of E_{max} (growth B_z) near t_0, thus the system gets into supercritical state.

Thus, when there is the continuous change of power consumption (E_{max}) of system and the rate of energy income (dE), the beginning avalanche can be related, after our definition, to stimulated events (globally or locally). Some delay between the moment of the minimum value of B_z and the beginning of avalanche t_0 can be connected to the fact that there was considered the transient of system from the empty state and before t_0 the system saturation was not critical. In Fig.2b for comparison we reproduce the statistical curves obtained in papers (Maltsev, 1998) and (Caan et al., 1978) by method of superimposed epoch for B_z IMF near the substorm onsets.

Comparison of the theoretical and experimental curves in Fig.2 shows the qualitative consent. Every curve has a region of minimum B_z values, and the minimum value of the average curve is reached before t_0 and values after this minimum a little below, rather than before it. We shall note, that in papers (Maltsev, 1998) and (Caan et al., 1978) the transfer time in the solar wind between the satellite and magnetopause was not calculated. In paper (Caan et al., 1978) after some suggestions the time between the beginning of the northward restoration of B_z IMF and time t_0 was estimated as about 10 minutes. On this basis the authors (Caan et al., 1978) have made a conclusion that there were triggerred substorms. In the paper (Maltsev, 1998) a conclusion was made, that the substorm had a spontaneous onset. Thus, after the results of statistical papers (Maltsev, 1998) and (Caan et al., 1978) it is impossible to authentically discuss the character of the substorm onset.

To demonstrate incorrect use of statistics, in Fig. 3 we present distributions of the average value of ($-E_{max}$) near avalanche onset obtained by different interval of averaging. Here we use the same set of avalanches as for Fig.2a. From this figure one can see the displacement of distribution toward B_z>0 with an increase of interval of averaging. In (Maltsev, 1998) a similar distribution of B_z near t_0 at averaging ±1 hour was interpreted as evidence that the part of substorms begins at the positive value of B_z. However for our model it is, obviously, only the effect of averaging.

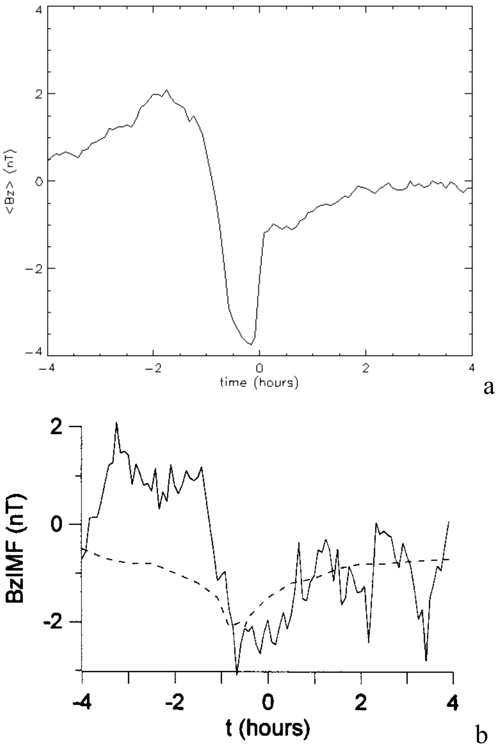

Рис.2. a). Average behavior of B_z near onsets of avalanches in SOC system. b). Average behaviour of B_z IMF near substorm onsets. Solid line - by (Maltsev,1998), dashed line - by (Caan et al.,1978).

Conclusions

1. Thus, using the analogy between dynamics of transients in a SOC system and preliminary-explosive phase of substorm, possible types of substorm onset can be classified. We want to mark the basic opportunity of truly spontaneous events, that is these events are not caused by any change of external parameters, but they are the result of the limited "volume" of the system. Also there are the globally stimulated events, caused by external influence simultaneously on all cells of the system, and locally stimulated ones, which from global point of view are possible to be considered as spontaneous ones.

2. It is shown, that there is the dependence of size of external disturbance, at which substorm begins, from internal state of magnetosphere in the sense that substorm can begin in a more filled magnetosphere under little external influence, and for stimulation of a less filled magnetosphere a stronger external influence is necessary. This conclusion agrees with earlier experimental results of paper (Kozelova et al., 1989).

3. Within the framework of the given analogy the explanations of statistical results of (Caan et al., 1978) and (Maltsev, 1998) concerning the behaviour of B_z IMF near substorm onset have been obtained. It is shown, that on the average dependence of B_z IMF the beginning of B_z growth occurs before the moment t_0 and this is not connected with stimulated or spontaneous character of the substorm. There is shown, a conclusion on the basis of statistics that a part of substorms that occur at positive Bz can be the result of averaging.

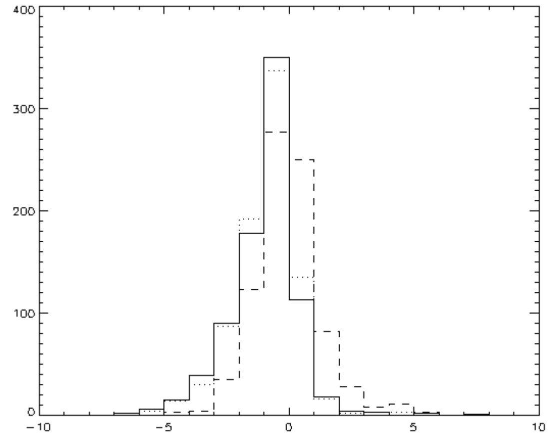

Fig.3. Distribution of the average value of ($-E_{max}$) near an avalanche onset. Solid line is calculated for ± 2 time steps (10 minutes), dotted line - for ±6 steps (30 minutes), dashed line - for ±36 steps (3 hours).

References

Akasofu, S.-I., Kamide, Y. (1976) Substorm energy, Planet.Space Sci., V.24, #3, P.223-227.

Bak P. (1997) How nature works. The science of self-organized criticality. Oxford University Press.

Caan M.N., McPherron R.L., Russell C.T. (1978) The statistical magnetic signature of magnetospheric substorms, Planetary and Space Science, V.26, #3, p.269-279.

Iyemori T. (1998) Substorm as a dissipation process in geomagnetic storms, SUBSTORM-4, Edited by S.Kokubun and Y.Kamide, Terra Scientific Publishing Company / Kluwer Academic Publishers. P.99-101.

Kozelova T.V., Pudovkin M.I., Lazutin L.L. (1989) Features of stimulated and spontaneous magnetospheric substorms obtained by satellite and groundbased data, Geomagnetizm i aeronomy, V. 18, #6, P.910-915 (in Russian).

Maltsev Yu.P. (1998) Are the substorm onsets triggered or spontaneous? Proc. XXI Annual Seminar "Physics of Auroral Phenomena", Apatity, PGI-98-03-106, pp.75-78.

Uritsky V.M., Pudovkin M.I. (1998) Low frequency 1/f-like fluctiations of the AE-index as a possible manifestation of self-organized criticality in the magnetosphere. Annales Geophysicae, V. 16, P.1580-1588.