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INTERCHANGE INSTABILITY IN PLASMA WITH CONTINUOUS DISTRIBUTION ON DRIFT VELOCITIES

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Abstract. As shown by *Volkov and Maltsev* [1986], the magnetosphere-ionosphere convection causes the interchange instability everywhere in the plasma sheet, even at its inner boundary. As a result, electric field and field-aligned current structures are formed. They are stretched approximately along the east-west direction. The paper by *Volkov and Maltsev* [1986] considered the magnetospheric plasma to be monoenergetic. Now we have taken into consideration the thermal spread. General dispersion equation for the interchange instability is obtained for an arbitrary distribution function. For special cases of rectangular and triangular functions, analytical solutions are found. Dependence of growth rate of the instability on the wave vector is investigated both for these two cases and for the case of Maxwellian distribution function. The time of development of the instability appeared to be about 5-10 minutes.

1. Introduction

Interchange instability in the magnetized plasma is analogous to the Rayleigh-Taylor instability in the hydrodynamics. The simplest case of the Rayleigh-Taylor instability is when a heavier liquid is situated above a lighter one (Fig. 1).

Gold [1959] have shown that the interchange instability evolves in the axially symmetrical magnetosphere under the following condition

$$\frac{d}{dr} pV^{\gamma} < 0 \tag{1}$$

where p is the plasma pressure, V is the volume of a tube with unit magnetic flux. According to (1), the outer boundary of a plasma belt is unstable (Fig. 2). *Swift* [1967] considered the interchange instability on the outer plasma sheet boundary as the causes of the poleward aurora bulge observed after break-up.

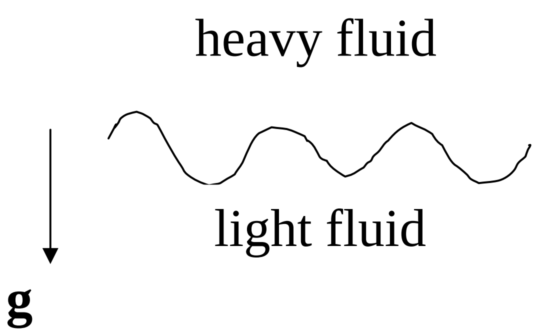

Fig. 1. Rayleigh-Taylor instability in the gravity field.

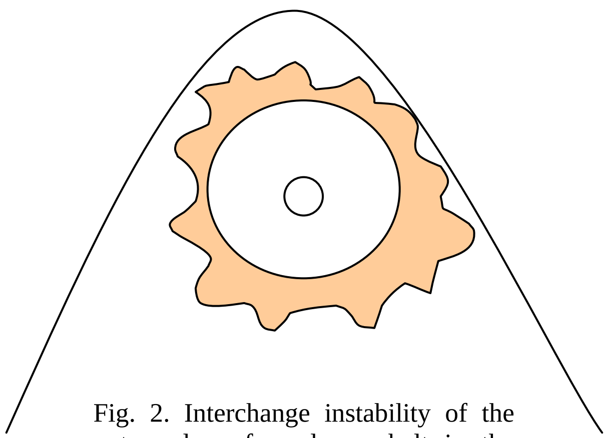

Fig. 2. Interchange instability of the outer edge of a plasma belt in the magnetosphere.

Volkov and Maltsev [1986] generalized (1) to a case of azimuthal asymmetry associated with the bulk magnetospheric convection. The instability was shown to evolve under the condition

$$([\mathbf{k} \times \mathbf{n}][\mathbf{k} \times \nabla(pV^{\gamma})]) < 0 \ , \tag{2}$$

where $\mathbf{k}$ is the wave vector, $\mathbf{n}$ is the unit vector from the curvature center of the magnetic field line. According to (2), the magnetosphere-ionosphere convection causes stratification of the magnetospheric plasma. As a result, electric field and field-aligned current structures are formed. They stretch approximately along the east-west direction. It is possible that this instability is responsible for generation of discrete auroral forms. The effect of finite conductivity along the magnetic field lines was investigated by *Volkov* [1988]. Structures with the scale of 10 km across magnetic field at the ionospheric level appeared to be mainly developed.

Conditions (1) and (2) are obtained for a case when drift velocities of different sorts of plasma particles are close. In reality the velocity spread can exceed the average velocity.

Ivanov and Pokhotelov [1987] studied the interchange instability in plasma consisting of several sorts of ions.
We consider the instability in plasma with continuous distribution of charged particles over drift velocities.

2. Dispersion equation

We will consider that characteristic time of the processes is larger then time of Alfven wave run between conjugate ionospheres. Plasma pressure is isotropic and constant along the magnetic field line. In this case system of the basic equations looks as follows:

$$j_z = -\frac{1}{V^\gamma}(\vec{e}_z[\nabla V \times \nabla f]) \qquad (3)$$

$$V = \int_0^l \frac{dz}{B}$$

$$j_z = -\mathrm{div}\hat{\Sigma}\vec{E} \qquad (4)$$

$$\vec{E} = -\nabla\varphi$$

$$\hat{\Sigma} = \Sigma_p$$

$$\frac{df}{dt} = 0 \qquad (5)$$

where $f = pV^\gamma$, V is the magnetic field tube volume of a unit cross-section at the ionospheric level, $\mathbf{e}_z$ is the unit vector along the magnetic field line, the current flowing from the ionosphere is considered positive. Equation (3) is obtained by *Vasyliunas* [1970] and *Tverskoy* [1982]. The unknown variables in the equations are considered as the sum of the undisturbed and disturbed values:

$$f = f_o + f_1\,, \quad \mathbf{v} = \mathbf{v}_o + \mathbf{v}_1$$

where $\mathbf{v}_1 = [\mathbf{E}_1\times\mathbf{B}]/B^2$ is the electric drift velocity disturbance. Variables change as $\exp(i\mathbf{kr} - i\omega t)$. Equation (5) may be rewritten as follows

$$-i\omega f_1 + (\mathbf{kv}_o) f_1 + (\mathbf{v}_1\nabla) f_0 = 0$$

Neglecting the disturbance of the magnetic field and linearizing system (3)-(5), we obtain the dispersion equation:

$$1 + \frac{([\mathbf{k}\times\nabla V][\mathbf{k}\times\nabla])}{k^2\Sigma_p V^\gamma B^2}\int\frac{f_0 dv}{-i\omega + i(\mathbf{kv})} = 0 \qquad (6)$$

3. Special cases

The cases of rectangular, triangular, and maxwellian distribution functions of particles over drift velocities will be considered below.

3.1. Rectangular distribution function

Substituting the rectangular distribution function

$$f_0 = \frac{pV^\gamma}{v_2 - v_1} \quad \text{for} \quad v_1 \le v \le v_2$$

$$f_0 = 0 \quad \text{for} \quad v < v_1\,,\ v > v_2$$

into equation (6), we obtain the following expression:

$$\omega = \frac{\mathbf{k}(\mathbf{v}_2 + \mathbf{v}_1)}{2} + i\frac{\mathbf{k}(\mathbf{v}_2 - \mathbf{v}_1)}{2}\cot\frac{\mathbf{k}(\mathbf{v}_2 - \mathbf{v}_1)}{2g_0} \qquad (7)$$

where

$$g_0 = \frac{([\mathbf{k}\times\nabla V][\mathbf{k}\times\nabla(pV^\gamma)])}{k^2\Sigma_P V^\gamma B^2}, \qquad (8)$$

According to (7), the plasma is unstable under any **k**. Sometimes the growth rate gets infinite. One can see that

$$\text{Im}\,\omega \to \infty \text{ when } \mathbf{k}(\mathbf{v}_2 - \mathbf{v}_1)/(2g_0) \to n\pi; \quad n = 1, 2, 3\ldots$$

The bold line in Fig. 3 shows the dependence of the growth rate (in s^{-1}) on the angle α for the rectangular distribution function, α being the angle between ∇V and **k**. The angle between ∇V and ∇pV^{γ} is assumed to be $\pi/9$. The gradient drift velocity is perpendicular to ∇V. We have performed calculations for the following parameters of the magnetospheric plasma: $k = 10^{-2}$ km^{-1}, p = 10 nPa, $V = 10^7$ km/T, $B = 0.3 \cdot 10^{-4}$ T, Σ_p = 1 S, $\nabla = 10^{-3}$ km^{-1}, v_1 = 1 km/s, v_2 = 2 km/s.

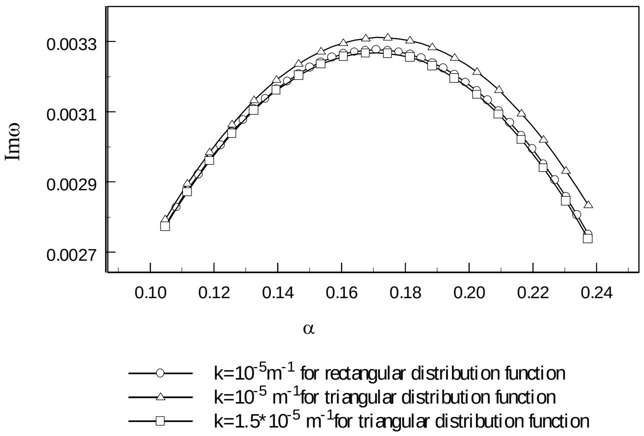

Fig. 3. The dependence of the growth rate on the angle between ∇V and **k** for the rectangular and triangle distribution functions.

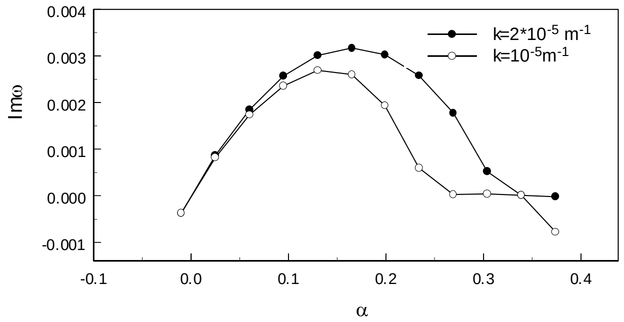

Fig. 4. The same as in Fig. 3, but for the Maxwellian distribution function.

When w_2 tends to w_1 the growth rate of instability becomes equal to g_0. It coincides with that obtained by *Volkov and Maltsev* [1986].

3.2. Triangular function of distribution

Such a function looks like

$$f_0 = \frac{2pV^{\gamma}(v - v_1)}{(v_3 - v_1)(v_2 - v_1)} \quad \text{for} \quad v_1 \le v \le v_2$$

$$f_0 = \frac{2pV^{\gamma}(v_3 - v)}{(v_3 - v_1)(v_3 - v_2)} \quad \text{for} \quad v_2 \le v \le v_3$$

$$f_0 = 0 \quad \text{for} \quad v < v_1, v > v_3$$

Substituting this function into (6), we obtain

$$1 - \frac{2ig_o}{(\mathbf{kv}_3) - (\mathbf{kv}_1)}\left[\frac{(\mathbf{kv}_1) - \omega}{(\mathbf{kv}_2) - (\mathbf{kv}_1)}\log\frac{(\mathbf{kv}_2) - \omega}{(\mathbf{kv}_1) - \omega} - \frac{(\mathbf{kv}_3) - \omega}{(\mathbf{kv}_3) - (\mathbf{kv}_2)}\log\frac{(\mathbf{kv}_3) - \omega}{(\mathbf{kv}_2) - \omega}\right] = 0 \tag{9}$$

It is not difficult to show that Im ω is finite always in this case.

We assumed v_1=0, v_2=1 km/s,v_3=2 km/s for calculations. The calculated growth rate is shown in Fig. 3 for the same parameters as in the case of the rectangular distribution function. The behavior of the growth rate is similar to that for the rectangular distribution function for the chosen parameters. The maximum of the growth rate instability is achieved for wave numbers directed along the bisector of the angle formed by ∇V and ∇pV^{γ}, i.e. disturbances are extended approximately in the west-east direction, this coincides with the result obtained by *Volkov and Maltsev* [1986]. The characteristic time of development of the instability is 5-10 minutes according to the calculations.

3.3. Maxwellian distribution function

The numerical calculation of the growth rate for the Maxwellian distribution function was carried out as follows. Let's define ω as $\omega = x + iy$. Substituting ω into the dispersion equation and dividing the real and imaging parts we shall receive the following system of the equations:

$$\int_0^{\infty} \frac{\upsilon^2 (x+(\mathbf{kv})) e^{-\frac{\upsilon^2}{\upsilon_0^2}} d\upsilon}{(x+(\mathbf{kv}))^2 + y^2} = 0 \ ,$$

$$1 + \frac{([\mathbf{k}\times\nabla V][\mathbf{k}\times\nabla p V^{\gamma}])}{k^2 \Sigma_p V^{\gamma} B^2} \frac{4}{\sqrt{\pi}} \int_0^{\infty} \frac{y \upsilon^2 e^{-\frac{\upsilon^2}{\upsilon_0^2}} d\upsilon}{(x+(\mathbf{kv}))^2 + y^2} = 0 \ ,$$

We solve this system by the iteration method. At first we substitute $x = 0$ into the first equation and find y, then we correct x from the second equation and so on. In Fig. 4, the dependence Im ω on α is shown. The calculations have been carried out for the same magnetospheric parameters and v_0= 1 km/s. The obtained results differ insignificantly from the results for the triangular distribution function.

4. Discussion

According to (2) and (6), favorable conditions for the interchange instability take place throughout the magnetosphere. This instability was attracted earlier for explanation of auroral arcs [*Volkov and Maltsev*, 1986; *Volkov*, 1988] as well as poleward auroral bulge [*Swift*, 1967; *Ivanov and Pokhotelov*, 1987].

Another possible effect of the instability is the observed stratification of the convective flow in the magnetosphere. The flow is not homogeneous. *Angelopoulos et al*. [1992, 1994]. Reports that the convection is bursty. The plasma in the plasma sheet is practically motionless for the most part of time. Short-time (~3 min) bursts of the fast (~500 km/s) earthward flow are detected with the probability 1.5-4 % [*Shiokawa et al*., 1997]. The bursts can be considered as fast narrow convective jets [*Maltsev and Golovchanskaya*, 1996]. Field-aligned currents must flow at the edges of the jets. The upward current can be responsible for the discrete auroral arc.

5. Conclusions

The dispersion equation for interchange instability in plasma with continuous distribution on the drift velocities is obtained. The cases of rectangular, triangular, and Maxwellian distribution functions are considered. As a result of the development of the interchange instability structures of the electric fields and field-aligned currents are formed. They stretch approximately along the east-west direction. The characteristic time of the instability development is about 5 minute.

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