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SEARCH OF A GOOD APPROXIMATION FOR THE MAGNETIC FIELD IN THE MAGNETOSPHERE

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Abstract. Numerous models of the magnetospheric magnetic field suggest the field approximation, which seems to be unsatisfactory. We used 37,000 experimental data in order to find an approximation of the magnetic field at distances $x > -40\ R_E$. The model field depends on the *Dst*, *Kp* indices, solar wind dynamic pressure, vertical IMF component, and dipole tilt angle. The approximation was carried out in two stages. At the first stage, the field was found in regions $|z| > 3\ R_E$ where electric currents are practically absent and the magnetic field is almost curl-free. The external field was presented as a sum of fields of simple distant sources. At the second stage, the field at the electric current layer $|z| < 3\ R_E$ was found by interpolation, the fields at $|z| = 3\ R_E$ being chosen as boundary conditions. Distribution of the field along *x* and *y* directions inside of the current layer was found by fitting to the experimental data. Criterions of correctness of the approximation are a small residual error and similarity of the calculated magnetic configuration with results of a direct magnetic tracing performed with the use of the experimental data.

1. Introduction

The magnetic field in the magnetosphere is the sum

$$\mathbf{B} = \mathbf{B}^{int} + \mathbf{B}^{ext} \tag{1}$$

where $\mathbf{B}^{int}$ is the field of the internal electric currents flowing inside the Earth, $\mathbf{B}^{ext}$ is the field of the external currents flowing in the magnetosphere. The field $\mathbf{B}^{int}$ is rather stable. Its variations for several years do not exceed one per cent. The field $\mathbf{B}^{ext}$ is very variable. It can change by 100 per cent for a few hours.

The main problem in the magnetic field modeling is to find a good approximation for the spatial distribution of $\mathbf{B}^{ext}$ as well as to understand how various geophysical parameters affect this distribution.

The external field is produced by four kinds of electric currents and correspondingly can be expressed as follows

$$\mathbf{B}^{ext} = \mathbf{B}_{mp} + \mathbf{B}_{rc} + \mathbf{B}_{ct} + \mathbf{B}_{fa} \tag{2}$$

where the right hand side of (2) contains the fields of the currents on the magnetopause, ring current, cross-tail current, and field-aligned currents, respectively. All these currents arise due to the solar wind-magnetosphere interaction, the nature of which is complicated so that the value of the currents depend on prehistory of the solar wind parameters for several previous hours.

Table 1 shows evolution of magnetic field models. Here P_{sw} is the solar wind dynamic pressure, B_y and B_zIMF are the corresponding components of the interplanetary magnetic field in GSM coordinates, ψ is the Earth dipole tilt angle, *Dst, Kp, AE,* and *AL* are geomagnetic indices, r_s is the geocentric distance to the subsolar point at the magnetopause, Λ_o is the latitude of the midnight equatorward boundary of the diffuse aurora, Λ_{ae} is the midnight latitude of the auroral electrojet maximum.

Table 1. Models of the magnetospheric magnetic field.

#	Author(s)	Input parameters (or conditions of validity)	Form of presentation of the field	Region of validity
1	*Hones* [1963]		Two dipoles	$r < 20\ R_E$
2	*Mead* [1964]	P_{sw}	$\mathbf{B}^{ext} = \mathbf{B}_{mp}$	$r < 20\ R_E$
3	*Williams and Mead* [1965]	quiet conditions	$\mathbf{B}^{ext} = \mathbf{B}_{mp} + \mathbf{B}_{ct}$	$r < 30\ R_E$
4	*Antonova and Shabansky* [1968]		Two dipoles + $\mathbf{B}_{ct}$	$r < 20\ R_E$
5	*Olson and Pfitzer* [1974]	quiet conditions	$\mathbf{B}^{ext} = \mathbf{B}_{mp} + \mathbf{B}_{rc} + \mathbf{B}_{ct}$	$r < 40\ R_E$
6	*Mead and Fairfield* [1975]	ψ, *Kp*	Polynomials of the 2nd order of r	$r < 17\ R_E$
7	*Tsyganenko and Usmanov* [1982]	ψ, *Kp*	$\mathbf{B}^{ext} = \mathbf{B}_{mp} + \mathbf{B}_{rc} + \mathbf{B}_{ct}$	$r < 30\ R_E$
8	*Tsyganenko* [1987]	*Kp*	$\mathbf{B}^{ext} = \mathbf{B}_{mp} + \mathbf{B}_{rc} + \mathbf{B}_{ct}$	$r < 40\ R_E$
9	*Tsyganenko* [1989]	ψ, *Kp*	$\mathbf{B}^{ext} = \mathbf{B}_{mp} + \mathbf{B}_{rc} + \mathbf{B}_{ct}$	$r < 40\ R_E$
10	*Hilmer and Voight* [1995]	ψ, *Dst*, Λ_o, r_s	$\mathbf{B}^{ext} = \mathbf{B}_{mp} + \mathbf{B}_{rc} + \mathbf{B}_{ct}$	$r < 50\ R_E$
11	*Tsyganenko* [1995, 1996]	ψ, *Dst*, P_{sw}, B_y, B_zIMF	$\mathbf{B}^{ext} = \mathbf{B}_{mp} + \mathbf{B}_{rc} + \mathbf{B}_{ct} + \mathbf{B}_{fa}$	$r < 80\ R_E$
12	*Alexeev et al.* [1996]	ψ, *Dst*, *AL*, P_{sw}, Λ_{ae}	$\mathbf{B}^{ext} = \mathbf{B}_{mp} + \mathbf{B}_{rc} + \mathbf{B}_{ct} + \mathbf{B}_{fa}$	$r < 40\ R_E$
13	*Ostapenko et al.* [1996]	*Dst, Kp, AE*, P_{sw}, B_zIMF	Polynomials of the 4th order of r	$3 < r < 10\ R_E$
14	*Ostapenko and Maltsev* [1997]	ψ, *Dst*, *Kp*, P_{sw}, B_zIMF	Polynomials of the 4th order of r	$3 < r < 10\ R_E$
15	*Feshchenko et al.* [1999]	ψ, *Dst*, *Kp*, P_{sw}, B_zIMF	Empirical field lines	$r < 40\ R_E$

First simplest models [*Hones*, 1963; *Mead*, 1964; *Williams and Mead*, 1965; *Antonova and Shabansky*, 1968] were used for rough estimates only. More complicated models [*Olson and Pfitzer*, 1974; *Mead and Fairfield*, 1975] were fitted to available magnetic measurements however the fitting technique was very simplified. More sophisticated models [*Tsyganenko and Usmanov*, 1982; *Tsyganenko*, 1987, 1989] were based on rather realistic suppositions about the spatial distribution of fields and currents however were insufficiently parameterized. The most developed models [*Hilmer and Voight*, 1995; *Tsyganenko*, 1995, 1996; *Alexeev et al*., 1996; *Ostapenko et al.*, 1996; *Ostapenko and Maltsev*, 1997] are fitted to a large database and depend on a large number of geophysical parameters. Field lines in the model by *Feshchenko et al*. [1999] are built directly from experimental data.

The fitting accuracy can be expressed with the help of residual sum of squares

$$RSS = \sum_{n=1}^{N} (\mathbf{B}_{\text{mod}}^{ext} - \mathbf{B}_n^{ext})^2 / \sum_{n=1}^{N} (\mathbf{B}_n^{ext})^2 \tag{3}$$

where $\mathbf{B}_{\text{mod}}^{ext}$ is the model field, $\mathbf{B}_n^{ext}$ is the field observed at the nth measurement, N is the total number of measurements in the region under investigation. The observed field can be taken from the database by *Fairfield at al*. [1994] that contains 79,000 three-component magnetic measurements at distances from 3 to 60 R_E. The model by *Tsyganenko* [1995, 1996] yields RSS = 0.37 at distances $x > -40\ R_E$ and RSS = 0.257 at distances from 3 $R_E < r < 10\ R_E$. The model by *Ostapenko and Maltsev* [1997] yields RSS = 0.20 at distances from 3 to 10 R_E.

The aim of this paper is to obtain a better approximation for the magnetic field at distances $x > -40\ R_E$ than that in the model by *Tsyganenko* [1995, 1996].

2. Description of the model

We assume that all electric currents are localized on the magnetopause and in the near-equatorial layer shown as a shaded area in Fig. 1. The layer is determined as follows

$$|z_{GSM} - z_{ns}| < 3\ R_E \tag{4}$$

where z_{ns} is the GSM coordinate of the neutral sheet determined from the paper by *Peredo et al*. [1993].

The condition curl **B** = 0 outside of the current layer allows to express the magnetic field as follows

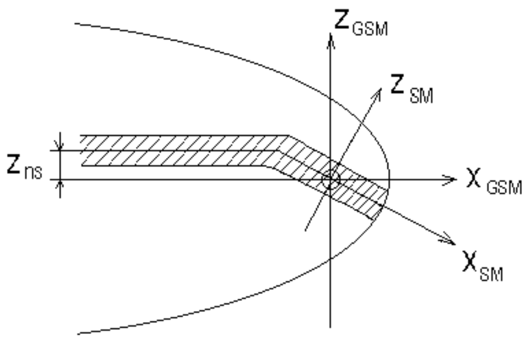

Fig. 1. The electric current layer (shaded) near the equatorial plane.

$$\mathbf{B}^{\text{outside}} = -\nabla\Psi \qquad \text{for } |z_{GSM} - z_{ns}| > 3\ R_E \tag{5}$$

where Ψ is the scalar magnetic potential. Since **B** is divergence-free we have $\Delta\ \Psi = 0$. A sum of multipole fields satisfies this equation. To find the field in the northern hemisphere, we have located the multipoles in the following three points:

$$x_1^{\text{GSM}} = 50\ R_E,\ y_1 = 0,\ z_1^{\text{GSM}} = 0;$$
$$x_2^{\text{SM}} = -2.5\ R_E,\ y_2 = 0,\ z_2^{\text{SM}} = -6\ R_E;$$
$$x_3^{\text{GSM}} = 200\ R_E,\ y_3 = 0,\ z_3^{\text{GSM}} = -30\ R_E;$$

To find the field in the southern hemisphere, one should replace $\mathbf{z_n}$ by $-z_n$. Correspondingly, the potential is the sum:

$$\Psi = \Psi_1 + \Psi_2 + \Psi_3$$

where the each potential Ψ_n is produced by sources located in the point x_n, y_n, z_n. As sources we assumed one monopole q_n and two dipoles p_{nx} and p_{nz} oriented along x and z axes. Each potential is

$$\Psi_n = \frac{q_n}{r_n} + p_{nx}\frac{x - x_n}{r_n^3} + p_{nz}\frac{z - z_n}{r_n^3} \tag{6}$$

where n = 1, 2, 3; $r_n = [(x - x_n)^2 + (y - y_n)^2 + (z - z_n)^2]^{1/2}$ is the distance to the source. For n = 2 and 3, both the monopoles q_n and dipoles p_nx and p_nz were presented as the following linear combination of the geophysical parameters

$$a_{nx} = a_{n\text{xo}} + p_{\text{nxDst}}\ \widetilde{Dst} + a_{\text{nxKp}}\ \widetilde{Kp} + a_{\text{nxPsw}}\ \widetilde{P}_{sw} + a_{\text{nxZIMF}}\ \widetilde{Z}_{IMF} + a_{\text{nx}\psi} \sin\psi \tag{7}$$

where the values with the tilde

$$\widetilde{Dst} = \frac{Dst + 16}{25},\ \widetilde{Kp} = \frac{Kp - 2.3}{1.3},\ \widetilde{P}_{sw} = \frac{P_{sw} - 2.2}{1.9},\ \widetilde{Z}_{IMF} = \frac{Z_{IMF} - 0}{3.7} \tag{8}$$

are the normalized geophysical parameters. The numerator of each fraction is the difference between the value of every parameter and its average value, the denominator is the standard deviation of this parameter. For n = 1, we assumed q_1 = 0, $p_{1\text{x}} = a_{1\text{x}} \sin\psi$, $p_{1\text{z}} = a_{1\text{z}} \cos\psi$ (positive ψ corresponds to summer conditions in the northern hemisphere). The coefficients a_{ni} were fitted from the database of *Fairfield et al*. [1994] by the least squares technique.

The field inside of the current layer (4) was presented as follows

$$\mathbf{B}^{\text{inside}} = \mathbf{B}^{\text{interp}} + \mathbf{B}^{\text{suppl}} \tag{9}$$

The field $\mathbf{B}^{\text{interp}}$ resulted from linear (with respect to z) interpolation of the outside field (5). The supplementary field $\mathbf{B}^{\text{suppl}}$ was expressed as

$$B_x^{suppl} = z(9-z^2)\left[s_{x0} + \frac{s_{x1}}{(x-10)^2+y^2+z^2+100} + \frac{s_{x2}}{(x+5)^2+y^2+z^2+100} + \frac{s_{x3}}{(x-20)^2+y^2+z^2+400}\right] \quad (10)$$

$$B_y^{suppl} = yz(9-z^2)\left[s_{y0} + \frac{s_{y1}}{(x-10)^2+y^2+z^2+100} + \frac{s_{y2}}{(x+5)^2+y^2+z^2+100} + \frac{s_{y3}}{(x-20)^2+y^2+z^2+400}\right] \quad (11)$$

$$B_z^{suppl} = (9-z^2)\left[s_{z0} + \frac{s_{z1}}{(x-10)^2+y^2+z^2+100} + \frac{s_{z2}}{(x+5)^2+y^2+z^2+100} + \frac{s_{z3}}{(x-20)^2+y^2+z^2+400}\right] \quad (12)$$

The coefficients s_{ik} are also presented in the form similar to (7). They were fitted from the database of *Fairfield et al.* [1994]. Table 2 contains the computed values of coefficients relating q, p, and s with the geophysical parameters.

Table 2. The relation coefficients

A	a_0	a_{Dst}	a_{Kp}	a_{Psw}	a_{IMFz}	a_ψ
q_1	0	0	0	0	0	0
$p_{1z}/\cos\psi$	-2.69E+01	-1.30E+00	2.01E+00	-9.50E+00	-2.30E+00	-1.39E+01
$p_{1x}/\sin\psi$	1.69E+01	-1.55E+00	2.12E-01	1.46E+00	-5.21E-01	2.54E+00
q_2	-1.59E-02	1.69E-03	-3.63E-03	5.05E-03	4.13E-04	-1.30E-02
p_{2z}	-1.11E-01	3.43E-02	1.05E-02	-4.12E-02	-7.96E-03	3.57E-02
p_{2x}	1.66E-03	4.53E-02	5.39E-04	1.08E-02	1.41E-03	-8.94E-02
q_3	-5.94E+00	-4.58E+00	6.29E+00	6.66E+00	-4.02E+00	-3.51E+01
p_{3z}	6.69E+02	1.04E+02	-2.71E+02	1.47E+02	8.48E+01	1.48E+03
p_{3x}	2.42E+02	-5.10E+02	7.51E+02	9.34E+02	-5.04E+02	-4.35E+03
s_{x0}	6.68E-01	2.69E-01	3.00E-01	-1.25E-01	-3.95E-01	-4.71E-01
s_{x1}	2.31E+02	1.86E+02	8.34E+01	-8.34E+01	-1.77E+02	-3.00E+02
s_{x2}	1.65E+02	-4.14E+00	-5.85E-01	3.13E+01	-1.60E+01	-6.50E+01
s_{x3}	-2.36E+03	-7.98E+02	-6.17E+02	2.47E+02	1.14E+03	1.91E+03
s_{y0}	-3.37E-02	-3.50E-02	9.62E-03	-2.26E-02	-1.11E-02	1.10E-02
s_{y1}	-3.91E+01	6.29E+00	1.53E+01	-1.32E+01	-1.63E+01	-5.31E+01
s_{y2}	-2.43E+01	2.04E+01	-7.10E+00	-6.85E-01	-1.22E+00	-2.63E+01
s_{y3}	2.33E+02	-3.91E+01	-2.45E+01	7.02E+01	6.31E+01	2.18E+02
s_{z0}	5.51E-01	2.65E-01	1.63E-02	4.17E-01	-4.13E-02	2.43E-01
s_{z1}	-3.32E+02	2.15E+02	-7.78E+01	2.17E+02	-1.19E+02	-2.47E+02
s_{z2}	-3.62E+02	3.94E+01	-1.02E+02	-1.33E+01	-5.04E+01	3.50E+01
s_{z3}	2.11E+03	-1.12E+03	7.34E+02	-1.10E+03	6.00E+02	3.64E+02

3. Discussion

The residual sum of squares calculated with the help of (3) appeared to be RSS = 0.245. That is less than RSS = 0.371 in the Tsyganenko [1995, 1996] model. Examples of magnetic field lines in our model are shown in Fig. 2 for average and storm conditions. The dipole tilt angle is zero. The lines are drawn in the noon-midnight meridian plane through 2° of latitude, beginning from 60°.

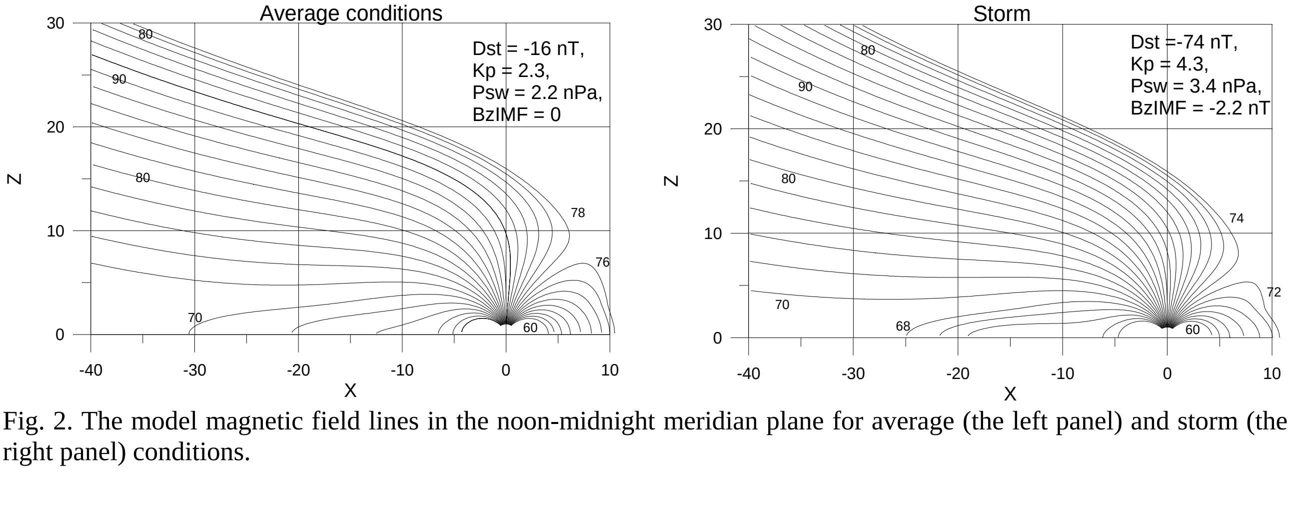

Fig. 2. The model magnetic field lines in the noon-midnight meridian plane for average (the left panel) and storm (the right panel) conditions.

The paper by *Feshchenko et al.* [1999] gives the purely empirical magnetic configuration for the same conditions as in Fig. 2 and compares it with the *Tsyganenko* [1995, 1996] model for these conditions. Our model under average conditions is similar to the both mentioned models. As for the storm conditions, our model appeared be intermediate in the dayside sector. For instance, the dayside polar cusp in our model is located at the latitude of 73°, whereas it is at 69° in the model by *Feshchenko et al.* [1999] and at 75° in the model by *Tsyganenko* [1995, 1996]. In the nightside, the model by *Feshchenko et al.* [1999] appears to be intermediate. It shows fast subsiding of the field with distance from $x = -5\ R_E$ to $x = -10\ R_E$ in the equatorial plane. The *Tsyganenko* [1995, 1996] model predicts much slower subsiding at distances from $x = -5\ R_E$ to $x = -40\ R_E$. Our Fig. 2 (the right panel) shows a minimum of the magnetic field at $x \approx -(10\text{-}15)\ R_E$. We think this is a defect of the model.

Another disadvantage of our model is non-zero divergence of the magnetic field in the current layer shown by the shaded area in Fig. 1. However it seems to be rewarded by the comparatively small residual error.

4. Conclusion

The magnetic field approximation is obtained for distances $x > -40\ R_E$. The approximation depends on the *Dst*, *Kp* indices, solar wind dynamic pressure, IMF *z* component, and dipole tilt angle. The approximation yields the residual error $RSS = 0.245$, which is smaller than $RSS = 0.371$ in the *Tsyganenko* [1995, 1996] model.

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