PRESSURE ANISOTROPY EFFECT IN THE MAGNETOSPHERIC FIELD-ALIGNED CURRENT DENSITY

O. V. Mingalev and I. V. Mingalev (*Polar Geophysical Institute, Apatity, Russia*)

Abstract. Formula for the parallel current density in the case of magnetohydrostatic plasma with anisotropic pressure is derived. This formula is deduced from the magnetohydrostatic equations without any additional assumptions. This formula expresses the parallel current density in a fixed point of the Earth's magnetosphere only in terms of the parameters occurring in the system of magnetohydrostatic equations. In the case of isotropic pressure the derived formula coincides with the well-known Vasyliunas–Tverskoy formula.

1. Introduction

In the case of plasma with isotropic pressure *Vasyliunas* [1970] and *Tverskoy* [1982] derived the formula for the parallel current density from the magnetohydrostatic equations. This formula expresses the parallel current density in a fixed point of the Earth's magnetosphere in terms of pressure gradient in this point and distribution of the absolute value of magnetic field along the magnetic field line. Meanwhile, it is well-known that the pressure tensor is essential anisotropic with ratio $p_\perp/p_\| \approx 2$ in the Earth's magnetosphere at the distance from the Earth less then $10R_E$ [*Lui and Hamilton*, 1992]. In this paper we consider the problem of generalization of the Vasyliunas–Tverskoy formula for the case of anisotropic pressure. It should be noted that in papers by *Heinemann* [1990] and *Heinemann and Pontius* [1991] the formula for the parallel current density were deduced from the magnetohydrostatic equations for plasma with anisotropic pressure and additional assumptions about special empirical representations for parallel and orthogonal pressure. As a result of this additional assumptions the deduced formulae for the parallel current density contain some empirical parameters which don't occur in the input system of one-fluid nondissipative magnetohydrodynamic equations.

In this paper we deduced the formula for the parallel current density only from the magnetohydrostatic equations for plasma with anisotropic pressure without any additional assumptions. As a result we express the parallel current density only in terms of the parameters occurring in the input system of one-fluid nondissipative magnetohydrodynamic equations.

2. The basic equations

The system of one-fluid nondissipative magnetohydrodynamic equations [*Chew et al.*, 1956] can be considered for plasma with both isotropic pressure and anisotropic pressure. Equation of motion of this system has the following form:

$$\rho\left(\frac{\partial \mathbf{v}}{\partial t} + (\mathbf{v}\,,\nabla)\mathbf{v}\right) = \operatorname{div}\widehat{\mathbf{P}} + [\,\mathbf{j}\,,\mathbf{B}\,]\,. \tag{1}$$

Here ρ is the mass density of plasma; $\mathbf{v}$ is the hydrodynamic velocity of plasma; $\mathbf{j}$ is the vector of electric current density; $\widehat{\mathbf{P}}$ is the pressure tensor; $\mathbf{B}$ is the magnetic induction. If the inertial terms, situated in the left–hand side of the equation (1) are small in comparison with the terms, situated in the right–hand side of this equation, one can consider the magnetohydrostatic equation

$$[\,\mathbf{j}\,,\mathbf{B}\,] = -\operatorname{div}\widehat{\mathbf{P}} \tag{2}$$

as a zero approximation for the equation (1). In the case of plasma with anisotropic pressure the pressure tensor has the form

$$\widehat{\mathbf{P}} = -p_\perp\widehat{\mathbf{I}} - \left(p_\| - p_\perp\right)\mathbf{b}\otimes\mathbf{b}\,, \tag{3}$$

where $\widehat{\mathbf{I}}$ is the identity tensor; $\mathbf{b} = \mathbf{B}/B$ is the unit vector in direction of the $\mathbf{B}$; $B = |\,\mathbf{B}\,|$; $\mathbf{b}\otimes\mathbf{b}$ is the dyadic tensor, formed by the vector $\mathbf{b}$; $p_\|$ and $p_\perp$ are parallel and orthogonal plasma pressure, respectively. From (3) it follows that

$$\operatorname{div}\widehat{\mathbf{P}} = -\nabla_{\!\perp} p_\perp - \left(p_\| - p_\perp\right)\left[\,(\mathbf{b}\,,\nabla)\mathbf{b} + \mathbf{b}\operatorname{div}\mathbf{b}\,\right] - \mathbf{b}\left(\mathbf{b}\,,\nabla p_\|\right)\,, \tag{4}$$

where $\nabla_{\!\perp} = \nabla - \mathbf{b}\,(\mathbf{b}\,,\nabla)$ is the orthogonal to the $\mathbf{B}$ component of the gradient operator ∇. For convenience, we denote by

$$\mathbf{a} = -\operatorname{div}\widehat{\mathbf{P}} + \mathbf{b}\,(\,\mathbf{b}\,,\operatorname{div}\widehat{\mathbf{P}}\,) = \nabla_{\!\perp} p_\perp + \left(p_\| - p_\perp\right)(\mathbf{b}\,,\nabla)\mathbf{b} \tag{5}$$

the orthogonal to the $\mathbf{B}$ component of the vector field $(-\operatorname{div}\widehat{\mathbf{P}})$. Substitution of the equality (4) into the equation (2) gives the following equations:

$$\mathbf{a} = \nabla_{\!\perp} p_\perp + \left(p_\| - p_\perp\right)(\mathbf{b}\,,\nabla)\mathbf{b} = [\,\mathbf{j}\,,\mathbf{B}\,]\,, \qquad \left(p_\| - p_\perp\right)\operatorname{div}\mathbf{b} = -\left(\mathbf{b}\,,\nabla p_\|\right)\,. \tag{6}$$

Also we shall use the Maxwell equation

$$\operatorname{div}\mathbf{B} \,=\, 0 \tag{7}$$

and the equation of current continuity

$$\operatorname{div}\mathbf{j} \,=\, 0\,. \tag{8}$$

Equations (6)–(8) form the system of magnetohydrostatic equations for plasma with anisotropic pressure.

3. Formula for the parallel current density in the case of the pressure anisotropy

Formula for the parallel current density $j_{\|} = (\mathbf{j}\,,\mathbf{b})$ can be deduced from the equations (6)–(8) without any additional assumptions. Let the Q be a bounded region (i. e. open bounded set) in space R^3. For convenience, we assume that the field $\mathbf{B}$ in the region Q satisfies the following simple natural condition executed in the Earth's magnetosphere by obvious way.

Condition 1. I. *The field* $\mathbf{B}(\mathbf{x})$ *in the region* Q *is smooth enough and* $|\,\mathbf{B}(\mathbf{x})\,| \geq B_0 > 0$ *for all* $\mathbf{x}\in Q$.
II. *In the region* Q *smooth enough surface* Σ *exists, such that*
1) the Σ *separates the region* Q *in two regions* Q^+ *and* Q^- : $\quad Q = Q^+\bigcup\Sigma\bigcup Q^-$;
2) for an arbitrary point $\mathbf{x}\in Q$ *the passing through* $\mathbf{x}$ *magnetic field line intersects surface* Σ *in the unique point* $\mathbf{z}(\mathbf{x})$ *remaining in the region* Q.
3) magnetic field line is not tangent to the Σ, *that is* $\big|\big(\mathbf{n}(\mathbf{x})\,,\mathbf{b}(\mathbf{x})\big)\big| > \alpha_0 > 0$ *for all point* $\mathbf{x}\in\Sigma$, *where* $\mathbf{n}(\mathbf{x})$ *is the unit normal to the surface* Σ.

In the Earth's magnetosphere in the capacity of region Q we ought to consider the bounded region with external boundary formed by closed magnetic field line and with internal boundary formed by the upper boundary of the ionosphere. We ought to consider the equator plane in the capacity of the surface Σ. Let the field $\mathbf{B}$ on the surface Σ be directed into Q^+. Then region Q^+ is situated North of the equator plane in the Earth's magnetosphere.

Equation (7) allows us to introduce curvilinear coordinates ξ^1, ξ^2 and ξ^3 connected by natural way with the magnetic field $\mathbf{B}$:

$$\mathbf{B} \,=\, [\,\nabla\xi^1\,,\nabla\xi^2\,]\,. \tag{9}$$

We denote by $\mathbf{g}_k = \dfrac{\partial\mathbf{x}}{\partial\xi^k}$ and $\mathbf{g}^k = \nabla\xi^k$ the vectors of the accompanying covariant basis and of the dual contravariant basis, respectively. This vectors are connected among themselves by the following standard relations:

$$(\mathbf{g}_k\,,\mathbf{g}^i) = \delta^i_k\,, \qquad i,\,k = 1,\,2,\,3\,, \tag{10}$$

where δ^i_k is the Kronecker symbol. Also we denote by

$$g_{i\,k} = (\mathbf{g}_i\,,\mathbf{g}_k)\,, \qquad g^{i\,k} = (\mathbf{g}^i\,,\mathbf{g}^k) \qquad \text{and} \qquad g = \det\|\,g_{i\,k}\,\| = \big(\mathbf{g}_1\,,[\,\mathbf{g}_2\,,\mathbf{g}_3\,]\big)^2 = \big(\mathbf{g}^1\,,[\,\mathbf{g}^2\,,\mathbf{g}^3\,]\big)^{-2} \tag{11}$$

covariant and contravariant components of the metric tensor and its determinant, respectively. For an arbitrary vector field $\mathbf{w}(\mathbf{x})$, we denote by

$$w_k \,=\, (\mathbf{w}\,,\mathbf{g}_k) \qquad \text{and} \qquad w^k \,=\, (\mathbf{w}\,,\mathbf{g}^k) \tag{12}$$

its covariant and contravariant components, respectively. Also we denote by $\mathbf{y}(s\,,\mathbf{x})$ the solution of the following Cauchy problem:

$$\frac{\partial\,\mathbf{y}(s\,,\mathbf{x})}{\partial\,s} \,=\, \mathbf{b}(\mathbf{y}(s\,,\mathbf{x})\,)\,, \qquad \mathbf{y}(0\,,\mathbf{x}) \,=\, \mathbf{x}\,. \tag{13}$$

Let us introduce the length $\sigma(\mathbf{x})$ of the magnetic field line from the surface Σ to the point $\mathbf{x}$ by the formula

$$\mathbf{y}(-\sigma(\mathbf{x})\,,\mathbf{x}) \,=\, \mathbf{z}(\mathbf{x})\in\Sigma\,, \qquad \text{which means that} \qquad \mathbf{y}(\sigma(\mathbf{x})\,,\mathbf{z}(\mathbf{x})\,) \,=\, \mathbf{x}\,. \tag{14}$$

From (14) it follows that, if point $\mathbf{x}\in Q^+$, the length $\sigma(\mathbf{x}) > 0$, and, if point $\mathbf{x}\in Q^-$, the length $\sigma(\mathbf{x}) < 0$.

Now we can formulate the result of this paper in the form of the following proposition.

Proposition 1. *Let functions* $p_{\|}(\mathbf{x})$, $p_{\perp}(\mathbf{x})$ *and vector fields* $\mathbf{B}(\mathbf{x})$, $\mathbf{j}(\mathbf{x})$ *be smooth enough in the region* Q *and satisfy here the equations (6)–(8). Let also the condition 1 be executed. Then the parallel current density* $j_{\|} = (\mathbf{j}\,,\mathbf{b})$ *in the region* Q *is determined by the following formulae:*

$$j_{\|}(\mathbf{x}) \,=\, j_{\|}(\mathbf{z}(\mathbf{x}))\frac{B(\mathbf{x})}{B(\mathbf{z}(\mathbf{x}))} \,+\, \big(\mathbf{b}(\mathbf{x})\,,[\,\nabla W_1(\mathbf{x})\,,\nabla\xi^1(\mathbf{x})\,]\big) + \big(\mathbf{b}(\mathbf{x})\,,[\,\nabla W_2(\mathbf{x})\,,\nabla\xi^2(\mathbf{x})\,]\big)\,, \tag{15}$$

where the point $\mathbf{z}(\mathbf{x})$ *is defined by (14) and functions* $W_k(\mathbf{x})$ *are defined by formulae*

$$W_k(\mathbf{x}) \,=\, \pm \int\limits_{\mathbf{z}(\mathbf{x})}^{\mathbf{x}} f_k \, ds \,=\, \int\limits_{-\sigma(\mathbf{x})}^{0} f_k(\mathbf{y}(s\,,\mathbf{x})\,)\, ds \,=\, \int\limits_{0}^{\sigma(\mathbf{x})} f_k(\mathbf{y}(s\,,\mathbf{z}(\mathbf{x})\,)\,)\, ds \;, \qquad \mathbf{x} \in Q^{\pm} \;, \qquad k = 1,\, 2 \;, \tag{16}$$

where $\mathbf{y}(s\,,\mathbf{x})$ *is the solution of the Cauchy problem (13) and functions* $f_k(\mathbf{x})$ *are defined by formulae*

$$f_1(\mathbf{x}) \,=\, \frac{a_1}{B} \,=\, \frac{a^1\, g^{22} \,-\, a^2\, g^{12}}{B^{\,3}} \;, \qquad f_2(\mathbf{x}) \,=\, \frac{a_2}{B} \,=\, \frac{a^2\, g^{11} \,-\, a^1\, g^{12}}{B^{\,3}} \;. \tag{17}$$

Here the vector field $\mathbf{a}(\mathbf{x})$ *is defined by formula (5) and components of the* $\mathbf{a}(\mathbf{x})$ *and the metric tensor are defined by the formulae (11), (12).*

For example, the formulae (15)–(17) can be used for determination of the parallel current density in the Earth's magnetosphere in that case, if there are empiric models of $\mathbf{B}$, $p_{\parallel}$ and $p_{\perp}$ in the magnetosphere, satisfying the second equation in (6), and there is empirical model of the parallel current density in the equator plane.

4. The case of isotropic pressure

In the case of plasma with isotropic pressure the pressure tensor has the form $\widehat{\mathbf{P}} \,=\, -p\,\widehat{\mathbf{I}}$. Then $\operatorname{div}\widehat{\mathbf{P}} \,=\, -\nabla p$, and equation (2) takes the form

$$\nabla p \,=\, [\,\mathbf{j}\,,\mathbf{B}\,] \;. \tag{18}$$

It means that in formulae (15)–(17) we are to consider the case $\mathbf{a} \,=\, \nabla p$. Since in the capacity of the first Euler potential $\xi^1(\mathbf{x})$ one can choose an arbitrary first integral $f(\mathbf{x})$ of the system (13), that is an arbitrary solution of the equation

$$(\,\mathbf{B}(\mathbf{x})\,, \nabla f(\mathbf{x})\,) \,=\, 0 \;, \qquad \mathbf{x} \in Q \;, \tag{19}$$

then taking into account equation (18) we can choose $\xi^1(\mathbf{x}) = p$, that gives $\mathbf{g}^1 \,=\, \nabla\xi^1 \,=\, \nabla p = \mathbf{a}$. Substituting this and (10) into the (12) we have the identities: $a_1 \,=\, (\mathbf{g}^1\,, \mathbf{g}_1\,) \,\equiv\, 1$, $a_2 \,=\, (\mathbf{g}^1\,, \mathbf{g}_2\,) \,\equiv\, 0$. Then substituting this identities into the (15)–(17) we receive the following formulae

$$j_{\parallel}(\mathbf{x}) = j_{\parallel}(\mathbf{z}(\mathbf{x}))\frac{B(\mathbf{x})}{B(\mathbf{z}(\mathbf{x}))} + \big(\,\mathbf{b}(\mathbf{x})\,, [\,\nabla V(\mathbf{x})\,, \nabla p(\mathbf{x})\,]\,\big)\,, \quad V(\mathbf{x}) = \pm\int\limits_{\mathbf{z}(\mathbf{x})}^{\mathbf{x}} \frac{ds}{B} = \int\limits_{-\sigma(\mathbf{x})}^{0} \frac{ds}{B(\mathbf{y}(s\,,\mathbf{x})\,)} \quad \text{for} \quad \mathbf{x} \in Q^{\pm}, \tag{20}$$

where $\mathbf{y}(s\,,\mathbf{x})$ is the solution of the Cauchy problem (13).

Formulae (20) generalize the formulae for the parallel current density received by *Vasyliunas* [1970] and *Tverskoy* [1982]. *Vasyliunas* [1970] derived the formula

$$j_{\parallel}(\mathbf{x}) = f(\mathbf{x})\, B(\mathbf{x}) \,+\, \big(\,\mathbf{b}(\mathbf{x})\,, [\,\nabla V(\mathbf{x})\,, \nabla p(\mathbf{x})\,]\,\big)\,,$$

where function $f(\mathbf{x})$ satisfies the equation (19). *Tverskoy* [1982] derived the formula

$$j_{\parallel}(\mathbf{x}) = \big(\,\mathbf{b}(\mathbf{x})\,, [\,\nabla V(\mathbf{x})\,, \nabla p(\mathbf{x})\,]\,\big)$$

for the case, when the magnetic field is symmetrical relatively the equator plane. In this case the parallel current density in the equator plane identically equals zero, as it follows from the Maxwell equation $\mathbf{j} \,=\, \dfrac{1}{\mu_0}\,\mathrm{rot}\,\mathbf{B}$.

5. Scheme of the proof of the formula for the parallel current density in the anisotropic case

Our method is a generalization of the method used by *Tverskoy* [1982] for isotropic case. Let functions $\xi^1(\mathbf{x})$ and $\xi^2(\mathbf{x})$ be some Euler potentials of the $\mathbf{B}$, that is (9) be satisfied. Let the point $\mathbf{x}_0 \in Q^+$. Let $\mathbf{z}(\mathbf{x}_0)$ be the point of intersection of the magnetic field line passing through the $\mathbf{x}_0$ with the surface Σ. Let us consider a fine flux tube K bounded by the four surfaces $\xi^1(\mathbf{x}) \,=\, \xi^1(\mathbf{x}_0) \pm \frac{1}{2}\,\Delta\xi^1$, $\xi^2(\mathbf{x}) \,=\, \xi^2(\mathbf{x}_0) \pm \frac{1}{2}\,\Delta\xi^2$ and two orthogonal with respect to the $\mathbf{B}$ planes, passing through the points $\mathbf{x}_0$ and $\mathbf{z}(\mathbf{x}_0)$. We denote by $\Gamma_k^{\pm}$ for $k = 1,\,2$ the lateral surfaces of the flux tube K lying on the surfaces $\xi^k(\mathbf{x}) = \xi^k(\mathbf{x}_0) \pm \frac{1}{2}\Delta\xi^k$, respectively; we denote by $J_k^{\pm}$ the current flowing out from the K through the surfaces $\Gamma_k^{\pm}$, respectively; we denote by $\Gamma_0^{\pm}$ the bases of the flux tube K passing through the points $\mathbf{x}_0$ and $\mathbf{z}(\mathbf{x}_0)$, respectively; we denote by $J_{\parallel}^{\pm}$ the current flowing out from the K through the bases $\Gamma_0^{\pm}$, respectively. Also we denote by $\mathbf{x}_1^{\pm}$ the points of intersection of the base Γ_0^{+} with two surfaces $\xi^1(\mathbf{x}) = \xi^1(\mathbf{x}_0) \pm \frac{1}{2}\Delta\xi^1$, $\xi^2(\mathbf{x}) = \xi^2(\mathbf{x}_0)$ respectively, and we denote by $\mathbf{x}_2^{\pm}$ the points of intersection of the base Γ_0^{+} with two surfaces $\xi^1(\mathbf{x}) = \xi^1(\mathbf{x}_0)$, $\xi^2(\mathbf{x}) = \xi_0^2(\mathbf{x}_0) \pm \frac{1}{2}\Delta\xi^2$ respectively. The cross-section of the tube K by orthogonal with respect to the $\mathbf{B}$ plane is close to parallelogram formed by the vectors $\frac{[\mathbf{g}^2, \mathbf{b}]}{|\mathbf{g}^2|}\Delta l_1$ and $\frac{[\mathbf{b}, \mathbf{g}^1]}{|\mathbf{g}^1|}\Delta l_2$, where lengths Δl_k of the lateral sides of the parallelogram are determined in terms of $\Delta\xi^k$ from the relations $\Delta\xi^1 = \frac{([\nabla\xi^2, \mathbf{b}], \nabla\xi^1)}{|\nabla\xi^2|}\Delta l_1 + \overline{\overline{o}}(\Delta\xi^1)$, $\Delta\xi^2 = \frac{([\mathbf{b}, \nabla\xi^1], \nabla\xi^2)}{|\nabla\xi^1|}\Delta l_2 + \overline{\overline{o}}(\Delta\xi^2)$, which give us that

$$\Delta l_1 = \frac{|\mathbf{g}^2|}{B}\left(\Delta\xi^1 + \overline{\overline{o}}(\Delta\xi^1)\right), \qquad \Delta l_2 = \frac{|\mathbf{g}^1|}{B}\left(\Delta\xi^2 + \overline{\overline{o}}(\Delta\xi^2)\right). \tag{21}$$

From here and (9) we derive the following equalities

$$J_{\parallel}^{+} = \frac{j_{\parallel}(\mathbf{x}_0)}{B(\mathbf{x}_0)}\Big(\Delta\xi^1\Delta\xi^2 + \overline{\overline{o}}(\Delta\xi^1\Delta\xi^2)\Big), \qquad J_{\parallel}^{-} = -\frac{j_{\parallel}(\mathbf{z}(\mathbf{x}_0))}{B(\mathbf{z}(\mathbf{x}_0))}\Big(\Delta\xi^1\Delta\xi^2 + \overline{\overline{o}}(\Delta\xi^1\Delta\xi^2)\Big), \tag{22}$$

$$\mathbf{x}_1^{+} - \mathbf{x}_1^{-} = \frac{[\mathbf{g}^2, \mathbf{b}]}{|B|}\left(\Delta\xi^1 + \overline{\overline{o}}(\Delta\xi^1)\right), \qquad \mathbf{x}_2^{+} - \mathbf{x}_2^{-} = \frac{[\mathbf{b}, \mathbf{g}^1]}{|B|}\left(\Delta\xi^2 + \overline{\overline{o}}(\Delta\xi^2)\right). \tag{23}$$

The currents $J_k^{\pm}$ are determined by the formulae

$$J_1^{\pm} = \int_{\mathbf{z}(\mathbf{x}_1^{\pm})}^{\mathbf{x}_1^{\pm}} (\mathbf{j}, \mathbf{n}_1^{\pm})\Delta l_2\, ds, \qquad J_2^{\pm} = \int_{\mathbf{z}(\mathbf{x}_2^{\pm})}^{\mathbf{x}_2^{\pm}} (\mathbf{j}, \mathbf{n}_2^{\pm})\Delta l_1\, ds, \qquad \text{where} \quad \mathbf{n}_k^{\pm} = \pm\frac{\mathbf{g}^k}{|\mathbf{g}^k|}. \tag{24}$$

Using (5), (6), (9) and (21), and standard relations among the vectors of covariant and contravariant basis

$$\mathbf{g}_1 = \sqrt{g}\,[\mathbf{g}^2, \mathbf{g}^3], \quad \mathbf{g}_2 = \sqrt{g}\,[\mathbf{g}^3, \mathbf{g}^1], \quad \mathbf{g}_3 = \sqrt{g}\,[\mathbf{g}^1, \mathbf{g}^2], \quad \mathbf{g}^1 = \frac{[\mathbf{g}_2, \mathbf{g}_3]}{\sqrt{g}}, \quad \mathbf{g}^2 = \frac{[\mathbf{g}_3, \mathbf{g}_1]}{\sqrt{g}}, \quad \mathbf{g}^3 = \frac{[\mathbf{g}_1, \mathbf{g}_2]}{\sqrt{g}},$$

we receive the equalities in the formulae (17) and the following formulae:

$$(\mathbf{j}, \mathbf{n}_1^{\pm})\Delta l_2 = \mp f_2\left(\Delta\xi^1 + \overline{\overline{o}}(\Delta\xi^1)\right), \qquad (\mathbf{j}, \mathbf{n}_2^{\pm})\Delta l_1 = \pm f_1\left(\Delta\xi^2 + \overline{\overline{o}}(\Delta\xi^2)\right). \tag{25}$$

Substitution (25) to the (24) gives us the equalities:

$$J_1^{\pm} = \mp W_2(\mathbf{x}_1^{\pm})\left(\Delta\xi^2 + \overline{\overline{o}}(\Delta\xi^2)\right), \qquad J_2^{\pm} = \pm W_1(\mathbf{x}_2^{\pm})\left(\Delta\xi^1 + \overline{\overline{o}}(\Delta\xi^1)\right), \tag{26}$$

where the functions $W_k(\mathbf{x})$ are defined by the formulae (16), (17). Then from (26) and (23), we receive the following equalities:

$$J_i^{+} + J_i^{-} = -\frac{\left(\mathbf{b}(\mathbf{x}_0), [\nabla W_k(\mathbf{x}_0), \mathbf{g}^k(\mathbf{x}_0)]\right)}{B(\mathbf{x}_0)}\left(\Delta\xi^1\Delta\xi^2 + \overline{\overline{o}}(\Delta\xi^1\Delta\xi^2)\right), \qquad k \neq i, \quad i, k = 1, 2. \tag{27}$$

From the divergence theorem and the equation (8), it follows that the full current J flowing out from the flux tube K equals zero: $J = J_{\parallel}^{+} + J_{\parallel}^{-} + J_1^{+} + J_1^{-} + J_2^{+} + J_2^{-} = 0$. Substituting formulae (22), (27) into this identity and making passage to the limit for $\Delta\xi^1, \Delta\xi^2 \to 0$ in the identity $\frac{J}{\Delta\xi^1\Delta\xi^2} \equiv 0$ we receive the formulae (15)–(17).

References

Chew, G. F., M. L. Goldberger, and F. E. Low, The Boltzmann equation and the one-fluid hydromagnetic equations in the absence of particle collisions, *Proc. R. Soc.,* Ser. A, **236**, P. 112, 1956.

Heinemann, M., Representation of Currents and Magnetic Fields in Anisotropic Magnetohydrostatic Plasma, *J. Geophys. Res., 95,* 7789–7797, 1990.

Heinemann, M., and D. H. Pontius, Representation of Currents and Magnetic Fields in Anisotropic Magnetohydrostatic Plasma 2. General Theory and Examples, *J. Geophys. Res., 96,* 17609–17626, 1991.

Lui, A. T. Y., and D. C. Hamilton, Radial profiles of quiet time magnetospheric paramiters, *J. Geophys. Res., 97,* 19325–19332, 1992.

Tverskoy, B. A., Longitudinal currents in the magnetoshpere (in Russian), *Geomagn. Aeron., 22,* 991-995, 1982.

Vasyliunas, V. M., Mathematical model of magnetospheric convection and its coupling to the ionosphere, *in Particles and Fields in the Magnetosphere,* edited by B. M. McCormac, p. 60-71, D. Reidel, Hingham, Mass., 1970.