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TRANSMISSION OF FAST MAGNETOSONIC WAVE THROUGH ROTATIONAL DISCONTINUITY

A.A. Lubchich *(Polar Geophysical Institute, Apatity, Russia)*
M.I. Pudovkin *(Institute of Physics, State University, St-Petersburg, Russia)*

Introduction

The way of magnetohydrodynamic (MHD) waves of solar-wind origin is not easy. In order to penetrate into the magnetosphere, they have to cope with two serious obstacles. These are the earth's bow shock and the magnetopause. The transmission of MHD waves through the bow shock has been studied rather thoroughly. As an example we can point out *McKenzie and Westphal* [1970]. Reflection, transmission and generation of MHD waves at the magnetopause for a closed magnetosphere has been investigated by *Wolfe and Kaufmann* [1975] *et al.*. The open magnetopause with a nonzero normal component of magnetic field is assumed to be a rotational discontinuity. Under an isotropic pressure, the magnetic fields on either side of it are known to have the same magnitude and subtend equal angles ψ with the normal, but can rotate at an arbitrary angle ΔA in the azimuthal direction. Fluid flow across the boundary is at the speed of the normal component of the Alfven velocity. Most noteworthy of all is the existence of a coordinate system, the so-called de Hoffman frame, in which fluid flow will be along the magnetic field and at the local Alfven speed V_A. The thermal pressure P, the fluid density ρ and other thermodynamic quantities are continuous across the discontinuity. We only know two theoretical papers, where transmission of MHD waves through such a discontinuity is considered. *Lee* [1982] has studied the transmission of Alfvén waves for a rotational discontinuity in which the magnetic field rotates by $\Delta A=180°$ across the boundary. However, the assumption that the discontinuity is flat like that extremely restricted the application domain of the results obtained. *Kwok and Lee* [1984] have studied the transmission of any of the incident waves at a rotational discontinuity of an arbitrary magnetic field configuration. However, correctness of the results obtained in the last paper (for example, existence of the very high amplification regions and nonzero amplitude of an emanating entropy wave for the case of an incident non-entropy wave) is doubtful. We performed more accurate consideration and got different results, which seem to be more correct. Our problem formulation is virtually the same as that used by Kwok and Lee. There is one distinction only: all possible incident angles of MHD waves are included into our consideration as well as those with which the refracted fast magnetosonic wave turns into the inhomogeneous surface wave. Because of bulk limitations we only consider here the transmission of fast magnetosonic waves through arbitrary rotational discontinuity.

Method of approach

The small-amplitude waves transmission problem is solved, as a rule, using the perturbation theory methods. As far as we know, in the MHD method was first applied by *Kontorovich* [1958]**.** The general strategy can be summarized as follows.

MHD equations determine the properties of MHD waves on the two sides of the rotational discontinuity. The familiar one-fluid MHD equations with an isotropic plasma pressure are used to describe the otherwise uniformly magnetized, homogeneous medium. Equations may be written as

$$\left.\begin{aligned}&\mathrm{div}\vec{B}=0\\&\frac{\partial\vec{B}}{\partial t}=\mathrm{rot}\left[\vec{V}\times\vec{B}\right]\end{aligned}\right\}\qquad\text{Maxwell equations}$$

$$\frac{\partial\rho}{\partial t}+\mathrm{div}\rho\cdot\vec{V}=0\qquad\text{mass conservation law}$$

$$\frac{\partial\vec{V}}{\partial t}+\left(\vec{V}\cdot\nabla\right)\cdot\vec{V}=-\frac{1}{\rho}\cdot\nabla P-\frac{1}{4\cdot\pi\cdot\rho}\cdot\left[\vec{B}\times\mathrm{rot}\vec{B}\right]\qquad\text{Euler equation}$$

$$\frac{\partial S}{\partial t}+\left(\vec{V}\cdot\nabla\right)S=0\qquad\text{entropy conservation law}$$

$$P=P(\rho,S)\qquad\text{equation of state}$$

The conservation laws of mass flux, momentum flux, energy flux, tangential electric field and normal magnetic flux determine all properties of the unperturbed rotational discontinuity.

The first order perturbations of the conservation laws give the relationship between perturbed quantities on the two sides of the discontinuity. *Kontorovich* [1958] has found that the eight conservation laws can be reduced to only seven equations. The eighth equation is linear combination of the seven equations. In the case of rotational discontinuity, the relations are as follows:

$$\delta p_1=\delta p_2 \tag{1a}$$

$$\delta\rho_1=\delta\rho_2 \tag{1b}$$

$$\vec{V}_1 \cdot \delta\vec{v}_{A1} = \vec{V}_2 \cdot \delta\vec{v}_{A2} \tag{1c}$$

$$\left(\delta\vec{v}_1 - \delta\vec{v}_{A1}\right) + \frac{1}{2}\cdot\left(\vec{V}_1 - \vec{V}_2\right)\cdot\frac{\delta\rho_1}{\rho} = \delta\vec{v}_2 - \delta\vec{v}_{A2} \tag{1d}$$

$$\delta v_{sur} = \frac{1}{2}\cdot V_x \cdot \frac{\delta\rho_1}{\rho} + \delta v_{x1} - \delta v_{A\,x1}, \tag{1e}$$

where $\delta\vec{v}_A$ is perturbation of the Alfven velocity. The subscripts 1 and 2 relate to upstream and the downstream region, respectively. The interaction of the incident wave with MHD discontinuity perturbs the discontinuity surface. Equation (1e) allows to determine the velocity δv_{sur} of small displacements of the surface by the perturbed quantities in the upstream region.

In the rotational discontinuity, there can be six eigenwaves emanating from the boundary. They are the fast magnetosonic wave in the upstream region, fast magnetosonic wave, slow magnetosonic wave, convected slow magnetosonic wave, Alfven wave and the entropy wave in the downstream region. The convected wave is the wave which propagates upstream in the fluid's rest frame. Convection of a slow magnetosonic wave makes it a downstream propagating one in the de Hoffmann frame. The amplitude of perturbed shock speed is the seventh unknown. The equation (1e) determines this amplitude. The perturbed quantities on each side of discontinuity presented in the equation (1a-d) are expressed in terms of amplitude and wave vector of the incident one and the six emanating waves. The perturbations of pressure and density are known to be associated with magnetosonic waves. With entropy wave are associated the perturbations of density, but not of pressure. The perturbations of pressure and density are continuous across the shock surface (the equations (1a) and (1b)). This immediately infers that the emanated entropy wave appears only in the case of the incident entropy wave. In this paper we deal with the case of an incident fast magnetosonic wave. In the case the downstream modes consist of all the modes mentioned above except the entropy wave. We note that this mandatory condition was not observed in the paper by *Kwok and Lee* [1984]. This is well enough to conclude that there is a mistake in their paper. Allowing for the absence of the emanating entropy wave, we obtain a 5×5 system of linear inhomogeneous equations (1a,c,d) in the five unknowns, where these five unknowns are associated with the two fast magnetosonic waves, the two slow magnetosonic waves and the Alfven wave. However, we do not know so far the propagation angles of the diverging waves.

The frequency ω in the de Hoffmann frame and the component of the $\vec{k}$ wave vector tangent to the shock surface remain continuous across this surface when MHD waves are transmitted through the shock. Let the y-z plane of an (x, y, z) coordinate system form the discontinuity interface, while the shock normal is collinear with the x direction. The wave vector of incident wave, $\vec{k}$, lies in the x-y plane. The continuity of the above two quantities ω and $\vec{k}$ may be rearranged as the continuity of the tangential phase velocity $c_p \equiv \frac{\omega}{k_y}$. This condition is derived out of the assumption of small amplitude of the incident wave only. It has nothing to do neither with boundary conditions nor with a MHD equations system used. For the Alfven wave, the refraction angle is determined by

$$\frac{k_{x\,al.}}{k_y} = \frac{1}{V_x}\cdot\left(\frac{1}{2}\cdot c_p - V_{y2}\right) \tag{2a}$$

Always, the emanating Alfven wave is the homogeneous harmonic linear polarized plane wave. There are two methods which might be employed for determination of propagation angles of the emanating magnetosonic waves. *Kontorovich* [1958] presented a geometrical method for construction of diverging waves. Another method is the obtaining of the Snell's law for this problem (*Kwok and Lee* [1984]). However, the use of these methods meets some difficulties. We propose our solution of this problem. The magnetosonic mode may be rendered in the form $\delta f = |\delta f| \cdot exp(\chi \cdot x + i \cdot k_y \cdot y - i\cdot\omega\cdot t)$, where χ can be complex. Here, δf is the wave amplitude of a certain physical quantity associated with the wave motion. The linearized MHD equations to describe this perturbations associated with magnetosonic modes may be solved for $\varphi \equiv \frac{\chi}{i\cdot k_y}$**.** We obtain the fourth–degree polynomial, which may be written as

$$A_0\cdot\varphi^4 + B_0\cdot\varphi^3 + C_0\cdot\varphi^2 + D_0\cdot\varphi + E_0 = 0 \tag{2b}$$

$$A_0 = V_x^4 - V_x^2\cdot\left(c_s^2 + V_A^2\right) + V_{Ax}^2\cdot c_s^2$$

$$B_0 = 2\cdot\left[V_x\cdot\left(c_p - V_y\right)\cdot\left(c_s^2 + V_A^2 - 2\cdot V_x^2\right) + V_{Ax}\cdot V_{Ay}\cdot c_s^2\right]$$

$$C_0 = -\left(V_x^2 + \left(c_p - V_y\right)^2\right)\cdot\left(c_s^2 + V_A^2\right) + c_s^2\cdot\left(V_{Ax}^2 + V_{Ay}^2\right) + 6\cdot\left(c_p - V_y\right)^2\cdot V_x^2$$

$$D_0 = 2\cdot\left[V_x\cdot\left(c_p - V_y\right)\cdot\left(c_s^2 + V_A^2 - 2\cdot\left(c_p - V_y\right)^2\right) + V_{Ax}\cdot V_{Ay}\cdot c_s^2\right]$$

$$E_0 = \left(c_p - V_y\right)^4 - \left(c_p - V_y\right)^2\cdot\left(c_S^2 + V_A^2\right) + c_S^2\cdot V_{Ay}^2$$

where c_s is the sound velocity. This equation describes the magnetosonic waves in the medium separated by the discontinuity with nonzero mass flux across surface. The discontinuities are the fast shock wave, the slow shock wave and the rotational discontinuity. Coefficients A_0, B_0, C_0, D_0 and E_0 depend on the tangential phase velocity (or the incident angle) and unperturbed quantities on the one side of discontinuity. For wave from upstream (downstream), we must employ the unperturbed quantities with subscript 1 (2). For physically corrected situations, this equation has either four real solutions or two real solutions and two conjugate complex solutions. Real solution describes the familiar homogeneous harmonic linear polarized plane magnetosonic wave. One of complex solutions describes the inhomogeneous plane surface wave. On side 1, it is solution with $Re\chi>0$. On side 2, it is solution with $Re\chi<0$. The wave amplitude of a physical quantity, associated with the inhomogeneous wave, subsides with distance from of the shock (approximatelly 3 times at a wave-length).

Solving the equation (2b) on the two sides of the discontinuity one can find reflected angle of convected fast magnetosonic wave from upstream and angles of propagation of three emanating magnetosonic wave from downstream region. Equations (2a) and (2b) give the propagation angles of all emanating waves as a function of the incident angle of an incoming wave. In case of incoming fast magnetosonic wave only refracted fast magnetosonic mode may be inhomogeneous surface wave. The perturbations associated with this surface wave may be written into the form:

$$\delta v_{Ax} = \left[-\left(1+\varphi^2\right)\cdot V_{Ax} + \varphi\cdot\left(V_{Ay}+\varphi\cdot V_{Ax}\right)\right]\cdot A; \qquad \delta v_x = \left[-\frac{V_{Ay}+\varphi\cdot V_{Ax}}{V_y+\varphi\cdot V_x - c_p}\cdot\left(1+\varphi^2\right)\cdot V_{Ax} + \varphi\cdot\left(V_y+\varphi\cdot V_x - c_p\right)\right]\cdot A$$

$$\delta v_{Ay} = \left[-\left(1+\varphi^2\right)\cdot V_{Ay} + \left(V_{Ay}+\varphi\cdot V_{Ax}\right)\right]\cdot A; \qquad \delta v_y = \left[-\frac{V_{Ay}+\varphi\cdot V_{Ax}}{V_y+\varphi\cdot V_x - c_p}\cdot\left(1+\varphi^2\right)\cdot V_{Ay} + \left(V_y+\varphi\cdot V_x - c_p\right)\right]\cdot A$$

$$\delta v_{Az} = -\left(1+\varphi^2\right)\cdot V_{Az}\cdot A; \qquad \delta v_z = -\frac{V_{Ay}+\varphi\cdot V_{Ax}}{V_y+\varphi\cdot V_x - c_p}\cdot\left(1+\varphi^2\right)\cdot V_{Az}\cdot A$$

$$\frac{\delta\rho}{\rho} = -\frac{\delta v_y + \varphi\cdot\delta v_x}{V_y+\varphi\cdot V_x - c_p}; \qquad \delta p = c_S^2\cdot\delta\rho \qquad (3)$$

For real φ, expressions (3) determine**,** as expected**,** the famliar linear polarized wave. For complex φ, we have elliptical polarized perturbations of Alfven velocity and flow velocity associated with the inhomogeneous surface wave. In the common case, perturbations $\delta\vec{v}_A$ and $\delta\vec{v}$ lying in the different plane of polarization.

Now we know all the necessary properties of diverging waves including the propagation angles and the polarization. By using (2) and (3), the system of relations (1) may be solution to obtain the amplitude coefficients all emanating waves and the perturbation of the shock surface. However, such a solution involves tedious algebra, and we should limit ourselves to numerical calculations. The analytical solution may be obtained only in certain special cases. We obtain these solutions for two cases. First, we consider incident wave propagation parallel to the discontinuity normal. Second, we suppose $\beta<<1$, where β is the thermal pressure to the magnetic pressure ratio. This analytical solutions are used for the verification of our numerical results. Because of bulk limitations these solutions aren't presented here.

Numerical results

The amplitudes of the emanating waves depend on the elevation angle ψ of the magnetic fields, on the azimuthal angle A_1 between wavevector of incident wave and magnetic field in upstream region, on azimuthal angle ΔA between magnetic fields on the two sides of the discontinuity, on polytropic index γ, on defined earlier β. To obtain numerical and analytical solutions presented here, the parameters used are $\psi=60°$, $A_1=30°$, $\Delta A=45°$, $\gamma=5/3$, $\beta=1.5$. The propagation angle λ for the waves is angle between wavevector and the x-axis. We consider the cases of incident fast magnetosonic waves with upstream and downstream regions. In the last case, the incident wave is a convected fast magnetosonic wave.

When the wavevector of incident mode is perpendicular to shock surface, the analytical solution gives the following normalized amplitude coefficients (the ratio of perturbations of the velocity associated with emanating and incident wave):

$\delta\overline{v}_{f1} = -0.02;\ \delta\overline{v}_{f2} = 0.98;\ \delta\overline{v}_{S+} = \delta\overline{v}_{S-} = -0.08;\ \delta\overline{v}_{a2} = 0.21$ - in case with an incident fast magnetosonic wave in the upstream region.

$\delta\overline{v}_{f1} = 1.00;\ \delta\overline{v}_{f2} = 0.00;\ \delta\overline{v}_{S+} = \delta\overline{v}_{S-} = 0.02;\ \delta\overline{v}_{a2} = 0.05$ - - in case with an incident convected fast magnetosonic wave in the downstream region.

It has been shown that the coefficient for transmitted fast wave is about one. The interaction of an incident wave with the shock surface gives rise to four new emanating waves, but their amplitudes are small.

Figure 1 shows variations of the propagation angles ((1A) and (1B)) and the normalized velocity perturbations ((1C) and (1D)) of all divergent waves as a function of incident angles of the fast magnetosonic waves. Figure (1A) and (1C) shows the case of incident fast magnetosonic wave from the upstream region; to be contrasted with figures (1B) and (1D), which show the case of incident convected fast wave from the downstream region. The normalized coefficients for the refracted fast magnetosonic wave (denoted by f1 or f2), the reflected fast wave (denoted by f2 or f1), the emanating Alfven wave (denoted by A), the slow magnetosonic wave (denoted by S+), the convected slow wave (denoted by S-) are plotted. The shock speed is labeled with Sur. In the case of normal incident angle, the result of the analytical solution is labeled with asterisks. In the case of small incident angles, the transmission coefficient of the fast wave is around 1, and the amplitudes of other emanating waves are small (around zero). For large incident angles the absolute value of the reflected coefficient tends to 1, and the amplitudes of other modes tend **to** 0. It is a total internal reflection. The group velocity of the incident and reflected waves directs parallel to the shock surface. In the case of large incident angle (λ_{in}<145°) of convected fast wave (for chosen quantity β, γ, A_1, ΔA and ψ), the refracted fast wave transforms into the inhomogeneous surface wave. For surface wave, Figure 1 is a plot of the absolute value of the complex amplitude coefficients according to sign of the phase.

Summary

In general, a small-amplitude fast magnetosonic wave incident on a rotational discontinuity has proved to give rise to reflected and refracted fast waves, as well as to all theoretically possible emanating modes except the entropy wave. Usually transmission coefficients of emanating waves are small. For small incident angles, the normalized amplitude of refracted wave is around 1. It has been found that maximal amplitude of the refracted wave differs only by the factor 1.5 in comparison with that of the incident waves. In case of a large incident angle of the fast wave, the refracted fast wave turns into the surface wave, which subsides with the distance from the shock.

In contrast to the result of *Kwok and Lee* [1984], it has been found that perturbations behind the front lead to a small amplification. Therefore for the perturbation of solar-wind origin, the rotational discontinuity is not a considerable barrier.

References

Kontorovich, V.M., On the interaction between small perturbations and the discontinuities in magnetohydrodynamics and the stability of shock waves, *JETPh, t.35*, p.1216-1225, 1958 *(in Russian)*.

Kwok, Y.C., and L.C. Lee, Transmission of magnetohydrodynamic waves through the rotational discontinuity at the earth's magnetopause, *JGR, v.89*, p. 10697-10708, 1984.

Lee, L.C., Transmission of Alfven waves through the rotational discontinuity at magnetopause, *Planet.Space Sci., 30*, p. 1127-1132, 1982.

McKenzie, J.F., and K.O. Westphal, Interaction of hydromagnetic waves with hydromagnetic shocks, *The Physics of Fluids, t13*, p. 630-640, 1970.

Wolfe, A., and R.L. Kaufmann, MHD wave transmission and production near the magnetopause, *JGR, v.80*, p. 1764-1775, 1975.

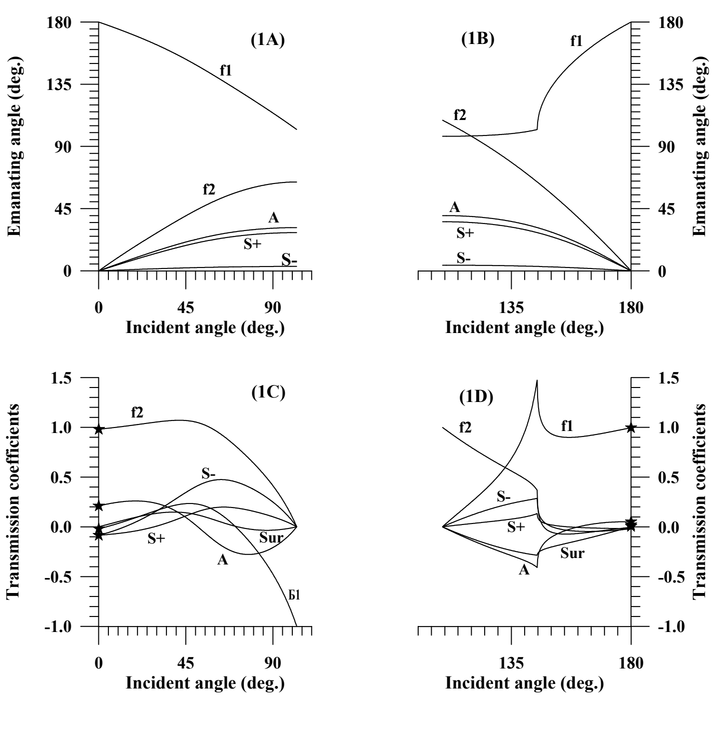

Figure 1.