

AN ANALYTICAL MODEL FOR NON-CONJUGATION OF THE EQUIVALENT CURRENTS IN THE DAYSIDE HIGH-LATITUDE IONOSPHERE

L.Benkevich, W.Lyatsky (*Polar Geophysical Institute, Apatity, Russia*)

Abstract. An analytical model for three-dimensional current systems has been developed, allowing for significant difference in the structure of ionospheric conductivity between the northern and southern hemispheres. The model may be used for qualitative investigation of equivalent ionospheric currents and thereby ground-observed magnetic effects. The equivalent current systems in either hemisphere have substantially different patterns due to secondary field-aligned currents emerging on the boundary of the ionospheric conductivity discontinuity and flowing between the conjugate hemispheres. The results obtained are applied to such phenomena as the magnetic impulse events and travelling convection vortices in the high-latitude ionosphere.

Introduction

The high-latitude convection has two characteristic features: meso-scale convection disturbances and appearance of the secondary field-aligned currents between the northern and southern hemispheres on the boundary of sunlit and dark ionosphere regions.

Among meso-scale disturbances, travelling convection vortices (TCV) have been studied by many researchers [Friis-Christensen *et al.*, 1988; McHenry *et al.*, 1990; Heikkila, 1989; Glassmeier, 1992; Luhr *et al.*, 1996; Vorobjev, 1993; Yahnin and Moretto, 1996; Jacobsen and Lyatsky, 1998, submitted] A possible cause for the events may be surface waves propagating on the magnetopause [Sibeck, 1990], the Kelvin-Helmholtz instability [McHenry *et al.*, 1990; Clauer and Ridley, 1995] or surface wave at the LLBL inner edge [Lyatsky and Sibeck, 1997]. TCV in the convection from SuperDARN measurements have been studied recently by Lyatsky *et al.* [submitted to JGR, 1998] who have shown that the vortices in convection and in equivalent ionospheric currents do not coincide. The effect might be a result of the inhomogeneity of the ionospheric conductivity resulted from different sunshine and particle precipitation conditions. One of the predictions of the paper is magnetic non-conjugation of the events. It is commonly accepted to locate magnetospheric position of a disturbance by its equivalent current streamlines, which are available from ground magnetic observations. However, these streamlines may appear to be non-coincident with the contours of the potential when the conductivity of the ionosphere is non-uniform, especially if it changes abruptly.

Another important feature of the dayside high-latitude ionosphere is emergence of the secondary field-aligned currents flowing between the northern and southern hemispheres on the boundary of the sunlit and dark ionosphere regions. Theoretical consideration of the problem is contained in [Lyatsky and Maltsev, 1983]. The secondary currents have to appear to satisfy the continuity condition for the currents in the ionosphere [Lyatsky *et al.*, 1998 submitted]. The expected effects of the secondary field-aligned currents are non-conjugation in equivalent ionospheric currents and in convection, as well as appearance of accelerated particles in the region of the upflowing field-aligned current on the terminator.

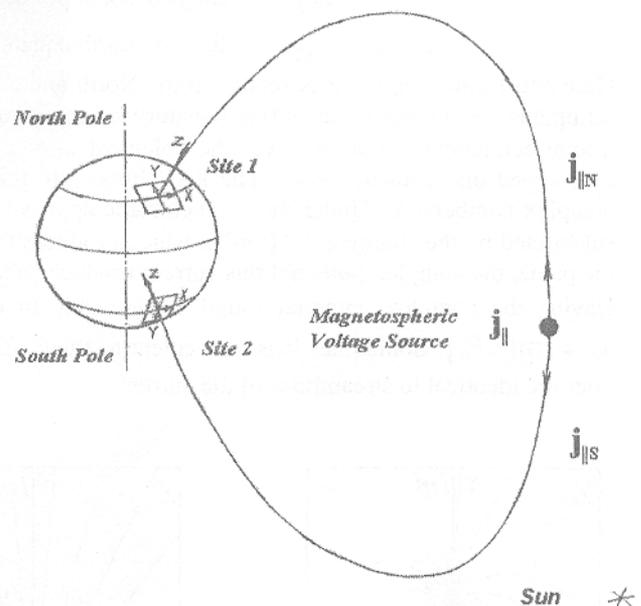


Fig.1. The Earth's ionosphere and a voltage source in the magneto-sphere producing two current filaments. The source is situated on a closed geomagnetic field line and the current flow along the high-conductive field lines into conjugate points of the ionosphere. Two sites in conjugate areas with their Cartesian co-ordinate systems are shown. The voltage source projections are in both sites. The ionospheric current systems within these sites are considered.

One purpose of this work was to solve the problem of calculating the equivalent ionospheric current system of a TCV taking into account the effects of the secondary field-aligned currents between both the hemispheres. Another one was finding those conditions under which an apparent vortex current appears at the conductivity discontinuity line that lies in lower latitudes with respect to the incident FAC.

Formulation

Obtaining the analytical solution required a number of simplifying assumptions. Figure 1 gives a general notion of the problem. The ionosphere is assumed to be a plane with height-integrated conductivity. We consider two sites on it: one in the north high-latitude hemisphere and the other in the south one. The Equator is supposed infinitely distant and the sites are considered to be infinite planes. Cartesian co-ordinate system is adopted. In the north site the X axis points to the east, Y axis points to the north and Z axis is upward. In the south site the same co-ordinate system is used, as if it was taken in the north and moved to the south with the Z axis always directed along a geomagnetic field line. Thus, in the south site the X axis points to the east again, Y axis points now to the south and Z is downward. The points in the sites with the same co-ordinates are conjugate ones and, hence, may be thought of as identical because they are connected through high-conductive geomagnetic field lines. Either site is assumed to be an infinite plane divided by the Y axis into two semiplanes, each one with different though uniform conductivity. A voltage source is located somewhere high in the magnetosphere, on a closed magnetic field line, so that it produces a pair of field-aligned currents incident upon both of the sites.

Our computations have been based upon the technique of complex function theory [Maltsev, 1973; Lyatsky et al., 1974; Lyatsky, 1978; Lyatsky and Maltsev, 1983; Belova et al., 1997]. The X axis is considered to be the real one, the Y axis is for the imaginary one. Thus, the conductivity is a complex value, the real part being the Pedersen conductivity and the imaginary one being the Hall conductivity. We have four semiplanes with the following conductivities:

$$\begin{aligned} \Sigma_{N1} &= \Sigma_{N1P} - i\Sigma_{N1H} && \text{for } y>0, \text{ north poleward semiplane,} \\ \Sigma_{N2} &= \Sigma_{N2P} - i\Sigma_{N2H} && \text{for } y<0, \text{ north equatorward semiplane,} \\ \Sigma_{S1} &= \Sigma_{S1P} - i\Sigma_{S1H} && \text{for } y>0, \text{ south poleward semiplane,} \\ \Sigma_{S2} &= \Sigma_{S2P} - i\Sigma_{S2H} && \text{for } y<0, \text{ south equatorward semiplane.} \end{aligned}$$

Here and further the letter N relates to the North and S to the South. The numeral "1" relates to the $y>0$ or poleward semiplane, "2" relates to the $y<0$ or equatorward semiplane for the both sites. While the sites are considered as identical, a total conductivity is introduced: the poleward $\Sigma_1 = \Sigma_{N1} + \Sigma_{S1}$ and equatorward $\Sigma_2 = \Sigma_{N2} + \Sigma_{S2}$, as if the sites were overlapped one onto the other. The FAC flows into the points on either site with the same co-ordinates (0,d) or, in complex numbers, id . Under the magnetostatic approximation (when the current is unchanged in time) the current I is substituted by the charge $q = I/(4\pi\Sigma_p)$, which produces the same electric field. If the conductivity were uniform all over the plane, the complex potential this current produced should be derived as $F_0 = -2q \ln(z-z_1)$, where $z = x + iy$ and $z_1 = id$. Having the complex potential found we are able to obtain the real potential $\phi = \text{Re}[F_0]$ and current function $\psi = \text{Im}[\Sigma F_0]$. Sometimes it is convenient to think of a current in terms of the current function because its contour lines are identical to streamlines of the current.

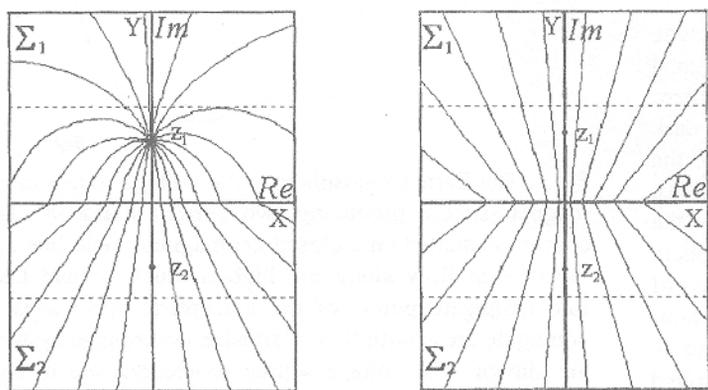


Fig.2. A single site with the conductivity discontinuity coincident with the Imaginary axis or the $y=0$ line. Real currents (left panel). Equivalent currents (right panel). An incident currents flow into the point indicated as z_1 . The sheet current streamlines are shown. The pattern of equivalent currents resembles a "broom" on either side of the discontinuity.

First we solve the problem for the single site obtained by overlapping the two above-mentioned ones. The case is shown in [Figure 2](#). The incident current I is flowing into a point z_1 . According to the technique of electrical images we introduce the «reflected» charge $q' = [(\Sigma_1 - \Sigma_2) / (\Sigma_1^* + \Sigma_2)] \cdot q$ (in the other semiplane) and suppose the potentials in both the semiplanes to be $F_1 = F_0 + F'_1$ and $F_2 = F_0 + F'_2$, where $F'_1 = -2q' \ln(z - z_2)$, $F'_2 = -2(q')^* \ln(z - z_1)$, and (*) means complex conjugation. As long as we suppose the sites overlapped, the real currents take the form $\psi_1 = \text{Im}[\Sigma_1^* F_1]$ in the poleward semiplane and $\psi_2 = \text{Im}[\Sigma_2^* F_2]$ in the equatorward one. Now we can solve the same problem for the two ionospheric sites whose points are connected via the high-conductive environment of the magnetosphere by «splitting» the overlapped site back into two separate ones. The potential in either site remains the same as in the single one. The currents are distributed among them in proportion to their conductivities. The secondary current will flow between the sites through their conductivity disruption line. This means that on this line the current function undergoes discontinuity. The equivalent currents, unlike the real ones, have to be continuous and will take the form (true for either site):

$$\psi^{eq} = \psi - \psi^{eq1} - \psi^{eq1}, \quad (1)$$

where ψ^{eq1} and ψ^{eq2} are compensative currents whose magnetic effects are equal and opposite to those of the incident I and the secondary J_{\parallel} currents. It is evident that $\psi_N^{eq1} = \text{Im}[\Sigma_{N1P} F_0]$ for both semiplanes. The compensative currents ψ^{eq1} are assumed to be caused by some dummy conductivity Σ'_N in the north and Σ'_S in the south. To find them, we construct the equations on the boundary $y=0$:

$$\text{Im}[\Sigma'_N F'_1]_{y=0} = \frac{1}{2} \text{Im}[\Sigma_{N2}^* F_2 - \Sigma_{N1}^* F_2]_{y=0}, \quad (2)$$

$$\text{Im}[\Sigma'_S F'_1]_{y=0} = \frac{1}{2} \text{Im}[\Sigma_{S2}^* F_2 - \Sigma_{S1}^* F_2]_{y=0}. \quad (3)$$

For convenience, we introduce two more potentials: $F'_{11} = -2q \ln(z - z_2)$ and $F'_{22} = -2q \ln(z - z_1)$. In the $x > 0$ semiplane the currents are as though due to the reflected charge, $\psi_{N1}^{eq1} = \text{Im}[\Sigma'_N F'_{11}]$, and in the $x < 0$ semiplane, ψ_{N2}^{eq1} is obtained from the condition $\psi_{N1}^{eq1}(z) = -\psi_{N2}^{eq1}(z^*)$. While converting (2) and (3), the relations $(F'_1)_{y=0} = F'_{2,y=0}$ and $(F'_2)_{y=0} = F'_{1,y=0}$ are also used. We omit some computations and give the results immediately. The compensative conductivity in the north is

$$\Sigma'_N = \frac{1}{2} [\Sigma_{N1} - \Sigma_{N2} - (\Sigma_1 - \Sigma_2) \frac{\Sigma_{N1}^* + \Sigma_{N2}}{\Sigma_1^* + \Sigma_2}]. \quad (4)$$

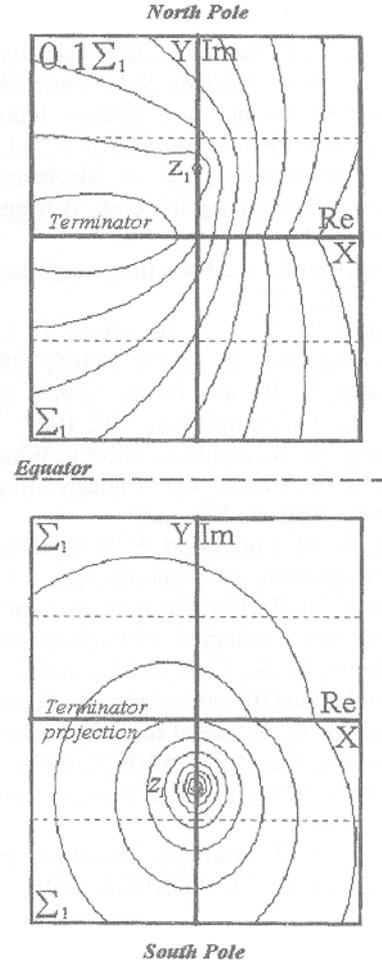
Due to the symmetry, in the south we get

$$\Sigma'_S = \frac{1}{2} [\Sigma_{S1} - \Sigma_{S2} - (\Sigma_1 - \Sigma_2) \frac{\Sigma_{S1}^* + \Sigma_{S2}}{\Sigma_1^* + \Sigma_2}]. \quad (5)$$

The equivalent current systems in both of the hemispheres are composed of:

$$\begin{aligned} \psi_{N1}^{eq} &= \text{Im}[\Sigma_{N1}^* (F_0 + F'_1) + \Sigma'_N F'_{11} - \Sigma_{N1P} F_0], \\ \psi_{N2}^{eq} &= \text{Im}[\Sigma_{N2}^* (F_0 + F'_2) + (\Sigma'_N)^* F'_{22} - \Sigma_{N1P} F_0], \\ \psi_{S1}^{eq} &= \text{Im}[\Sigma_{S1}^* (F_0 + F'_1) + \Sigma'_S F'_{11} - \Sigma_{S1P} F_0], \\ \psi_{S2}^{eq} &= \text{Im}[\Sigma_{S2}^* (F_0 + F'_2) + (\Sigma'_S)^* F'_{22} - \Sigma_{S1P} F_0]. \end{aligned} \quad (6)$$

Now that the current functions are available, the currents in all of the four parts are calculated using the formula $\mathbf{J} = [e_z, \nabla\psi]$, where e_z is a unit vector directed along the Z axis.



[Fig.3.](#) Non-conjugate equivalent current patterns in the conjugate hemispheres. The equivalent sheet current streamlines in the ionospheric sites lying in conjugate areas are shown. The sheet currents are caused by the magnetospheric incident FAC flowing into the conjugate points z_1 in either site. The conductivity model is $\Sigma_{S1} = \Sigma_{S2} = \Sigma_{N2} = \Sigma_1$; $\Sigma_{N1} = 0.1\Sigma_1$. An apparent circular current with its focus on the line of the conductivity disruption (on the terminator) appears in the north site.

Result

We have modelled the distribution of the ionospheric equivalent currents in two conjugate sites. The case of uniform conductivity in the south site and non-uniform conductivity in the north one was considered, the north site being divided into two semiplanes with the conductivity ratio $\Sigma_{N1} / \Sigma_{N2} = 0.1$. Hence, the conductivity to the north from the discontinuity line was ten times lower than that to the equator. [Figure 3](#) shows the equivalent current streamlines obtained both in the south and in the north. As it has been predicted, we can see an apparent circular current with its focus on the line of conductivity disruption. This corroborates the idea that locations of foci of the TCVs observed in magnetic field do not coincide to those of the incident current sources in the magnetosphere. As a result of the ionospheric conductivity discontinuity, the apparent position of the vortex may be at any distance from the ionospheric footprint of its field aligned current.

References

- Belova, E., E. Pchelkina, W. Lyatsky and A. Pashin, The effect of ionospheric inhomogeneity on the magnetic pulsation polarization, *J. Atmos. Solar-Terr. Phys.*, 59 (12), 1425-1434, 1997.
- Clauer, C. R., and A. J. Ridley, Ionospheric observations of magnetospheric low-latitude boundary layer waves on August 4, 1991, *J. Geophys. Res.*, 11, 21873-21884, 1995.
- Friis-Christensen, E., M. A. McHenry, C. R. Clauer and S. Vennerström, Ionospheric travelling convection vortices observed near the polar cleft: A triggered response to sudden changes in the solar wind, *Geophys. Res. Lett.*, 15, 253-256, 1988
- Glassmeier, K. H., Travelling magnetospheric convection twin-vortices: observations and theory, *Ann. Geophysicae*, 10, 547-565, 1992.
- Heikkila, W. J., T. S. Jorgensen, L. J. Lanzerotti, and C. G. MacLennan, A transient auroral event on the dayside, *J. Geophys. Res.*, 94, 15291-15305, 1989.
- Lyatsky, W. B., and D. G. Sibeck, Central plasma sheet disruption and the formation of poleward-moving auroral events, *J. Geophys. Res.*, 102, 1997.
- Lyatsky, W. B., and D. G. Sibeck, What are the dayside auroras?, *Physics and Chemistry of the Earth*, 1998, in press.
- Lyatsky, W. B., and Y. P. Maltsev, Magnetosphere-Ionosphere Interaction, *monograph, Publ. House "Nauka", Moscow*, 1983, 192 p. (in Russian).
- Lühr, H., M. Lockwood, P. E. Sandholt et al.: Multi-instrument ground-based observations of a travelling convection vortices event, *Ann. Geophysicae*, 14, 162-181, 1996.
- Maltsev, Y. P., Electric field and current system of the magnetospheric substorm, *PhD thesis, the Leningrad State University, Leningrad*, 1973 (in Russian).
- McHenry, M. A., C. R. Clauer, and E. Friis-Christensen, Relationship of solar wind parameters to continuous, dayside, high-latitude travelling ionospheric convection vortices, *J. Geophys. Res.*, 95, 15007-15022, 1990.
- Sibeck, D. G., A model for the transient magnetospheric response to sudden solar wind dynamic pressure variations, *J. Geophys. Res.*, 95, 3755-3771, 1990.
- Vorobjev, V. G., The Hall vortices dynamics in the day-side high-latitude area, *Geomagnetism and Aeronomy*, 33, 58-68, 1993.
- Yahnin, A., T. Moretto: Travelling convection vortices in the ionosphere map to the central plasma sheet, *Ann. Geophysicae*, 14, 1025-1031, 1996.