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MODELING OF NONSTATIONARY ELECTRON PRECIPITATIONS BY THE WHISTLER CYCLOTRON INSTABILITY

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1. Introduction

For comparison of theoretical results with experimental data, it is obviously useful to employ simplified models which enable to obtain quick estimates and thus to get quick choice of parameters providing the best agreement with experiment. In this paper one of the models for a nonstationary regime of the whistler cyclotron instability is discussed. It is based on the so-called multi-level set of equations for the cyclotron maser [1], taking into account nonlinear modulation of the pitch-angle distribution of trapped particles. To get the model closer to experiment, its generalized version with slow time dependence of the coefficients is considered. Such dependence can exist, e.g., due to variation of the number density, energy or anisotropy of energetic particles or background plasma density in the instability region. On the base of numerical calculations, we investigate the dependencies of characteristics of energetic electron precipitation pulsations on those parameters and their possible time evolution. An attempt is made to compare the obtained results with observational data.

2. Basic equations

We start from the so-called multilevel equations for the whistler cyclotron instability (CI) [1]:

$$\frac{dF_i}{dt} = -D_0 E \delta_i F_i + J_i \qquad (1a)$$

$$\frac{dE}{dt} = (h_i F_i - \nu) E \qquad (1b)$$

Here E is the whistler wave energy density; F_i (i = 1, 2, …) are amplitudes in the expansion of the pitch-angle distribution function of energetic electrons of complete set of eigenfunctions Z_i of the quasi-linear diffusion operator; δ_i are the corresponding eigenvalues; J_i are amplitudes of the source function harmonics; h_i are the coefficients of the growth rate expansion; ν is the wave damping rate and $D_0 \approx \omega_{BL}\beta_*/N_c W$ where $\beta_* = \omega_{pL}^2 v^2/\omega_{BL}^2 c^2$, ω_{pL} and ω_{BL} are the electron plasma frequency, and gyrofrequency, respectively, index L refers to the equatorial plane, $W = mv^2/2$ is the electron energy, N_c is the cold plasma number density. This system of equations can be derived from the general quasi-linear theory if (1) diffusion over energy is neglected; (2) wave energy spectrum is narrow in frequency; and (3) weak pitch-angle diffusion regime is accomplished. The 1st and the 2st assumptions are valid in region with sufficiently high background plasma density, when the frequency of whistler waves is low, $\omega \sim \omega_{BL}/\beta_* << \omega_{BL}$. Weak pitch-angle diffusion regime exists for sufficiently low values of the wave energy density.

Calculations show that the eigenvalues δ_i are the roots of the following characteristic equation:

$$J_0\left(x_c^{(i)}\right)Y_1\left(px_c^{(i)}\right) - Y_0\left(x_c^{(i)}\right)J_1\left(px_c^{(i)}\right) = 0 \qquad (2)$$

where $x_c^{(i)} = 2\delta_i^{1/2}æ_c$, $p = æ_m/æ_c$, $æ_c = \sin\theta_c = \sigma^{-1/2}$ is the sine of equatorial loss cone angle (σ is the mirror ratio); $æ_m$ is the upper boundary of the interaction region, $æ_m \approx \sqrt{1-\beta_*^{-1}}$. The eigenfunctions Z_i have the form

$$Z_i = C_i\left[J_0\left(x_c^{(i)}\right)Y_0(x) - Y_0\left(x_c^{(i)}\right)J_0(x)\right] \qquad (3)$$

$C_i = \pi\delta_i$, $x = 2\delta_i^{1/2}æ_c$. The coefficients h_i are written as

$$h_i = (\omega_{BL}/N_c)\left[æ_m^2 Z_i\left(x_m^{(i)}\right) - 1\right] \qquad (4)$$

Analysis of the system (1) with constant coefficients showed [1] that it can have oscillatory solutions corresponding to pulsating precipitations of energetic electrons. To get pulsating regimes, it is necessary to consider at least two components in the distribution function expansion (i = 1, 2). Further we take into account only two first components; it is possible because the eigenvalues δ_i rapidly increase with the number.

3. Results and discussion

It is interesting to discuss characteristics of the pulsating regimes under realistic assumptions of cold and hot plasma parameters. These parameters are usually not constant in time, thus we should consider possible changes in the cyclotron instability regime by suitable variation of the coefficients in (1). In this paper we will assume sufficiently slow evolution of the external parameters, so that we can use same equations with correspondingly varying coefficients.

Qualitatively, consequences of variation of each parameter can be understood basing on known analytical results [2]. Period of pulsations is determined by the time τ_J of energetic particle accumulation to the instability threshold. Thus it depends mainly on the particle source intensity and anisotropy and on the reflection coefficient $R\left(T \sim \tau_J \propto \sum_i h_i J_i / \nu\right)$; also it depends weakly on N_c and W. Duration of an impulse is proportional to the wave growth time which is of order of ν^{-1}.

We consider the variations that may occur during a passage of a cloud of energetic electrons through the cyclotron interaction region. In this case we can expect variation of the energetic particle source amplitude (which is proportional to the electron density in a cloud), pitch-angle anisotropy, and characteristic energy of electrons. Other quantities that can vary are: background plasma density and an effective reflection coefficient R which determines the damping rate ν according to the formula $\nu = 2|\ln R|/T_g$, $T_g = \oint dz/v_g$ is the period of wave packet oscillations between conjugate ionospheres. The most substantial variation can be expected in energetic electron density and anisotropy. This is an agreement with recent observations with recent analyses of substorm injection events [3, 4]. Relevant properties of pulsating regimes of CI are:

1. Increase of the source power leads to a decrease of the pulsating period T and further to transition to the damped pulsation regime.

2. Increase of the 2nd harmonic in the source pith-angle distribution leads to increase in T.

Figure 1 and 2 show two examples of numerical solution of the equations (1) which demonstrate the importance of these factors. In both cases, the source intensity $J \equiv J_1 + J_2$ and the ratio $A = J_2/J_1$ varied according to the equations

$$J = J_0\left[1 + \Delta_J \sin(\pi t/2T_J)\right] \quad (5)$$

$$A = A_0\left[1 + \Delta_A \sin(\pi t/2T_A)\right] \quad (6)$$

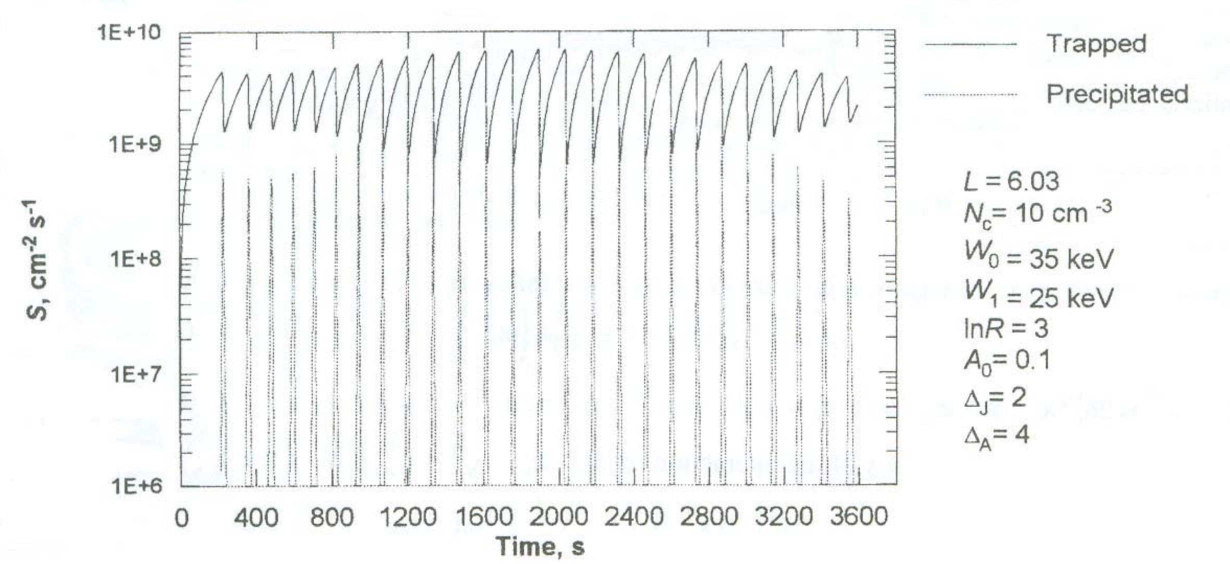

Fig. 1. Result of numerical calculations of the equations (1) with parameters corresponding to a substantial variation of the pitch-angle distribution in the source.

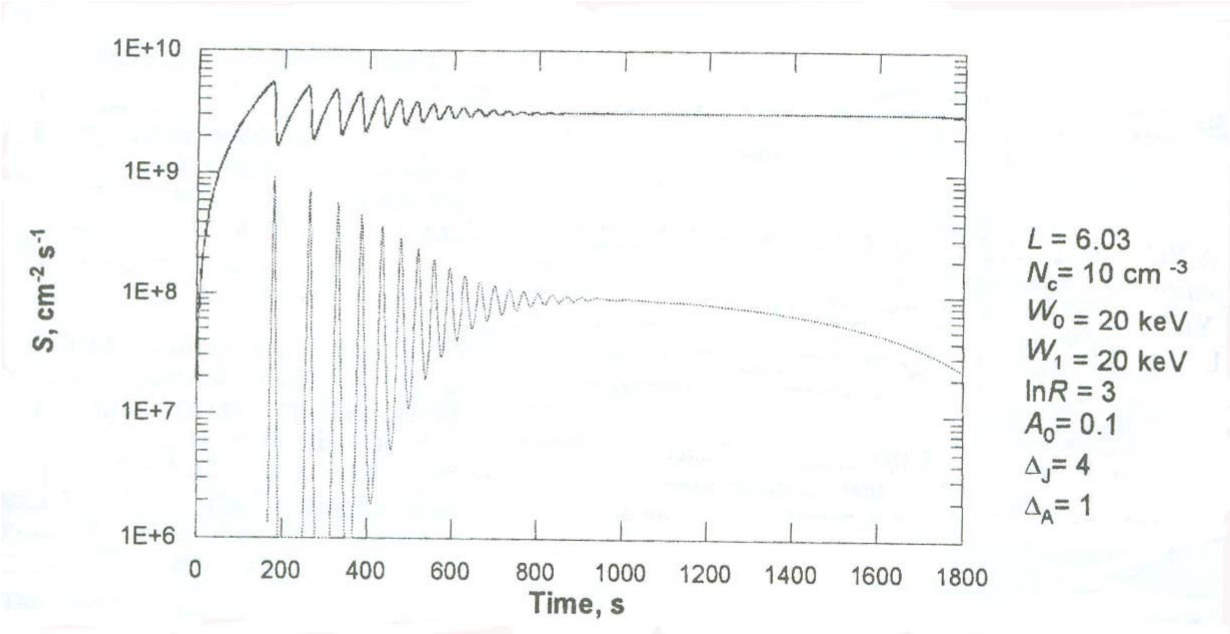

Fig. 2. Same as in Fig. 1 but for stronger and shorter injection with smaller variation of the pitch-angle distribution in the source.

Characteristic times T_J and T_A were chosen equal: $T_J = T_A$. Figure 1 corresponds to a slower variation (T_{J1} = 3600 s) with larger change in pitch-angle distribution (Δ_{A1} = 4) and smaller variation of the source intensity (Δ_{J1} = 2) as compared with the parameters chosen for Figure 2 (T_{J2} = 1800 s, Δ_{A2} = 2 and Δ_{J2} = 4).

From these numerical results, we can see that different time evolution of pulsating regimes of CI can be seen depending on the parameters of the injected energetic electron cloud. In particular, pulsating precipitation can cease when the energetic electron density is at maximum (see Fig. 2). Increase in the 2nd harmonic in the energetic electron pitch-angle distribution can result in net increase in the pulsation period even if the simultaneous growth of the energetic electron density leads to decrease in the period (Fig. 1).

We note that the parameters of energetic electrons for numerical calculations were chosen close to the values derived from case studies of two substorm injections accompanied by pulsating electron precipitation [3, 4]. Fig. 1 represents a localized injection which occurred at some distance from the observation point; substantial variation of the pitch-angle distribution due to pitch-angle dependence of the drift velocity is expected in this case, and such a variation has been indeed recorded [3]. During that injection, one could observe pulsations during the whole event, and the period increased near the maximum density of the energetic electron cloud. Fig. 2 corresponds to the case of the shorter and stronger injection which occurred in a large area; thus one expects only small variation of the anisotropy due to the drift velocity dispersion. For that case, pulsations were seen only in the beginning of the event. We see from the above discussion that both these facts can be in principle explained using our simple model.

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