

ON THE FREQUENCY MODULATION OF VLF EMISSIONS

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Abstract. Efficiency of the VLF wave frequency modulation by a magnetosonic low frequency wave as a function of magnetosphere plasma parameters is studied. Weak pitch angle diffusion of electrons into the loss cone is assumed. It is shown that in the quiet magnetosphere the disturbance of the maximum intensity frequency would be of order of the relative magnetic field disturbance in the equatorial plane. The upper cutoff frequency modulation is also considered. The frequency modulation seen in the calculated theoretical sonagrams may be related to the "non-dispersive" and to the "inverted V" structures of QP hiss. The relationship between the frequency modulation in the equatorial plane and on the ground is calculated.

1. Introduction

In quasi-periodic (hereafter QP) VLF emissions the variations of both maximum intensity frequency and upper/lower emission bound frequency are clearly seen (Sazhin and Hayakawa [1994], and references therein). It is natural to suggest these variations to be due to the modulation of the VLF growth rate by magnetic pulsations. The idea the magnetosonic wave can lead to the amplitude and frequency modulation of VLF waves had been claimed by Sato and Fukunishi [1981], but quantitative aspects of the problem have not been clarified. The investigation on the emission upper cutoff response at a given point of the field line to the magnetic field change under the weak and strong diffusion regime has been carried out by Haugstad [1976]. The dependence of the modulation efficiency on the magnetosphere plasma parameters have not been earlier examined, to our knowledge. The goal of the actual study is just to clarify what frequency features are appropriate to the ground-observed QP emission modulated by the magnetosonic wave and how they are controlled by the magnetosphere plasma parameters.

2. Model

A plane magnetosonic ULF wave propagating radially to the Earth in the magnetosphere equatorial plane modulates the VLF growth rate in the thin flux tube. (Fig.1). The main distinction of our model from that of Sato and Fukunishi [1981] is that no wide range of L-shell sequentially excited by the ULF wave is considered. The expression for the local VLF growth rate used in the paper is given by Cuperman and Landau [1974]:

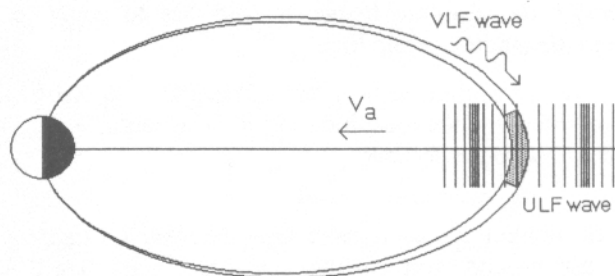


Fig.1. Geometry of the problem.

$$\gamma = \frac{1}{\alpha} \sqrt{\frac{\pi(1-x)}{2\alpha\beta_{\parallel}x}} \left(D - \frac{1}{1-x}\right) (1-x)^3 \exp\left(-\frac{(1-x)^3}{2\alpha\beta_{\parallel}x}\right), \quad (1)$$

where $\gamma = Im\omega/\Omega_e$, $x = Re\omega/\Omega_e$, ω is the VLF wave frequency, Ω_e is the electron gyrofrequency, $D = T_{\perp}/T_{\parallel} \equiv A+1$, the electron anisotropy factor, α is the total-to-warm electron content ratio and $\beta_{\parallel}$ is the electron parallel beta. Integrating (1) along the magnetic field line yields the expression for a path integrated growth rate Γ :

$$\Gamma = \int_{-s}^{+s} \frac{\gamma(s)}{V_{VLF}} ds \quad (2)$$

where V_{VLF} is the VLF wave group velocity, s is the equatorial distance along the magnetic field line. The dipole geomagnetic field model is used. The VLF wave is supposed to propagate strictly along the field line crossing the equator and to be amplified in a small region near the equatorial plane. The wavelength of the ULF perturbation is chosen to be large enough to be sure that both in the middle and edges of the amplification region the perturbation phase is almost the same. Then (1) and (2) were used to get numerically the frequency of maximum intensity and the upper cutoff frequency in the equatorial plane and on the ground.

Numerical simulation has been made under the conditions typical for quiet magnetosphere at $L=5-7$: electron number density is 1 cm^{-3} , warm electron content $\alpha^{-1} = 0.1$, electron parallel beta $\beta_{\parallel} = 0.1$, electron anisotropy $A=0.5$, field-aligned Alfvén velocity is 5400-2100 km/s (assumed to be constant along the field line). The

perturbation amplitude is suggested to be 15 nT (what corresponds to the relative amplitude, ε , of the magnetic field perturbation in the equatorial plane, equal to 0.06-0.17) and the ULF wave period is 30s.

3. Results

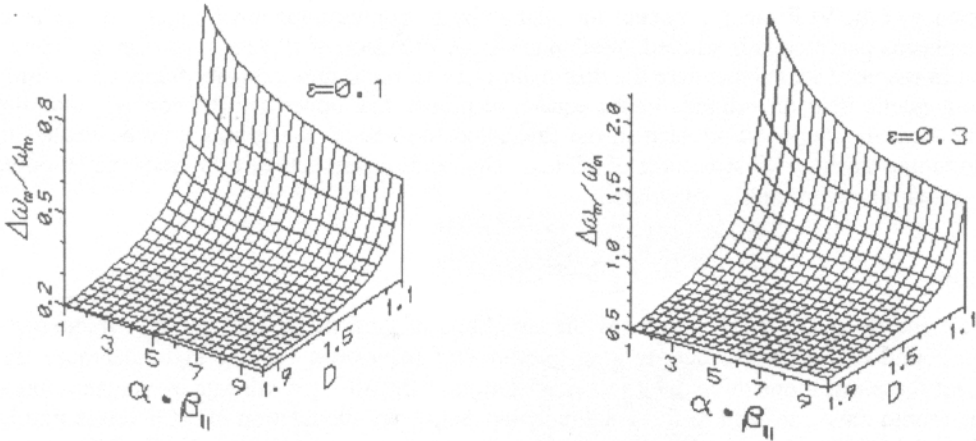


Fig.2 (a,b). The relative shift of the maximum growth rate frequency ω_m in the equatorial plane as a function of $\alpha\beta_{||}$ product and electron anisotropy factor $D=A+1$ for moderate ($\varepsilon=0.1$) and strong ($\varepsilon=0.3$) magnetic field perturbation in the equatorial plane (see text for references).

1. It was found since the upper cutoff frequency being defined via zero VLF growth rate changes by simple low $\omega_u/\Omega_e = 1 - (A+1)^{-1}$ the absolute frequency shift has to conserve itself along the magnetic field line.

2. The maximum growth rate frequency x_m (normalised) determined by the condition $dy/dx=0$ is obtained in linear approximation as follows:

$$x_m = (\alpha\beta_{||} + 1 + (A+1)/A)^{-1}$$

After calculating the curves relating the relative frequency shift on the ground and in the equatorial plane $(\Delta\omega_m/\omega_m)_{\Gamma}/(\Delta\omega_m/\omega_m)_{eq}$ as a function of electron plasma parameters $\alpha, \beta_{||}$ and A in the equatorial plane (Fig.2) the expected ground-observed shift of the emission maximum frequency can be estimated (Fig.3).

3. The study of the emission upper/lower boundary modulation is held in the frame of the VLF generated frequency band (Fig.4) cut at the level of 0.5 emission intensity maximum. However, the upper emission edge observed in ground sonagrams may also correspond to the so-called duct cutoff $\omega_{duct} \approx 0.5\Omega_{e(eq)}$, and in this case it is determined by the equatorial $\varepsilon\Omega_e$ product value only.

4. It is shown (Fig.4) that the VLF frequency bandwidth grows as the warm electron anisotropy A increases, but the frequency modulation is the deeper the lower the A value is. The generated frequency band is shifted as a whole by the magnetic field variation when the warm electron content is high (of order of 10% or more) and thence the modulation type looks like the "inverted-V" structure; otherwise, for the low warm plasma abundance, the emission maximum frequency shows no reaction to the perturbation, in contrast with the emission upper edge behavior, and then the "undispersed" QP-hiss formation is expected.

5. Sample theoretical sonagrams of the ground-detected VLF signal under the given magnetosphere conditions are created (Fig.5). The frequency variations were found to oscillate in phase with the magnetic field perturbation in the equatorial plane (not shown): from 0 till 10 s no perturbation occurs, then the perturbation (sinusoidal by assumption) reaches its phase of $\varphi=\pi/2$ at 17.5s, $\varphi=\pi$ at 25 s, $\varphi=-\pi/2$ at 32.5s and $\varphi=2\pi$ at 40s.

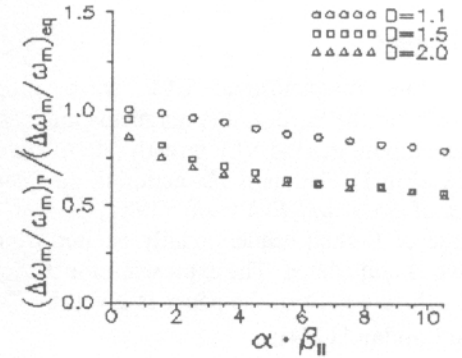


Fig.3. The relationship between relative shift of the VLF emission maximum on the ground and in the magnetosphere equatorial plane as a function of $\alpha\beta_{||}$ product ($\beta_{||}=0.1$ is fixed) for $D \approx 1$ and for $1 < D < 2$, where $D=A+1$ (see text for references).

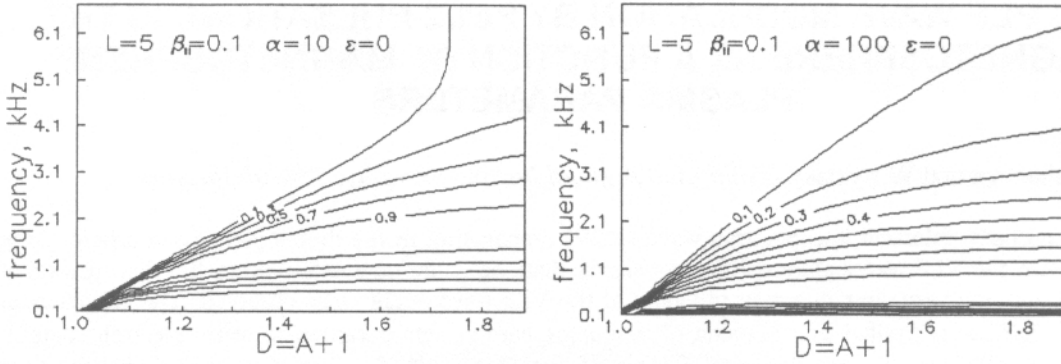


Fig.4 (a,b). The undisturbed generated frequency band as a function of electron anisotropy factor D for two values of warm electron content: 10% (left panel) and 1% (right panel). $\beta_{\parallel}=0.1$. Curve labels denote the logarithm of the VLF emission intensity normalized to the maximum intensity logarithm.

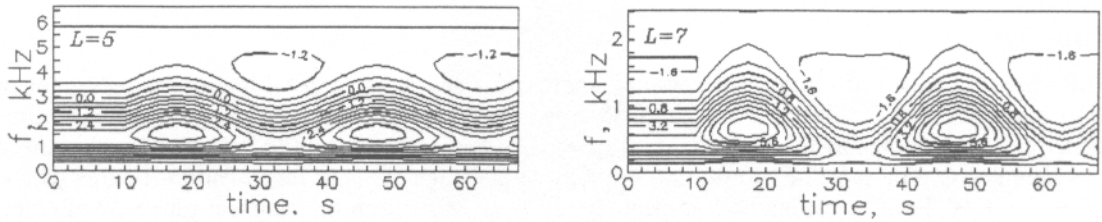


Fig.5 (a,b). The theoretical sonagrams simulated for typical quiet magnetosphere conditions at $L=5$ (left) and $L=7$ (right). Total electron number density $n=1\text{cm}^{-3}$, electron anisotropy $A=0.5$, warm electron content $\alpha^{-1}=n_w/n=0.1$, $\beta_{\parallel}=0.1$. The perturbation equatorial value $\epsilon=\Delta B_{eq}/B_{eq}=0.06$ (left) and 0.17 (right).

4. Conclusions

1. It is shown that the ground-observed modulation of the VLF maximum intensity frequency is of order of or more than the relative magnetic field perturbation in the equatorial plane and slightly weaker, but not twice as less as the modulation of the maximum growth rate frequency in the equatorial plane.
2. It is shown that i) the VLF generated frequency band expands as the electron anisotropy grows and ii) the modulation type is controlled by the warm electron content: when the warm electron abundance is high, the generated frequency band shifts itself as a whole, and the QP inverted-V structures formation is expected; when otherwise, the frequency shift occurs in the upper emission boundary only, in contrast to the emission maximum frequency stability, that corresponds to the undispersed QP hiss.
3. Under the typical quiet magnetosphere conditions at $L=5$ and by the perturbation equatorial amplitude of order of 15 nT variation of the upper emission boundary at level 0.5 would reach 10-25% for anisotropy $A=0.5$.
4. The simulated theoretical sonagrams show the modulation type intermediate between the inverted V-structures and undispersed QP-hiss.

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