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# ALFVEN WAVE GENERATION BY IONOSPHERIC INHOMOGENEITY IN THE FIELD-ALIGNED CURRENT REGION

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## 1. Introduction

The problem of Alfven wave generation by an ionospheric inhomogeneity located in a region of ambient ionospheric electric field was solved by *Maltsev et al.*[1974, 1977], *Mallinckrodt and Carlson* [1978], *Lyatsky and Maltsev* [1983], *Kan and Sun* [1985], *Lysak* [1990] and many other authors. It was shown that the inhomogeneity is polarized in the background electric field and the polarization electric field propagates into the magnetosphere along the magnetic field lines as the field of the outgoing Alfven wave. At the boundary of the inhomogeneity the field-aligned currents appear; those currents are closed by the inertia transverse currents at the front of the propagating Alfven wave. In the papers mentioned above no background field-aligned current is considered, and the background electric field is assumed to be homogeneous.

Our aim is to study the Alfven wave generation by ionospheric inhomogeneity in the field-aligned current region.

## 2. Formulation of the problem and basic equations

Let us consider a field-aligned current, $j_{z0}$, flowing into the ionosphere and being closed there by ionospheric current. The ionosphere is assumed to be homogeneously conducting. The height-integrated ionospheric current, $\mathbf{J}_{i0}$, is assumed to increase linearly with $x$. Value of the ionospheric electric field, $\mathbf{E}_0$, increases linearly with $x$ too. Let us now assume that in some area of the ionosphere the conductivity changes. The conductivity disturbance leads to disturbances in the electric field, what leads to the Alfven wave generation and appearance of the field-aligned currents associated with the wave. As shown in *Maltsev et al.* [1977] and *Lyatsky and Maltsev* [1983], the problem of the Alfven wave generation by the ionospheric inhomogeneity turns into solution of the following equation system:

$$\nabla_\perp \cdot \mathbf{J}_i = \nabla_\perp \cdot \hat{\Sigma}\mathbf{E} = j_z \qquad (1)$$

$$j_z' = -\nabla_\perp \cdot \mathbf{J}_A = -\Sigma_A \nabla_\perp \cdot \mathbf{E}' \qquad (2)$$

where $\hat{\Sigma}\mathbf{E} = \Sigma_P\mathbf{E} + \Sigma_H \mathbf{e}_z \times \mathbf{E}$, $\mathbf{J}_i$ is the height-integrated ionospheric current, $j_z$ is the field-aligned current, $\mathbf{E}$ is the ionospheric electric field, $\Sigma_P$ and $\Sigma_H$ are the Pedersen and Hall height-integrated ionospheric conductivities, $\mathbf{J}_A$ is the transversal inertia current of the propagating Alfven wave height-integrated across the wave front, $\Sigma_A$ is an effective conductivity of the magnetospheric plasma for the propagating Alfven wave, $\Sigma_A=(\mu_0 V_A)^{-1}$, where $\mu_0$ is the vacuum magnetic permeability, and $V_A$ is the Alfven velocity, the unit $\mathbf{e}_z$-vector is directed along the magnetic field, $\mathbf{E}'$ is the transverse electric field of the Alfven wave and $j_z'$ is the field-aligned current of the wave. The values in (1-2) are the sums of the undisturbed ones and disturbances:

$$\mathbf{J} = \mathbf{J}_0 + \mathbf{J}', \mathbf{E} = \mathbf{E}_0 + \mathbf{E}, \hat{\Sigma} = \hat{\Sigma}_0 + \hat{\Sigma}', j_z = j_{z0} + j_z'.$$

The equation (1) is the continuity equation for electric current in the ionosphere, the equation (2) is the continuity equation for electric current at the Alfven wave front: it shows that the field-aligned current of the Alfven wave is closed by the transverse inertia currents, $\mathbf{J}_A$, at the front of the wave, where

$$\mathbf{J}_A = \Sigma_A \mathbf{E}' \qquad (3)$$

The equation set (1-2) is the same as that used by *Maltsev et.al.* [1974] and *Lyatsky and Maltsev* [1983]; however we assume that $j_{z0} \neq 0$ and $\mathbf{E}_0$ to be not homogeneous, but increasing along the $x$-axis.

## 3. Solution of the problem

Following *Maltsev et al.* [1974], we assume transverse electric field of the Alfven wave to be potential in the plane perpendicular to the magnetic field, i.e.

$$\mathbf{E}' = -\nabla_\perp \varphi' \qquad (4)$$

where $\varphi'$ is an electric potential of the generated Alfven wave. From the (1-4) set we obtain the following equation for $\varphi'$ :

$$(\Sigma'_P + \Sigma_{P0} + \Sigma_A)\nabla_\perp^2\varphi' = \frac{\Sigma'_P}{\Sigma_{P0}} j_{z0} + (\mathbf{E}_0 - \nabla_\perp\varphi')\cdot(\nabla_\perp\Sigma'_P + \nabla_\perp\Sigma'_H \times \mathbf{e}_z) \qquad (5)$$

From equations (2) and (5) we obtain the formula for calculation of the field-aligned current of the Alfven wave:

$$j'_z = \frac{\Sigma_A}{\Sigma'_P + \Sigma_{P0} + \Sigma_A}\left[\frac{\Sigma'_P}{\Sigma_{P0}} j_{z0} + (\mathbf{E}_0 - \nabla_\perp\varphi')\cdot(\nabla_\perp\Sigma'_P + \nabla_\perp\Sigma'_H \times \mathbf{e}_z)\right] \qquad (6)$$

From (6) we see that the field-aligned currents of the outgoing Alfven wave appear not only at the boundary of the inhomogeneity, where $\nabla\Sigma'\neq 0$, but inside the inhomogeneity too. If $j_{z0}= 0$, the disturbances of the field-aligned currents take place only at the boundary of the inhomogeneity.

## 4. Alfven wave over circle-like inhomogeneity

We introduce the complex framework at the $(x,y)$ plane $w=x+iy=|w|e^{i\lambda}$ and the complex conductivity $\Sigma=\Sigma_P - i\Sigma_H$. In this case the vectors in horizontal plane may be written as $E=E_x+iE_y$. The radius of the circle is $a$, the centre of the circle is in the point $w=0$. Inside the circle the conductivity is homogeneous and equals to $\Sigma$. Outside the circle the conductivity is undisturbed and equals to $\Sigma_0$. The background electric field, $E_0$, is assumed to change along $x$-axis only, $E_0(0)$ is the background electric field in the centre of the circle.

The electric field of the outgoing Alfven wave over the circle-like inhomogeneity may be presented in the form:

$$E' = E_d + E_q + E_m \qquad (7)$$

where $E_d$, $E_q$ and $E_m$ are the terms describing the electric fields for two-dimensional dipole, quadrupole and screened monopole sources, respectively. The values inside the circle (when $|w|<a$) are determined by the following expressions:

$$E_d^{in} = -\frac{(\Sigma-\Sigma_0)E_0(0)}{\Sigma+\Sigma_0^*+2\Sigma_A}, \quad E_q^{in} = -\frac{j_{z0}}{2\Sigma_{P0}}\frac{(\Sigma-\Sigma_0)w^*}{\Sigma+\Sigma_0^*+2\Sigma_A}, \quad E_m^{in} = -\frac{j_{z0}}{2\Sigma_{P0}}\frac{(\Sigma_P-\Sigma_{P0})w^*}{\Sigma_P+\Sigma_A} \qquad (8)$$

The values outside the circle (when $|w|>a$) are determined by the following expressions:

$$E_d^{out} = \left[-E_d^{in}\frac{a^2}{w^2}\right]^*, \quad E_q^{out} = \left[-E_q^{in}\frac{a^4}{w^2|w|^2}\right]^*, \quad E_m^{out} = 0 \qquad (9)$$

Symbol * means the complex conjugation. $E_d$ is homogeneous inside the inhomogeneity, and it is a two-dimensional dipole-like field outside the inhomogeneity. $E_q$ outside the inhomogeneity have the form of the field of the two-dimensional quadrupole. $E_m$ is the electric field of the homogeneously charged screened cylinder. For the maximum values of the dipole and quadrupole terms the following relationship takes place:

$$\frac{|E_d^{\max}|}{|E_q^{\max}|} = \frac{2\Sigma_{P0}|E_0(0)|}{j_{z0}a} \approx 2\frac{d}{a} \qquad (10)$$

where $d$ is of order of the width of the background field-aligned current region (from its northern or southern edge up to inhomogeneity location). The ratio of $E_d$ to $E_m$ is approximately of the same order of magnitude.

Field-aligned currents of the Alfven wave consist of a homogeneous current inside the circle, $j_z^{in}$, and a sheet current, $J_z^{ed}$ at the boundary of the circle. Inside the circle we obtain

$$j_z^{in} = \frac{\Sigma_A}{\Sigma_{P0}}\frac{\Sigma_P-\Sigma_{P0}}{\Sigma_P+\Sigma_A} j_{z0} \qquad (11)$$

The sheet current at the circle boundary is:

$$J_z^{ed} = -\Sigma_A \operatorname{Re}\left\{\frac{2(\Sigma-\Sigma_0)E_0(0)e^{-i\lambda}}{\Sigma+\Sigma_0^*+2\Sigma_A} + \frac{aj_{z0}}{\Sigma_{P0}}\frac{(\Sigma-\Sigma_0)e^{-2i\lambda}}{\Sigma+\Sigma_0^*+2\Sigma_A} + \frac{aj_{z0}}{2\Sigma_{P0}}\frac{\Sigma_P-\Sigma_{P0}}{\Sigma_P+\Sigma_A}\right\} \qquad (12)$$

Three terms in the brackets correspond to the two-dimensional dipole, quadrupole and screened monopole sources, respectively.

## 5. Strip-like inhomogeneity

Now we consider an Alfven wave over a strip-like inhomogeneity. The strip is stretched along the y-axis, its centre is in $x=x_0$, the width of the strip is $2a$. The conductivity inside the stripe is $\Sigma$. Value of the x-component of the ionospheric electric field, $E_{x0}$, increases linearly with x whereas $E_{y0}$ is constant. As it follows from the formulation of the problem, all derivatives $\partial/\partial y$ are equal to zero and $E_y'=0$.

The electric field of the outgoing Alfven wave inside the strip inhomogeneity is the sum of the homogeneous and linearly-changing electric fields:

$$E_x' = -\frac{\left[(\hat{\Sigma} - \hat{\Sigma}_0)\mathbf{E}_0(0)\right]_x}{\Sigma_P + \Sigma_A} - \frac{j_{z0}}{\Sigma_{P0}}\frac{\Sigma_P - \Sigma_{P0}}{\Sigma_P + \Sigma_A}\left(x - x_0\right) \qquad (13)$$

With no background field-aligned current only the homogeneous field takes place. The electric field of the wave outside the strip equals to zero.

Field-aligned currents of the outgoing Alfven wave over the strip-like inhomogeneity consist of a homogeneous current inside the strip which is determined by the same equation as for a circle-like inhomogeneity (11), and the sheet currents at the edges of the strip:

$$J_z^{strip} = \pm\Sigma_A \frac{\left[(\hat{\Sigma} - \hat{\Sigma}_0)\mathbf{E}_0(0)\right]_x}{\Sigma_P + \Sigma_A} - a\frac{\Sigma_A}{\Sigma_{P0}}\frac{\Sigma_P - \Sigma_{P0}}{\Sigma_P + \Sigma_A} j_{z0} \qquad (14)$$

when $x = x_0 \mp a$.

## 6. Discussion and application to auroral arc

As noted above, the field-aligned current generated directly over a strip of increasing conductivity is of the same direction as the background one. As a result, the total field-aligned current over the strip increases what yields positive feedback which may lead to instability development in the magnetosphere-ionosphere system. This positive feedback may play some role in the auroral arc generation. In the papers by *Atkinson* [1970], *Holzer and Sato* [1973], *Sato and Holzer* [1973], *Sato* [1978], *Leontyev and Lyatsky* [1982], *Watanabe and Sato* [1988] mechanisms of the auroral arc generation due to the outgoing Alfven wave were suggested. However authors of those papers didn't take into consideration the Alfven wave produced by the background field-aligned current existence. With no background field-aligned current the field-aligned currents of outgoing Alfven wave are generated at the strip edges only, and for explanation of the ionization increase inside the strip it is necessary to have some additional phase shift between the conductivity disturbance and the field-aligned current. This phase shift is explained by reflection of the Alfven wave from the conjugated ionosphere [*Sato and Holzer*, 1973; *Sato*, 1978; *Watanabe and Sato*, 1988] or by introduction of an accelerating double electric layer at some distance from the ionosphere [*Leontyev and Lyatsky*, 1982]. For appearance of the necessary phase shift, a relative movement of the arc with respect to the magnetospheric convection is required. In fact, the relative movement appears to be very small [*Haerendel et al.*, 1993] what yields a serious problem in realization of the mechanism. In the case under consideration, the increase in the field-aligned current occurs not only at the edges, but also inside the arc independently of any relative movement of the arc with respect to magnetosphere convection that may be an essential factor for the auroral arc generation theory.

## 7. Summary

The problem of the Alfven wave generation over an ionospheric inhomogeneity embedded to the homogeneously conducting ionosphere in the region of ambient homogeneous field-aligned current has been solved. It has been shown that:

1) In the region of background field-aligned current the field-aligned current of the outgoing Alfven wave appears not only at the boundary, but inside the inhomogeneity too.

2) When the conductivity in the inhomogeneity increases the current over the inhomogeneity is directed as the background one, and the value of the total field-aligned current increases.

3) In the region of electron precipitation (upward field-aligned current) increase in the field-aligned current over the strip leads to positive feedback occurrance that might be applied for explanation of the auroral arcs generation. This mechanism allows the arc to move with convection.

4) Electric field and the field-aligned currents of the outgoing Alfven wave over the circle inhomogeneity may be presented as the sum of the two-dimensional dipole, quadrupole and screened monopole sources. With no background field-aligned current only the dipole source takes place.

5) Electric field of the outgoing Alfven wave inside the strip inhomogeneity is the sum of the homogeneous and linearly-changing electric fields. With no background field-aligned current only the homogeneous field takes place.

6) The ratio of maximum values of the monopole and dipole electric fields (or, in the case of the strip, of the homogeneous and linearly-changing electric fields) is of order of ratio of the spatial scale of the inhomogeneity to the width of the background field-aligned current region.

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